Chapter 10: Hedonics: The Demand and Supply of Goods with Multiple Attributes

I. More Realistic Models of Consumer Choice and Production

Most of the models of consumer choice that we've used so far have assumed that the nature of the goods on offer where fixed. Not only was it known how a particular good would affect one's utility (e.g. advance one's purposes), but each good was unique onto itself. An apple was an apple. But there are many varieties of apples, some sweeter than others, some juicier than others, some are read, others green or yellow. It turns out that "apples" have multiple attributes that individuals may take account of when the purchase a single apple or a bag of apples to make an apple pie. The same is true of nearly all consumer products. For example, a cell phone is not a cell phone, but an electronic device with many attributes that are valued by consumers—e.g. that consumers find useful or satisfying to have. However, every cell phone "model" is a bit different from others produced by the same company or others produced by other companies. Thus, most words that we assign to goods are actually words that describe a collection of goods with more or less similar collections of attributes. A cell phone may be larger or small, may be of different colors, have a faster or slower processor, more or less memory, come with different operating systems and apps, or be foldable or not. The same sort of variation in attributes is associated with most of the goods that consumers purchase, and firms produce—and importantly, not all attributes are assessed in the same way by all individuals. Part of the variation in demand for goods and services is due variation in the demand for various attributes of the goods on offer.

Goods within such collections are good substitutes for one another, but they are not perfect substitutes. Since every product is a bit different, the firms selling specific models of cell phones face downward sloping demand curves for their products with slopes dependent on the extent to which their overall utility for consumers differs significantly or not from other similar products. We have already, implicitly, began to analyze how different attributes affect demand in the section of chapter 7 on the demand for goods with uncertain quality and the r and d section of chapter 8. This chapter provides a series of models that focuses more narrowly on the effects of attributes themselves—without the stochastic aspects of the choice settings modeled in those sections.

In this chapter, we'll extend the basic neoclassical models developed in part one to take account of both how the attributes of goods affect demand for particular products and also how differences in the demand for attributes among consumers affect product designs and firm profits.

II. Simple Models of the Demand for Goods with Multiple Attributes

Attributes and the Consumer Choice: On the Net Benefits of Alternatives

Let us return to the first model we developed, the net benefit maximizing model of consumer choice and now imagine that an individual regards a product to be a combination of specific attributes. Suppose that the product of interest is an apartment to live in while a university student. Several attributes of apartments may be relevant for Al's decision. For example, Al may care about its distance from the university, the size of the apartment, and its newness (its condition). Thus, Al is willing to pay a higher rent for a new large apartment that is close to campus than for a run-down small apartment that is far away from campus—perhaps even in another town. Differences in these attributes affect Al's reservation price (total benefit) for the two apartments. His or her subjective net benefits for the two apartments varies with his or her reservation prices for the two apartments and with the rental cost of the apartments. If the rents are the same, Al would clearly realize greater net benefits with the larger, newer, apartment closer to campus. However, if the prices were significantly different, Al might rent the smaller, run-down, apartment that is farther from campus even though he or she in a sense prefers the newer larger apartment close to campus. The smaller run-down apartment "saves money," and those savings can be put to other uses that are more valuable to Al than the advantages associated the newer, larger and more convenient apartment.

This type of choice setting can be modeled by placing explicit values on the three attributes, distance from campus, size, and age of the apartments. Let apartment 1 be the newer apartment and apartment 2 be the older one with $B_1 > B_2$, where $B_1 = b_1S_1 - b_2D_1 – b_3A_1$ and $B_2 = b_1S_2 - b_2D_2 – b_3A_2$, where S_i, D_i, and A_i are the attributes of apartment "i" and b_1, b_2, and b_3 are Al's assessment of the "worth" or "value added" by the three attributes focused on. Note that each apartment comes with all three attributes and that the "b" valuations may differ quite a bit for Al (and also vary among other prospective renters). Distance might be much more important than size, for example.

As a net benefit maximizer, Al chooses the apartment with the higher net benefits $N_1 = B_1 – C_1$ and $N_2 = B_2\text{-}C_2$. If Al rents apartment 2 (the older one), it is not because he or she "likes" apartment 2 better than apartment 1 (B_1 is likely to be larger than B_2), but because the older apartment produces greater net benefits—it frees money that can be used for better meals, more textbooks, a better computer, or holiday travel, etcetera.

In Statistical estimates of the way in which characteristics of apartments affect their market rental rates, the "b" valuation terms are estimated using data on apartment rents and their characteristics. The b coefficients estimated are those of the "average consumer" in the apartment market of interest. These coefficients of valuation, of course, vary among persons in that market—indeed, there may be no single consumer with the average assessments of the value of the individual apartment attributes. Nonetheless, average values provide landlords and renters with ideas about what a "reasonable" rent would be—based on renter preferences and past market clearing prices.

Attributes and the Demand for Multi-Attribute Goods

Many economic choices are of the one or nothing variety, in which case all one can do to model such choices is to think carefully about the sources of the net benefits or utility associated with the alternatives available in the markets of interest as done above (and could be done in more detail). Nonetheless, there are also many choices in which more or less of a good is to be purchased, produced, or rented—and these choices can be modelled as well by extending the above model a bit.

