Chapter 10: On the Nature of Goods Sold in Markets: The Demand and Supply of Goods with Multiple Attributes—Hedonics

I. More Complete Models of Consumer Choice and Production

Most of the models of consumer choice that we've used so far have assumed that the nature of the goods on offer where fixed. Not only was it known how a particular good would affect one's utility (e.g. advance one's purposes), but each good was unique onto itself. An apple was an apple. However, there are many varieties of apples in today's grocery stores. Some apples are sweeter than others, some juicier than others, some are red, others green or yellow. It turns out that "apples" have multiple attributes that individuals take account of when they purchase a single apple to eat after lunch or a bag of apples to make an apple pie. The same is true of nearly all consumer products. For example, a cell phone is not a cell phone, but an electronic device with many attributes that are valued by consumers—e.g. that consumers find useful or satisfying to have. Every cell phone "model" is a bit different from others produced by the same company and by other companies. Thus, most words that we assign to goods are actually words that describe a collection of goods with more or less similar collections of attributes. A cell phone may be larger or small, may be of different colors, have a faster or slower processor, more or less memory, come with different operating systems and apps, or be foldable or not. The same sort of variation in attributes is associated with most of the goods that consumers purchase, and firms produce—and importantly, not all attributes are assessed in the same way by all individuals. Part of the variation in demand for the goods and services for sale in markets is due variation in individual demands for various attributes of the goods on offer.

Goods that fall into the same categories of products or services are substitutes for one another, but they are not perfect substitutes. Since every product is a bit different, the firms selling specific models of cell phones face downward sloping demand curves for their products with slopes dependent on the extent to which their overall utility for consumers differs significantly or not from other similar products selling for about the same price. We have already, implicitly, began to analyze how different attributes affect demand in the section of chapter 7 on the demand for goods with uncertain quality and the research and development section of chapter 8. This chapter provides a

series of models that focuses more narrowly on the effects of attributes themselves—without the stochastic aspects of the choice settings modeled in those chapters.

In this chapter, we'll extend the basic neoclassical models developed in part one to take account of how the attributes of goods affect demand for particular products and also how differences in the demand for attributes among consumers affect product designs and firm profits. We'll continue to assume that firms make only a single product in this chapter. Multiproduct firms are taken up in chapter 11.[1]

II. Lean Models of the Demand for Goods with Multiple Attributes

Attributes and Consumer Choice: On the Net Benefits of Alternatives

Let us return to the first model we developed, the net benefit maximizing model of consumer choice and now imagine that an individual regards a product to be a combination of specific attributes. Suppose that the product of interest is an apartment to live in while a university student. Several attributes of apartments may be relevant for Al's decision. For example, Al may care about its distance from the university, the size of the apartment, and its newness (its condition). Thus, Al is willing to pay a higher rent for a new large apartment that is close to campus than for a run-down small apartment that is far away from campus—perhaps even in another town. Differences in these attributes affect Al's reservation price (total benefit) for the two apartments. His or her subjective net benefits for the two apartments varies with his or her reservation prices for the two apartments and with the rental cost of the apartments. If the rents are the same, Al would clearly realize greater net benefits with the larger, newer, apartment closer to campus. However, if the prices were significantly different, Al might rent the smaller, run-down, apartment that is farther from campus even though he or she in a sense prefers the newer larger apartment close to campus. The smaller run-down apartment "saves money," and those savings can be put to other uses that are more valuable to Al than the advantages associated the newer, larger and more convenient apartment.

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[1] Firms that produce several varieties of the same type of product usually do so by varying the attributes of each variety of their product lines. Similarly, monopolistic competition often involves firms that produce similar products, but with each firm producing varieties of more or less the same product that include somewhat different mixes of attributes (features, appearance, colors, etc.) than those produced by other firms.

This type of choice setting can be modeled by placing explicit values on the three attributes, distance from campus, size, and age of the apartments. Let apartment 1 be the newer apartment and apartment 2 be the older one with $B_1 > B_2$, where $B_1 = b_1 S_1 - b_2 D_1 - b_3 A_1$ and $B_2 = b_1 S_2 - b_2 D_2 - b_3 A_2$, where S_i, D_i, and A_i are the attributes of apartment "i" and b_1, b_2, and b_3 are Al's assessment of the "worth" or "value added" by the three attributes focused on. Note that each apartment comes with all three attributes and that the "b" valuations may differ quite a bit for Al (and also vary among other prospective renters). Distance might be much more important than size, for example.

As a net benefit maximizer, Al chooses the apartment with the higher net benefits $N_1 = B_1 - C_1$ and $N_2 = B_2 - C_2$. If Al rents apartment 2 (the older one), it is not because he or she "likes" apartment 2 better than apartment 1 (B_1 is likely to be larger than B_2), but because the older apartment produces greater net benefits—it frees money that can be used for better meals, more textbooks, a better computer, or holiday travel, etcetera.

In Statistical estimates of the way in which characteristics of apartments affect their market rental rates, the "b" valuation terms are estimated using data on apartment rents and their characteristics. The b coefficients estimated are those of the "average consumer" in the apartment market of interest. These coefficients of valuation, of course, vary among persons in that market—indeed, there may be no single consumer with the average assessments of the value of the individual apartment attributes. Nonetheless, average values provide landlords and renters with ideas about what a "reasonable" rent would be—based on renter preferences and past market clearing prices.

Attributes and the Demand for Multi-Attribute Goods

Many economic choices are of the one or nothing variety, in which case all one can do to model such choices is to think carefully about the sources of the net benefits or utility associated with the alternatives available in the markets of interest as done above (and could be done in more detail). Nonetheless, there are also many choices in which more or less of a good can be purchased, produced, or rented—and these choices can be modelled as well by extending the above model a bit.

Consider Al's decision to purchase apples of a given type. The apples have size, S, tartness, T, and juiciness J. and the value of a single apple to Al can be represented in a manner similar to the above, as, for example: $B = b_1 S - b_2 T - b_3 J$. Assume that there is just one kind of apple, or that Al has already done a comparison among the available apple types and chosen a particular variety to purchase. How many will she purchase?

As characterized, the benefit function does not include the effect of diminishing marginal returns. This effect does not necessarily have to be taken into account for "one or nothing" types of choices, but for "how many" types of decisions diminishing returns tend to be important—at least according to economic theory after the marginal revolution took place in the late nineteenth century.

One way to incorporate diminishing marginal returns and the quantities purchased into the model is the following. Let $b(Q) = b_1(QS)^e - b_2(QT)^f - b_3(QJ)^g$ with exponents e, f, g < 1. The latter assures diminishing marginal returns for each attribute. A consumer still gets the (S, T, J) attributes with every apple purchased, so if one buys 2 apples one gets twice as much of each of the three attributes (2S, 2T, 2J); if one purchases three apples, one gets three times as much of each attribute (3S, 3T, 3J), and if one purchases Q apples, one gets (QS, QT, QJ) units of the three attributes. For multi-attribute goods, it is the attributes that produce the benefits or utility, not the quantities of apples by themselves. The models in earlier chapter basically assume that the attribute mix of each good is fixed and so can be ignored for purposes of analysis—which is often a reasonable assumption for products where the attribute mix is narrow and stable. However, it is not reasonable if the attribute mix varies substantially and changes through time. New varieties of apples are introduced every few years—and some of the old "standards" such as Delicious and Macintosh are often hard to find.

We can now model Al's decision about how many apples to purchase. Suppose that Al's favorite apples can be purchased at price P. The total cost of Q apples, C=c(Q) is PQ. Given the above, the net benefits associated with various quantities of apples are:

$$n(Q) = b(Q) - c(Q) = [b_1(QS)^e + b_2(QT)^f + b_3(QJ)^g] - PQ \tag{10.1}$$

Differentiating the net benefit equation with respect to Q and setting the result equal to zero characterizes the ideal (net- benefit maximizing) quantity of apples to purchase:

$$n_Q = b_Q - c_Q = [eb_1(QS)^{e-1} + fb_2(QT)^{f-1} + gb_3(QJ)^{g-1}] - P = 0 \: at \: Q^* \tag{10.2}$$

As usual, Al will purchase apples up to the point where his or her marginal benefits from them (the terms inside the brackets) equals the marginal cost of the apples (here P). Note that the marginal benefit of the last apple is the sum of the marginal benefits associated with each of the attributes of

the apple. Unfortunately, because the exponents all differ, there is no simple solution for Q as a function of P that can be worked out from equation 10.2.[2]

However, we can use the implicit function theorem to characterize Al's demand function for apples as:

$$Q^* = q(P, b_1, e, b_2, f, b_3 \cdot g, S, T, J) \tag{10.3}$$

Al's demand for apples varies with the price of the apples and the parameters of the benefit function that determine how the attributes of the apple (S, T, and J) generate benefits for Al.

Changes in price, the valuation factors, or in the attributes of the apples being purchased will alter the quantity of apples that Al's demands. Note that all the variables in lower case can be regarded as "taste variables" and the attributes (S, T, and J) can be regarded as factors that determine the quality of the apple for Al (given those taste factors: $b_1, e, b_2, f, b_3. \, and \, g$). Tastes matter as in all the previous models of consumer choice, but so do the attributes of the goods purchased. In this model, if the desirable attributes of a good increases, so will Al's demand for that good, which implies that the quantity purchased at a given price, P, increases as the perceived quality of the apples increase—an intuitive result, but not obvious in the standard model.

Utility Maximizing Choices of Goods with More than One Attribute

The net-benefit maximizing model of rational consumers has several advantages. It normally involves just one choice dimension, and so the simplest optimization methods can be used to characterize net-benefit maximizing choices and their implications for market demand. The results often are intuitive and clear. This is partly because many of our own choices are made one at a time, and thus the net-benefit maximizing model resembles the thought process most of us has used in our past decisions. (This is especially true of economics majors.) On the other hand, whenever

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[2] Had all the exponents been the same, as with e=f=g, a concrete solution for Al's ideal purchase of apples could have been worked out. In that case, $[gb_1(QS)^{g-1} + gb_2(QT)^{g-1} + gb_3(QJ)^{g-1}] - P = 0$ can be written as $[gQ^{g-1}][b_1(S)^{g-1} + b_2(T)^{g-1} + b_3(J)^{g-1}] = P$ which can be solved for Q* as $Q^* = \{\frac{P}{[g][b_1(S)^{g-1}+b_2(T)^{g-1}+b_3(J)^{g-1}]}\}^{1/(g-1)}$. Recall that g<1 which implies that $Q_P^* < 0$.

This is easier to see if you take account of one of the properties of negative exponents to rewrite the above as $Q^* = \{\frac{g][b_1(S)^{g-1}+b_2(T)^{g-1}+b_3(J)^{g-1}}{[P]}\}^{1-g}$. Note that in this case an increase in any of the attributes increases demand (they all have positive terms in the numerator).

constraints are important (as with budget constraints) and various tradeoffs associated with those constraints affect choices, the utility maximizing model provides additional useful insights into the factors that affect the choices of purposeful consumers.

To see how a utility maximizing model of choices with respect to multi-attribute goods, consider a minor extension of the above model. Assume that Al has a budget to allocate between two goods: apples with multiple attributes and some other good with only a single attribute, as might be claimed of peanuts. Assume that Al has a utility function defined over apples of a given type (e.g. with particular attributes) and peanuts. The modelling methods developed above for the net benefit maximizing model of goods with multiple attributes imply that Al's utility function with respect to apples and nuts can be written as U = u (QS, QT, QJ, N). Q is the quantity of Al's preferred type of apple, and S, T, and J are the attributes of Al's preferred type of apple as before. N is the quantity of nuts. Al's budget constraint is the usual one: $W = P^A Q + P^N N$, where W is Al's budget and P^A is the price of apples and P^N is the price of peanuts.

We can use the budget constraint to characterize the N as $N = (W - P^A Q)/P^N$. Substituting that relationship into the utility function yields:

$$U = u\left(QS, QT, QJ, \left(\frac{W - P^A Q}{P^N}\right)\right) \tag{10.4}$$

This function, as in most of the other cases for which we've used the substitution method has only a single choice variable (the quantity of apples), and evaluates the utility function along the budget constraint. If U is strictly concave and has positive first derivatives for all the characteristics in that function, then the quantity of apples, Q*, that satisfies the first order conditions of equation 10.4 with respect to Q is the quantity that maximizes utility, given the budget constraint and Al's preferences for apple characteristics.

Differentiating equation 10.4 with respect to Q and setting the result equal to zero yields:

$$U_Q = u_S S + u_D D + u_J J - u_N \left(\frac{P^A}{P^N}\right) = 0 \equiv H \ at \ Q^* \tag{10.5}$$

The first three terms are the marginal benefits from apples (now in marginal utility terms). Again, the marginal benefits are the sum of the marginal benefits from each of the three attributes of the typical apple. The consumer's ideal purchase of apples include consideration of the marginal opportunity cost of apples, which in this case is the purchase of fewer peanuts.