Consider Al's decision to purchase apples of a given type. The apples have size, S, tartness, T, and juiciness J. and the value of a single apple to Al can be represented in a manner similar to the above, as, for example: $B = b_1S - b_2T – b_3J$. Assume that there is just one kind of apple, or that Al has already done a comparison among the available apple types and chosen a particular variety to purchase. How many will she purchase?

As characterized, the benefit function does not include the effect of diminishing marginal returns. This effect does not necessarily have to be taken into account for "one or nothing" types of choices, but for "how many" types of decisions they tend to be important—at least according to economic theory after the marginal revolution took place in the late nineteenth century.

One way to incorporate diminishing marginal returns and the quantities purchased into the model is the following. Let $b(Q) = b_1(QS)^e - b_2(QD)^f – b_3(QJ)^g$ with exponents e, f, g < 1. The latter assures diminishing marginal returns to each attribute. A consumer still gets the (S, D, J) attributes with every apple purchased, so if one buys 2 apples one gets twice as much of each of the three attributes (2S, 2D, 2J), and if one purchases Q apples, one gets (QS, QD, QJ). For multi-attribute goods, it is the attributes that produce the benefits or utility, not the quantities of apples by themselves.

We can now model Al's decision about how many apples to purchase. Suppose that apples can be purchased at price P. The total cost of Q apples, C=c(Q) is PQ. Given the above, the net benefits associated with various quantities of apples are:

$$n(Q) = b(Q) - c(Q) = [b_1(QS)^e + b_2(QD)^f + b_3(QJ)^g] - PQ \quad (10.1)$$

Differentiating the net benefit equation with respect to Q and setting the result equal to zero characterizes the ideal (net- benefit maximizing) quantity of apples to purchase:

$$n_Q = b_Q - c_Q = [eb_1(QS)^{e-1} + fb_2(QD)^{f-1} + gb_3(QJ)^{g-1}] - P = 0 \; at \; Q^* \quad (10.2)$$

As usual, Al will purchase apples up to the point where his or her marginal benefits from them (the terms inside the brackets) equals the marginal cost of the apples (here P). Note that the marginal benefit of the last apple is the sum of the marginal benefits associated with each of the attributes of the apple. Unfortunately, because the exponents all differ, there is no simple solution for Q as a function of P that can be worked out from equation 10.2.[1]

However, we can use the implicit function theorem to characterize Al's demand function for apples as:

$$Q^* = q(P, b_1, e, b_2, f, b_3. g, S, D, J) \quad (10.3)$$

Al's demand for apples varies with the price of the apples and the parameters of the benefit function that determine how the attributes of the apple (S, D, and J) generate benefits for Al.

Changes in price, the valuation factors, or in the attributes of the apples being purchased will alter the quantity of apples that Al's demands. Note that all the variables in lower case can be regarded as "taste variables" and the attributes (S, D, and J) can be regarded as factors that

[1] Had all the exponents been the same, as with e=f=g, a solution for Al's ideal purchase of apples could have been worked out. $[gb_1(QS)^{g-1} + gb_2(QD)^{g-1} + gb_3(QJ)^{g-1}] - P = 0$ can be written as $[gQ^{g-1}][b_1(S)^{g-1} + b_2(D)^{g-1} + b_3(J)^{g-1}] = P$ which can be solved for Q* as $Q^* = \{\frac{P}{[g][b_1(S)^{g-1}+b_2(D)^{g-1}+b_3(J)^{g-1}]}\}^{1/(g-1)}$. Recall that g<1 which implies that $Q_P^* < 0$.

This is easier to see if you take account of one of the properties of negative exponents to rewrite the above as $Q^* = \{\frac{g][b_1(S)^{g-1}+b_2(D)^{g-1}+b_3(J)^{g-1}}{[P]}\}^{1-g}$. Note that in this case an increase in any of the attributes increases demand (they all positive terms in the numerator.

determine the quality of the apple for Al (given those taste factors: $b_1, e, b_2, f, b_3. and\ g$). Tastes matter as in all the previous models of consumer choice, but so do the attributes of the goods purchased. In this model, if the desirable attributes of a good increases, so will Al's demand for that good, which implies that the quantity purchased at a given price, P, increases.

Utility Maximizing Choices of Goods with More than One Attribute

The net-benefit maximizing model of rational consumers has several advantages. It normally involves just one choice dimension, and so the simplest optimization methods can be used to characterize net-benefit maximizing choices. The results often are intuitive and clear. This is partly because many of our own choices are made one at a time, and thus that model is similar to the thought process we have used in the past. (This is especially true of economics majors.) On the other hand, whenever constraints are important (as with budget constraints) and various tradeoffs associated with those constraints affect choices, the utility maximizing model provides additional useful insights into the factors that affect the choices of purposeful consumers.