The implicit function theorem, in turn, implies that Al's demand for this type of apple can be written as:

$$Q^* = q(P^A, S, T, J, W, P^N) \tag{10.6}$$

The result implies that the demand for this type of apple varies with its price, its values for the specific attributes (S, T, J), with the price of the other good of interest, and Al's budget constraint (W).

Comparative statics of Al's demand function can be characterized in the usual way, using the implicit function differentiation rule. We'll focus on the price of apples and one of the desirable attributes. (Keep in mind that each of the partial derivatives includes all of the arguments of the original utility function, so there are numerous cross partials that affect these two derivatives).

$$Q^*_{PA} = \frac{H_{PA}}{-H_Q} = \frac{[Su_{SN} + Tu_{DN} + Ju_{JN}]\left(\frac{-Q}{PN}\right) - u_N\left(\frac{1}{PN}\right) + u_{NN}\left(\frac{QP^A}{(PN)^2}\right)]}{-U_{QQ}} > 0 \tag{10.7}$$

$$Q^*_S = \frac{H_S}{-H_Q} = \frac{u_S + SQu_{SS} + QTu_{DS} + QJu_{JS} - u_{NS}\left(\frac{QP^A}{PN}\right)}{-U_{QQ}} > 0 \quad (?) \tag{10.8}$$

Equation 10.7 implies that the slope of the demand function is negative, as usual for a demand function. Strict concavity implies that the denominator is positive. The usual rule of thumb for the cross partials of strictly concave functions (e.g. all are greater than zero) and second derivatives (e.g. all are less than zero) imply that the utility function is strictly concave and that all of the terms in the numerator are negative. Thus, equation 10.7 is less than zero over the entire range of apple prices—other things being equal. Al's demand curve for apples of this type is downward sloping.

Equation 10.8 shows that the effect of the apple size attribute (or any other of the desirable attributes) is ambiguous, which is a different result than that associated with the net benefit maximizing model above. This ambiguity is due to effects of apple size on the second derivatives and cost functions. Diminishing marginal utility implies that the marginal utility from the last unit of S diminishes at Q*. If one has more S, then its value at "the margin" (e.g. at Q*) is lower than it was before S increased. (Intuitively, if the apples are larger, it takes fewer to satisfy one's appetite for apples.) This together with an increase in the subjective marginal cost of additional apples (lost utility from peanuts, implied by the positive cross partial) produces two negative terms. The other three terms in the numerator are positive. So, the overall effect of an increase in apple size on the quantity of applies purchased is unclear. It is quite possible that the improved quality will lead to greater

purchases of the apple, but it is not necessarily the case. It depends on the relative size of the two effects.

Note that this ambiguity is also associated with separable versions of utility functions (e.g. utility function with zero cross partials). If the utility function is assumed to be separable, all of the terms in the numerator disappear except the first two. However, the first is positive and the second is negative. So, ambiguity remains in the separable case unless there is very little in the way of diminishing returns, in which case the intuitive positive sign obtained in the net-benefit maximizing model is the result. Or, if it is known that diminishing returns are quite large (as when satiation sets in or storage costs are high) then Al's purchase of apples (in numbers, not in weight) decreases as their average size increases.

In the utility maximizing model, an increase in quality causes the last unit of the good to decline in marginal value. This implies that one can purchase fewer apples and get the same satisfaction as previously associated with Q^* apples. Thus, it is quite possible that Al, would purchase fewer apples after this increase in quality. Whether this is the case or not is, of course, quite important to firms that create the product—here apples—being sold. Indeed, this effect may cause apple sellers to price their apples by the pound rather than by the apple.

The same logic applies to all goods with variable attributes. In a net benefit maximizing model, the effect of an increase in quality will be an increase in unit sales. In a general utility maximizing model an increase in quality may increase unit sales, but does not necessarily do so even if the price of the good remains the same. Intuitive, we may believe that the positive terms dominate the negative ones, but this is not necessarily the case.

Alternatively, if only a single unit of the good is normally purchased (as, for example, with cell phones or houses), then an improvement in quality of one good rather than the other will increase the utility associated with the improved good, and so induce new purchasers to buy a unit of the improved good who would otherwise not have done so. In that case, unit sales of the higher priced good is likely to increase because more people purchase them rather than because of effects at the margin. This is an implication of the logic of utility maximization in cases where single units of a good are purchased rather than of conditions of the margin indicated by calculus. If $U=u(A,N)$ and initially that $u(0, 1) > u(1, 0)$, but that after the improvement in A, $u(0, 1) < u(1, 0)$, then one unit of A will be purchased rather than one unit of N after A has improved (e.g. its desired characteristics increased) assuming that only one or the other is affordable.

III. Creating Goods: Designing Profitable Products with More than One Attribute

In the models developed in previous chapters, the nature of goods and their value to consumers was "given," which is to say assumed to preexist. This allowed us to develop a theory of price determination without paying much attention to what purchasers actually wanted—the needs that they hoped to satisfy, the amusement or comfort they hoped to obtain, the aesthetics ideas that they hoped to advance, and so forth. Goods simply increased utility—e.g. increased the satisfaction of such goals as they were obtained from the things and services purchased in markets. But how much a particular good or service advances an individual's aims depends on the characteristics of the good or service of interest as well as the aims that an individual has.

In this section, we'll continue to assume that individual aims are "given," but now assume that the nature of the things and services on offer are designed to advance various consumer desires. Indeed, in some cases, goods and services are designed to advance latent interests that individuals do not know that they have—although an entrepreneur's or firm's believes about latent interest are often wrong. Many "improvement" or "new" products may be judged by consumers to be inferior to other products that are already on the market and so fail to sell. (Most new businesses fail within ten years of their origins and many new products as well.)

However, some new products do attract the purchases of consumers and so become part of the domain of things and services on offer in markets—e.g. and thus among the things and services modeled in previous chapters. Although there are many failures, the domain of goods sold in markets is always in flux and when economic progress is evident, it constantly increases. The failures are clearly because of information problems that firms face when trying to understand individual preferences over attributes. However, first, let us assume that firms know what individuals want—which is an assumption that simplifies that analysis and explains the existence of products with attributes that consumers are willing to pay for.

How to Design Products to Maximize Profits

The simplest model of product design is one where there is a single group of consumers with essentially identical tastes for product attributes that is well-known by firms attempting to sell to such consumers. In that case firms will tailor the product to consumer preferences and variants other than that "ideal" type of product will fail in the markets unless they are significantly less

expensive to produce and sell. Because some desired attributes are costly, firms anticipate tradeoffs that individuals will make between price and quality when they design their products.

The easiest of these cases is the monopolist case. Suppose that a single innovative firm—Apex—faces a downward sloping demand curve that is simply M times equation 10.6, because there are M consumers with essentially identical tastes and income. In that case the firm faces an optimization problem with more "controls" than in the problems previously examined. Apex can control output, price, and the characteristic of the good.

Given $Q = MQ^* = Mq(P^A, S, T, J, W, P^N)$ as the demand function, the inverse demand function can be written as: $P^A = q(Q^A, M, S, T, J, W, P^N)$. Apex's profit function is thus:

$$\Pi = P^A Q^A - c(Q^A, S, T, J) \tag{10.9}$$

Differentiating with respect to Q^A, S, T, and J yields a system of first order conditions all of which have to be satisfied if the quantity of the optimal product is to be produced:

$$\Pi_Q = P^A{}_Q Q^A + P^A - C_Q = 0 \text{ at } \Pi^* \tag{10.10a}$$

$$\Pi_S = P^A{}_S Q^A - C_S = 0 \text{ at } \Pi^* \tag{10.10b}$$

$$\Pi_T = P^A{}_T Q^A - C_T = 0 \text{ at } \Pi^* \tag{10.10c}$$

$$\Pi_J = P^A{}_J Q^A - C_J = 0 \text{ at } \Pi^* \tag{10.10d}$$

Notice that each of the first order conditions is similar—they each imply that the firms should set the marginal revenue generated by each of the four control variables equal to its marginal cost when selection the profit maximizing combination of output (Q*), size (S*), tartness (T*), and juiciness (J*). The same logic would apply to other goods with more characteristics. As a monopolist Apex would price the apples sold using the ideal vales for each of its control variables, as with:

$$P^{A*} = q(Q^{A*}, M, S^*, T^*, J^*, W, P^N) \tag{10.11}$$

Very similar results would apply to firms selling in perfectly competitive markets—the main difference would be the difference in the marginal revenue function, which would simply be the price of the good sold in each case—which would reflect the overall marginal cost of producing the "ideal" apple (or other product) in equilibrium, including the cost of obtaining the profit-maximizing sizes, tartness, and juiciness.

In both cases, firm's would maximize quality only in cases in which either quality (adjustments in characteristics of the good sold) was costless, or if the typical consumer's ideal levels were finite (e.g. satiation occurred) and it turned out that the marginal revenue associated with producing each characteristic equaled its marginal cost at exactly the point equal to the ideals of the typical consumer. In all the usual cases, there would be some divergence between the ideal qualities of a typical consumer's ideal apple (ignoring costs) and that produced by Apex because of the effects of marginal production costs.[3]

As true in many cases, the relatively simple model of product design developed in the subsection can be extended easily to more complex settings. For example, there may be dozens of relevant attributes rather than just three, as, for example, true of automobiles and houses. The market type may be monopolistically competitive rather than a simple monopoly as modelled. In that case, each firms product design would affect both its own demand and that of its rivals in the market of interest. There may be more than one type of consumer. In that case, different firms may attempt to service different sub-groups. If the groups and group sizes are "given," and the market type is either monopolistically competitive or monopolistic, then the basic model above can be applied for each subgroup and its associated market demand curve. Wealthy persons may purchase one goods with one set of attributes and poorer persons may purchase similar goods, but prefer different combinations of attributes, because they have different preferences or because they are more concerned about cost than wealthier persons.[4]

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[3] It bears noting that a consumer that produced his or her own apples would also produce less than "perfect" apples unless he or she could achieve perfection for free. In that case the marginal utility of each of the four control variables would be set equal to its marginal opportunity cost in terms of expenditures on other goods and/or uses of time in other pursuits. Only a perfect apple zealot would pursue the absolutely ideal apple unless it was essentially costless to achieve.

[4] The latter is one explanation for the existence of "inferior" and "superior" goods. Demand for the former falls as income increases and demand for the latter increases more than proportionately as income rises. Buses, taxis, and limousines are all methods of urban ground transportation, but as income rises, people tend to use buses somewhat less and taxis and limousines more. Note that these modes of transportation are good substitutes for each other as far as urban transportation is concerned, but each mode has a different combination of attributes.

The next subsection shows how one can extend the model to take account of consumer heterogeneity when the groups are not sufficiently large to be supported by separate firms or groups of firms.

Designing Products for Heterogeneous Consumers

Consumers and potential consumers of most goods, of course, vary in many respects. They may have different preferences over attributes (color, speed, size, sound level, sound fidelity, tartness, sweetness, saltiness, vitamins, protein, etc., etc.). They may have different incomes, be of different ages, have different weights, and health, as well as be members of different peer groups, and so forth. They may know more or less about the characteristics of particular goods and services. All these differences may affect their demand for goods and services by generating differences in their marginal utilities for various attributes, their cross partials among them, and their rates of diminishing marginal utility for the products brought to market. Indeed, some things that are "goods" for one group may be "bads" for others—as with types of music, food, art, clothing, and the sound of motors in vehicles.

There is often a broad variation in preferences for the various attributes of most types of product categories. In some cases, this variation in demand produces a wide variety of products that fall within a particular category, as with restaurants, clothing, and automobiles. In others, it produces a compromise. Producers take account of differences in preferences and income and produce a product that is unlikely to be ideal for anyone, but is serviceable for a wide range of consumers.

The general effect of heterogeneity can be illustrated with a case in which there are just two varieties of consumers. Suppose that Apex is selling its product to two homogeneous groups that make choices that can be characterized with a utility-maximization problem similar to equation 10.4 above. In this case there will be one group—group "a" with M^a members—solving a problem like: maximize $U = u^a\left(QS, QT, QJ, \left(\frac{W^a - P^A Q}{P^N}\right)\right)$ and another group—group "b" with M^b members— solving a problem like maximize $U = u^b\left(QS, QT, QJ, \left(\frac{W^b - P^A Q}{P^N}\right)\right)$. The utility functions and incomes of each member of group a and b are different, but the two groups are both focused on the same attributes and face similar opportunity costs for purchases of the good of interest (the last term). We'll continue to refer to the good of interest as apples—but it can really be nearly any good sold in markets.