To see how a utility maximizing model of choices with respect to multi-attribute goods, consider a minor extension of the above model. Assume that Al has a budget to allocate between two goods: apples with multiple attributes and some other good with only a single attribute, as might be claimed of peanuts. Assume that Al has a utility function defined over apples of a given time (e.g. with particular attributes) and peanuts. The modelling methods developed above for the net benefit maximizing model of goods with multiple attributes imply that Al's utility function with respect to apples and nuts can be written as U = u (QS, QD, QJ, N). Q is the quantity of apples, and S, D, and J are the attributes of a typical apple as before. N is the quantity of nuts. Al's budget constraint is the usual one: $W = P^AQ + P^NN$, where W is Al's budget and P^A is the price of apples and P^N is the price of peanuts.

We can use the budget constraint to characterize the N as $N=(W-P^AQ)/P^N$. Substituting that relationship into the utility function yields:

$$U = u(QS, QD, QJ, \left(\frac{W-P^AQ}{P^N}\right)) \qquad (10.4)$$

This function, as in most of the other cases in which we've used the substitution method has only a single choice variable (the quantity of apples), and evaluates the utility function along the budget constraint. If U is strictly concave and has positive first derivatives for all the goods in that function, then the quantity of apples, Q*, that satisfies the first order conditions of equation 10.4 with respect to Q is the quantity that maximizes utility, given the budget constraint.

Differentiating equation 10.4 with respect to Q and setting the result equal to zero yields:

$$U_Q = u_S S + u_D D + u_J J - u_N \left(\frac{P^A}{P^N}\right) = 0 \ \equiv H \ at \ Q^* \qquad (10.5)$$

The first three terms are the marginal benefits from apples (now in utility terms). Again the marginal benefits are the sum of the marginal benefits from each of the three attributes of the typical apple. The implicit function theorem, in turn, implies that Al's demand for Apples of this type of apple can be written as:

$$Q^* = q(P^A, S, D, J, W, P^N) \qquad (10.6)$$

The result implies that the demand for this type of apple varies with its price, its specific attributes (S, D, J), with the price of the other good of interest, and Al's budget constraint (W).

Comparative statics of Al's demand function can be characterized in the usual way, using the implicit function differentiation rule. We'll focus on the price of apples and one of the desirable attributes. (Keep in mind that each of the partial derivatives includes all of the arguments of the original utility function, so there are numerous cross partials that affect these two derivatives).

$$Q^*_{P^A} = \frac{H_{P^A}}{-H_Q} = \frac{[Su_{SN} + Du_{DN} + Ju_{JN}](\frac{-Q}{P^N}) - u_N\left(\frac{1}{P^N}\right) + u_{NN}\left(\frac{QP^A}{(P^N)^2}\right)]}{-U_{QQ}} > 0 \qquad (10.7)$$

$$Q^*_S = \frac{H_S}{-H_Q} = \frac{u_S + SQu_{SS} + QDu_{DS} + QJu_{JS} - u_{NS}\left(\frac{QP^A}{P^N}\right)}{-U_{QQ}} > 0? \qquad (10.8)$$

Equation 10.7 implies that the slope of the demand function is negative, as usual for a demand function. Strict concavity implies that the denominator is positive. The usual rule of thumb for the cross partials of strictly concave functions (e.g. all are greater than zero) and second derivatives (e.g. all are less than zero) imply that all of the terms in the numerator are negative. Thus, equation 10.7 is less than zero over the entire range of apple prices—other things being equal. Al's demand curve for apples of this type is downward sloping.

Equation 10.8 shows that the effect of the apple size attribute (or any other of the desirable attributes) is ambiguous, which is a different result than that associated with the net benefit maximizing model above. This ambiguity is due to effects of apple size on the second derivatives and cost functions. Diminishing marginal utility implies that the marginal utility from the last unit of S diminishes at Q*. One has more S and so its value at "the margin" (e.g. at Q*) is lower than it was before S increased. This together with an increase in the marginal cost of additional apples (in terms of lost utility from peanuts) produces two negative terms. The other three terms in the numerator are positive. So, the overall effect of an increase in apple size on the quantity of applies purchased is unclear.

Note that this ambiguity is also associated with separable versions of utility functions (e.g. utility function with zero cross partials). If the utility function is assumed to be separable, all of the terms in the numerator disappear except the first two. However, the first is positive and the second is negative. So, ambiguity remains unless there is very little in the way of diminishing returns, in which case the intuitive positive sign obtained in the net-benefit maximizing model is the result. Or, if it is known that diminishing returns are quite large (as when satiation set in or storage costs are high) then Al's purchase of apples decreases as their average size increases.

In the utility maximizing model, an increase in quality causes the last unit of the good to decline in marginal value. This implies that one can purchase fewer apples and get the same satisfaction as previously associated with Q* apples. Thus, it is quite possible that Al, would purchase fewer apples after this increase in quality. Whether this is the case or not is, of course, quite important to firms that create the product—here apples—being sold. Ultimately, this is likely to vary among attributes and consumers.

III. Designing Profitable Products with More than One Attribute

(to be written up in the future)