The mathematics employed at the beginning of the chapter implies that each group has a somewhat different demand function.

$$Q^a = M^a Q^{a*} = M^a q^a(P^A, S, T, J, W^a, P^N) \text{ and}$$

$$\mathbf{Q^b} = M^b Q^{b*} = M^b q(P^A, S, T, J, W^b, P^N)$$

Which implies that the overall demand function is simply:

$Q = Q^a + Q^b$, which the implicit function theorem implies can be written as:

$$Q = q(P^A, S, T, J, M^a, M^b, W^a, W^b, P^N) \tag{10.12}$$

Figure 1 illustrates the geometry of equation 10.12 for the case where the two demand curves are linear and start at the same point on the vertical (price) axis. Note that the sum is in the price dimension rather than the price dimension. Note also that the inverse of the Q function is not the equal to the sum of the inverses of the individual demand functions. That sum would take place in the P dimension and lie well above the Q function for much of its range (see the red dashed line). The Q function goes from P to Q, as with P' to Q' and the inverse goes from Q to P, as from Q' to P'.

Figure 10.1: Sum of Demand Functions and its Inverse
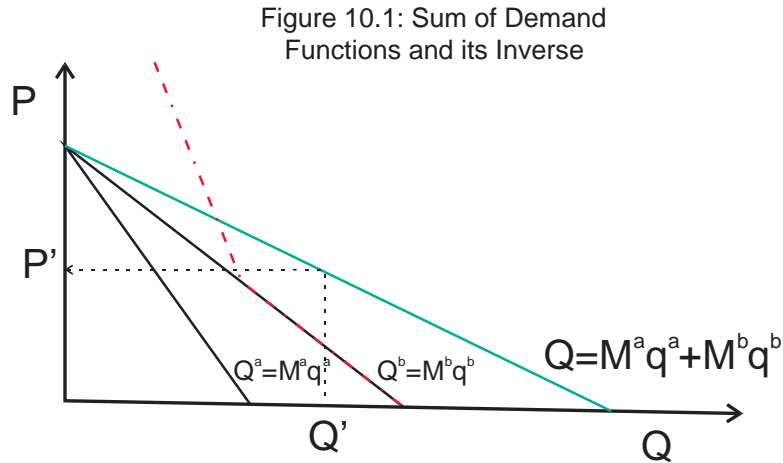
The implicit function also implies that the inverse of Q in the PxQ plane can be written as:

$$P^A = p(Q, S, T, J, M^a, M^b, W^a, W^b, P^N) \tag{10.13}$$

This is the price function that is used by Apex to determine it's profit maximizing combination of output and attributes of its product.

The rest of the mathematics in this general form looks very similar to the single group case—except that there are now two Ms and two Ws in both the demand and inverse demand functions—both of which may look quite different than either of the group demand curves or their inverse demand functions. Apex's profit function is now:

$$\Pi = P^A Q^A - c(Q^A, S, T, J) \tag{10.14}$$

Differentiating with respect to Q^A, S, T, and J yields a system of first order conditions all of which are satisfied when the profit-maximizing quantity of the optimal product is produced:

$$\Pi_Q = P^A{}_Q Q^A + P^A - C_Q = 0 \; at \; \Pi^* \tag{10.15a}$$

$$\Pi_S = P^A{}_S Q^A - C_S = 0 \; at \; \Pi^* \tag{10.15b}$$

$$\Pi_T = P^A{}_T Q^A - C_T = 0 \; at \; \Pi^* \tag{10.15c}$$

$$\Pi_J = P^A{}_J Q^A - C_J = 0 \; at \; \Pi^* \tag{10.15d}$$

Notice that, as in the one group case, each of the first order conditions is similar—they each imply that the firms should set the marginal revenue generated by each of the four control variables equal to its marginal cost in order to select the profit maximizing combination of output (Q*), size (S*), tartness (T*), and juiciness (J*). Although not obvious from the notation, the marginal revenue functions now include demand effects on both groups of consumers.

As a firm with a downward sloping demand curve, Apex would use the inverse demand function to price the apples sold using the ideal vales for each of its control variables, as with:

$$P^{A*} = q(Q^{A*}, M, S^*, T^*, J^*, M^a, M^b, W^a, W^b, P^N) \tag{10.16}$$

Apex's tradeoffs among groups tends to be hidden with the mathematical representations used in general models, but their effects are present in both the shape of the demand curve and in its arguments. The shape requires taking partial derivatives with respect to the parameters of the demand function, which because of the role of the inverse demand function tends to be quite messy as true of the simpler model reviewed when we introduced firms with downward sloping demand curves.

A Concrete Functional Form Illustration

To provide a bit more insight into the tradeoffs that profit maximizing firms will make among groups, suppose that the demand functions are $Q^a = M^a(a'S+b'T+c'J+d'W)/P$ and $Q^b =$

$M^b(a"S+b"T+c"J+d"W)/P$. These resemble demand functions derived from exponential multiplicative utility functions, where a fixed amount (the numerator) is spent on a good by each member of the group. However, in this case the "fixed amount" is jointly determined by the attributes of the good brought to market as well as the consumer's budget constraint. We'll again assume that the individual groups are two small to justify two different products. (Multi-product firms are taken up in the next chapter.) Given these assumptions, Apex's demand function is:

$Q^A = Q^a+Q^b$ or

$Q^A = [(M^a a'+ M^b a'')S+ (M^a b'+ M^b b'')T+ (M^a c'+ M^b c'')J+(M^a d'+ M^b d'')W]/P^A$

and its inverse demand function is:

$P^A = [(M^a a'+ M^b a'')S+ (M^a b'+ M^b b'')T+ (M^a c'+ M^b c'')J+(M^a d'+ M^b d'')W]/Q^A$

To simply the notation below, we'll denote the numerator as $(M^a Z^a +M^b Z^b)$ with $Z^a=(a'S+b'T+c'J+d'W)$ and $Z^b = (a"S+b"T+c"J+d"W)$, which allows P^A to be written as

$$P^A = (M^a Z^a +M^b Z^b)/Q^A \qquad (10.16)$$

Apex's profit function is again:

$$\Pi = P^A Q^A - c(Q^A,S,T,J) \qquad (10.17)$$

Keep in mind that Q^A for a price-making firm is a control variable rather than a function, but its pricing equation, P^A, is a function based on the demand for its product(s).

Differentiating with respect to Q^A, S, T, and J yields the following four first order conditions that will be simultaneously satisfied if Apex maximizes its profits. The derivatives with respect to S, T, and J are calculated from the M^a and M^b functions above. Note that in terms of general notation, the results are the same as above. This illustrating example is after all a special case of the more general characterization of a firm's product design and production decisions. However, the concrete functional forms attributed to the demand functions imply particular functional forms for the first order conditions. These illustrate the sorts of tradeoffs across consumer groups that Apex will make in a manner that is clearer than in the general case.

$$\Pi_Q = P^A{}_Q Q^A + P^A - C_Q = 0 \ at \ \Pi^*$$

$$\Pi_{Q^A} = -\frac{M^a Z^a + M^b Z^b}{(Q^A)^2} Q^A + \frac{M^a Z^a + M^b Z^b}{Q^A} - C_Q = 0 \qquad (10.18a)$$

$$\Pi_S = P^A{}_S Q^A - C_S = 0 \text{ at } \Pi^*$$

$$\Pi_S = (M^a a' + M^b a'') - C_S = 0 \tag{10.18b}$$

$$\Pi_T = P^A{}_T Q^A - C_T = 0 \text{ at } \Pi^*$$

$$\Pi_T = (M^a b' + M^b b'') - C_T = 0 \tag{10.18c}$$

$$\Pi_J = P^A{}_J Q^A - C_J = 0 \text{ at } \Pi^*$$

$$\Pi_J = (M^a c' + M^b c'') - C_J = 0 \tag{10.18d}$$

In this case, it is clear that the marginal revenue effects (the first terms) depend on the relative size of the two groups and also of their respective tastes (the a, b, and c terms) for the good being designed by the firm. The characteristics chosen are, in effect, weighted averages of the preferences of the two groups.

One odd property associated with the assumed demand functions is that total revenue does not vary with quantity, thus the first term in this case is irrelevant. Changes in revenue are entirely the result of the types of products produced and sold. This odd feature—which is common to the demand curves derived from Cobb-Douglas and similar utility functions—implies that product attributes and production costs are the only control variables that matter. Note that the firm's profit maximizing product design reflects both the effects of attributes on each groups demand and also the number of each type of consumer. Large groups have greater impact on product design than small groups. In addition, changes in technology that alter the marginal cost (and the marginal cost) of incorporating the various characteristics of the product designed will affect the profit maximizing design—other things being equal.

Thus, the term "new and improved" often simply means that changes in input prices or in the methods of production have changed the profit maximizing combination of the inputs. This usually makes most consumers better off in that profits are higher but selling costs are lower than they would otherwise be. Moreover, although such changes in product attributes are induced by changes in production methods, consumer preferences for characteristics still partly determine the ultimate design of the product Apex brings to market. Reductions in the cost of incorporating a particular attribute into its product usually encourages more of that quality to be incorporated into the product, which tends to make consumers better off even if selling price is not significantly reduced by such effects. Indeed, selling price might increase because of increased demand.

IV. Choices to Purchase or Produce New Products Under Imperfect Information

The core models of neoclassical economics in Part I of this textbook implicitly assume that consumers know all the relevant attributes of the products on offer and that all products have been optimized by firms with the interests and numbers of consumers in mind. Products are "given" rather than conceived, designed, produced, offered for sale, and then purchased under conditions of competition and limited information. The nature of products and the product mix are among the unanalyzed factors that are held constant—they are elements of the "other things being equal" expression that partial equilibrium analysis uses to facilitate analysis. This simplifies the analysis of market prices at a moment in time, but it not a reasonable way to analyze the things that are produced and priced in markets through time.

In practice, the product mix of markets are neither unique nor permanent. It is clear that from at least the late nineteenth century onwards, new products were continually introduced in well-functioning market-based societies. The list of formerly new products that are now commonplace in middle-class households includes: various forms of electric lightbulbs, refrigerators, washing machines, toilets, dishwashers, televisions, stereos, computers, software, various communication devices, furnaces, air conditioners, and automobiles—to name just a few examples of new products successfully brought to market during that period. Within each of these categories of goods and services, a wide range of refinements were subsequently adopted and brought to market in the periods after the product categories were first introduced. Even long-standing, relatively simple products such as apples, tomatoes, pasta, wine, chairs, tables, beds, mattresses, blankets, carpets, cooking pans, tents, and backpacks, were subjects of ongoing refinements. Changes in the nature (attributes) of the things and services sold in product categories are commonplace events in well-functioning commercial societies. Such changes affect market circumstances. New discoveries of inputs useful for producing a product's attributes may lower the cost of providing particular attributes, which induces product reform. Contrariwise, as new uses for existing inputs are discovered, the price for those ingredients tend to rise, which increases the cost of an attribute or class of attributes, inducing refinements in the products and changes in output and posted prices.

The previous section demonstrated that product designs tend change when various market circumstances change. Many of these are beyond the control of any firm or small group of firms and difficult to predict. Input prices may change. The production methods of rivals may change. The relative size of the groups served by a firm may change. Consumer tastes for various attributes may

change. Other changes are generated by changes that are controlled—at least to some degree—by the firms themselves. A firm's estimates of the course of future prices, number or future mix of consumers and their "tastes" may change through time, because new (and hopefully better) estimation methods are adopted. The new estimates, in turn, may affect both the nature and price of the goods and services brought to market in future periods. Similarly, other innovations in the mode of production or decision to add or subtract from the vector of attributes that determine the nature of the product(s) sold, will have effects on demand and thereby on prices, outputs, and the ultimate nature of the goods brought to market.

Fluctuations in all of the external parameters of a firms product design decision and the firm's own choices make product design a more complex an uncertain process than the mathematics in the first part of this chapter suggests—because the models have implicitly assumed that firms know a great deal more about future consumers and input costs than they actually do. If the various parameters of the choice problem are or have been stable, such information may be available. But if not, there will be significant uncertainty (in the statistical rather than Knightian sense) that the frim has to take account of. If the firm is risk neutral, and is able to calculate expected values, then the previous models can be interpreted as expected profit maximizing exercise where each of the external factors that affect its decisions are estimated values (expected values). If the firm is not risk neutral, then more than expected values will be required for the firm to make informed choices.

Likewise, products with multiple attributes often include features that consumers cannot readily observe by looking or tasting a product. Are their unnoticed features that make a particular version of a product better than another? How long will the product continue to produce useful services? Are their effects associated with continued use of a product that will affect one's health in the long run? How does the product interact without others that one already possesses? And so on. The more attributes a product has, the more details a consumer has to take into account to maximize utility, and thus the more informational and computational challenging a decision tends to be (assuming for now, that consumers actually try to maximize utility). An apple is not an apple— macintosh apples differ from delicious apples which differ from honey crisp apples and so on—so which apple is best? Or, perhaps some apples better for one thing than another, as with making apple pies or as health snack after lunch. In such cases, the type of apples purchased—as true of many other things—varies not only with one's preferences over attributes but with one's expected use of the apple-type purchased, which may vary from day to day or with the seasons. Changes in

the conclusions reached and approximations made will affect demand and thereby indirectly affect the attributes of the products on offer and its price through effects on firm demand functions.

Uncertainties associated with both supply and demand is why Buchanan and Vanberg (xxxx) argue that markets are experimental laboratories. To plan whether a good will sell or not—one has to bring it to market. If it sells, good, a firm's calculations and estimates have proven correct (or at least not entirely wrong). Once brought to market, for a consumer know whether a new or revised product is as useful or not (e.g. advances his or her interests or not), one has to purchase it, put it into service, and observe the results. If it "delivers" then a "correct" purchase decision has been made—which is to say that no serious errors in one's assessment were made.

In settings where one has experience selling and buying very similar goods, uncertainties are clearly smaller—but whenever new products are introduced or old ones significantly refined, uncertainties are commonplace on both sides of the market. In many cases, imagining that a particular attribute of a particular product can be added, subtracted, or significantly changed to make it more attractive is itself an innovation, but of course such ideas (hypotheses) are not always true.

Creating "New" Products by Altering Attributes

This section models the supply and demand for new multi-attribute products in settings where uncertainties exist about consumer demand for them. Apples are not "apples" they are a category of product, with a good deal of variety because of the varying apple attributes of the apples brought to market. The same is true of nearly every "good" and its associated "attributes" that one imagines being in utility functions. Successfully inducing consumers to purchase a new product requires both a correct assessment about consumer tastes for attributes and the provision of sufficient information—whether in the product design itself or advertising—to induce a subset of consumers to try the new product. Not all such products require major innovations. Most product innovations—even significant ones—involve understanding how several pre-existing products can be combined to make a new one that consumers will purchase.

For example, take a look at the two photos below (from Wikipedia) of washing machines. The first is a Finnish washing machine patented in 1766, which is basically a barrel, with a hand crank and a more or less waterproof door. The second is a more advanced version of that idea, more than a century later, sold by Miele, a German appliance firm, in 1923. Note that the washing barrel has been moved inside another wooden tub, and turning it is now powered by an electric motor

rather than by a hand crank. In addition to that innovation (one that, of course, required an electric motor to have been invented), rollers have been added to the washing machine to expedite the removal of most of the water in the wet clothing, and thus speed up the drying.
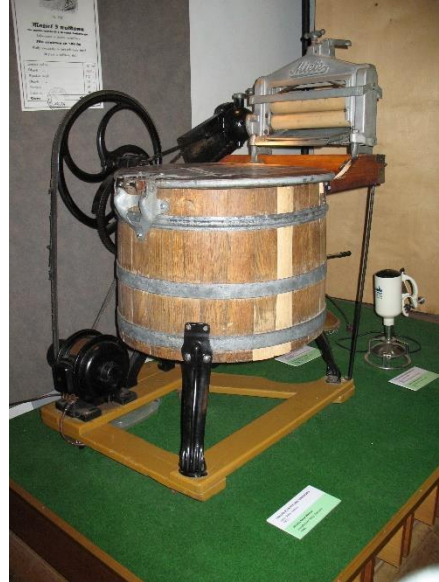
Rollers continued to be part of washing machines until the "spin" cycle was worked out after WWII. (The roller idea was patented in 1843. The first patent for an electric powered washing machine was issued around 1900.) Refinements in that period clearly took quite a while to be worked out, and were spurred on by other innovations (e.g. the electric motor) as well as gradual increases in household incomes.

Ideas that in retrospect look obvious often take a long while to occur to someone, to be implemented, patented, and successfully placed on the market for sale. To actually sell the product, it has to be deemed "worth its price" by a subset of consumers—which is partly a matter of the selling price and partly of expectations about the product itself. By 1960, according to Wikipedia, 60% of electrified households had an electric-motor powered washing machine. So, as the product was refined (e.g. new attributes added and alternative mixes of attributes tried out), the washing machine gradually became very popular and affordable.

Nonetheless, its long period of development suggests that Schumpeterian quantum leaps in technology or product designs are quite rare—although even refinements can be consequential. The washing machine freed a lot of time from washing that could be (and was) applied to other activities. And, people more commonly and routinely came to wear clean cloths. Such modest innovations are

easier to model than major ones, because fewer attributes are varied and so it is easier to anticipate the average consumer response to changes in what might be called relevant or salient attributes.

The Implicit Demand for a New Product

The demand for a new product cannot be directly observed—even by the consumers themselves. They have no experience with a non-existent good or service, so they can only imagine (estimate) how such a product will compare with the ones that they are familiar with. Utility functions, in their revealed preference sense, have holes in them in the places where various products did not previously exist. Interpolations across holes and extrapolations to parts of attribute spaces never experienced are possible, but these are estimates of utility rather than the real thing. Thus, risk aversion and the ease of undertaking the estimation both are factors that influence decision to try a new product. Errors in both directions are possible. One may be pleasantly surprised or disappointed when one actually experiences a new good, service, location, or period of life.

Suppose that the consumer has direct experience with varieties V_1 and V_2 and is attempting to appraise the anticipated utility of and thereby reservation price for variety V_3, which has not been previously experienced. Varieties V_1 and V_2 are still on the market and so the new product V_3 and price must yield a higher utility level than either of the two familiar products for a subset of consumers if it is to sell.

To model this choice, we'll again assume that just two products are purchased and that our consumer, Al, has allocated W dollars to spend on the two products, V and Z with known prices. However, Al will purchase only one of the three varieties of the multi-attribute good. We'll denote the attributes of each variety as A_i, B_i, C_i, and their Price as P_i with $i \in (1,2,3)$. Given these assumptions, $U_1 = u(V_1A_1, V_1B_1, V_1C_1, Z) = u(V_1A_1, V_1B_1, V_1C_1, [W-V_1P_1]/P_Z)$, and $U_2 = u(V_2A_2, V_2B_2, V_2C_2, Z) = u(V_2A_2, V_2B_2, V_2C_2, [W-V_2P_2]/P_Z)$, given the assumed budget constraint.

If V_3 is between V_1 and V_2 in terms of its attributes and its price is between P_1 and P_2, then AL might estimate the utility of V3 as:

$$U_3^e = (\alpha) u(V_1A_1, V_1B_1, V_1C_1, [W-V_1P_1]/P_Z) + (1-\alpha) u(V_2A_2, V_2B_2, V_2C_2, [W-V_2P_2]/P_Z) \qquad (10.19)$$

with $0 \leq \alpha \leq 0$. Notice that this method of interpolation generates a number—expected value—similar to that of the probabilistic cases examined in chapter xx when we examined risky choices and risk aversion.

Only the variety deemed "best" by Al will be purchased, which in a rational choice model based on utility maximization will be the one with the greatest anticipated utility. After the best variety (all things considered, including its price) is determined, Als demand for it can be determined in the manner that his/her demand for apples was characterized near the beginning of this chapter. Determining the "best", however, requires calculation of total utility rather than first order conditions and marginal utilities. Among the known varieties, either $U_1 > U_2$ or $U_2 > U_1$ or $U_1 = U_2$. Let us assume that $U_2 > U_1$ or that $U_1 = U_2$. In ether case, we can use U_2 as our base for comparison. Al will switch to the new product only if $U_3^e > U_2$. Al will be indifferent between variety 2 and the new variety 3 if and only if $U_3^e = U_2$.

Assume that Al always purchases the utility maximizing levels of the goods purchased, and that those combinations of goods are denoted with a "*" as with V_1^*, V_2^*, V_3^* and Z^*. If we assume that the attributes or goods and that satiation does not occur with any of the attributes, then a P_3 exists that will make $U_3^e (V_3^*, Z^*) = U_2(V_2^*, Z^*)$. That is Al's reservation price for the new variety, which is to say the highest price that Al will ever pay for the new variety. At any price lower than that price, Al will switch to the new variety.

We can use the implicit function theorem to characterize Al's reservation price because in that case $U_3^e - U_2 = 0$ and both U_2 and U_3 are continuous functions in the domain of interest as per our usual assumptions about utility functions.

$$P^R = p(A_3, B_3, C_3, A_2, B_2, C_2, P_Z, W, \alpha) \qquad (10.20)$$

Al's the effects of changes in the attributes of the two varieties and his/her budget constrain have the intuitive properties when the utility functions satisfy our usual assumptions. Al's reservation price falls as the quality of variety 2 increase and rises as the expected quality of the new variety increases. It also falls as the price of variety 2 increases and rises with the budget constraint. It also falls as α increases if $U_1 < U_2$. That is to say, if the information about the new product or its design indicates that it is of relatively high quality but is priced below the currently most attractive product, Al is more likely to purchase it.[5] However it is clear that α must be greater than zero for this occur—which is to say that some aspects of the new product or some favorable information must

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[5] These partial derivatives can be calculated using the implicit function differentiation rule.

be available about the new product for it to sell—even if it potentially very attractive to consumers like Al.

The Supply of New and Refined Products

Given an estimate of a significant group of consumer reservation price function, it is possible for a firm such as new entrant such as Acme to determine whether profits can be realized by producing an alternative version of a product already sold and also of entirely new products. For a new entrant, this is simply a matter of whether profits can be realized by producing a product that targets a neglected group of consumers or not. For a firm that already sells a product in the category of interest, the threshold is a bit higher. The revised product must be more profitable than its original one. In either cases, the firm has to calculate whether absolute profits can be increased by producing a new or revised product. To do this, it calculates the profit maximizing attributes, informational campaign, and output of the product and determines whether the profit realized is either greater than zero (new entrant) or larger than that of the existing product (current producer). Notice that in the second case, the existing producer, a revision makes sense only if circumstances have changed, since it already producing a product and output level that maximizes its profits.

For a new entrant, the production choice looks very similar to the previous design problem except now there is an additional informational variable that needs to be taken into account. (The main change is the use of a reservation price equation rather than an inverse demand curve and the notation used for quantity of the product produced, V, it's attributes (A_3, B_3, C_3) and the existence of an informational control variable, α.

$$\Pi = P^R V_3 - c(V_3, A_3, B_3, C_3, \alpha) \tag{10.21}$$

Differentiating with respect to Acme's control variables (output, attributes, and information) yields:

$$\Pi_v = P^R{}_V V^3 + P^R - C_V = 0 \; at \; \Pi^* \tag{10.22a}$$

$$\Pi_A = P^R{}_A V^3 - C_A = 0 \; at \; \Pi^* \tag{10.22b}$$

$$\Pi_B = P^R{}_B V^3 - C_B = 0 \; at \; \Pi^* \tag{10.22c}$$

$$\Pi_C = P^R{}_C V^3 - C_C = 0 \; at \; \Pi^* \tag{10.22d}$$

$$\Pi_\alpha = P^R{}_\alpha V^3 - C_\alpha = 0 \; at \; \Pi^* \tag{10.22e}$$

Given profit maximizing values for the output, three attributes, and informational cost (whether a design feature or advertising campaign), Acme can determine whether profits are greater than zero (e.g. greater than its opportunity cost use of resources). If so, it will enter the market. If not, it will not.

Similarly, if Apex and existing producer undertakes the calculation, it will revise its product (change the attribute mix and spend on information) if profits are greater under this revision than with its existing product, given the current (or anticipated future) choice setting.

As modelled, the above consumer purchase decision were assumed to have unbiased estimates of the new variety—partly because of past experience and partly because of α—and to be risk neutral. The supplier decision was assumed to have an unbiased estimate of the consumer reservation price schedule, and also assumed to be risk neutral.

If either or both groups are risk averse, they would be somewhat less inclined to purchase the new good and/or to produce it. However, the basic choice problem is not significantly changed by risk aversion per se—although calculating the risk does add some complexity to the utility and profit maximizing decisions because the error term (the distribution of the estimates) around the expected values would have to be take into account. Risk aversion would imply that the expected utility and expected profits are lower at the expected values of the control variables than in the model developed, which makes switching to a new or revised product less likely.

V. Conclusions

Very few markets involve products that are single dimensional with unalterable characteristics. This chapter has shown that a more realistic model of a firm's production decision can be easily developed that accounts for the designs of the products that we observe in developed economies. Goods within a product class tend to be varied and firms often serve somewhat different groups of consumers—rather than compete head-to-head for all of their sales. Moreover, the character of the products produce change through time as circumstances change. These include refinements in the products that are produced by other firms that sell similar products and also include changes in the marginal production costs of product attributes.

Models of markets that neglect the multi-attribute character of essentially all goods and services can not explain why firms produce similar but not identical goods, nor how the character of goods change through time—except through a bit of intuitive hand waving that implicitly

acknowledges that goods modeled as pre-determined are actually generated by market forces similar to those that account for production and sale of such goods. That is to say, the pursuit of profits accounts for both the characteristics of the goods on sale and their refinement through time to reflect different conditions demand for and production costs of the products brought to market. Contemporary markets do not endless churn out identical products, but rather the nature of the products churned out varies from time to time and from place to place reflecting differences in the preferences of the groups served and in the marginal production costs of the goods brought to market.

Some instances of economic progress occur as production technologies improve, but many others occur as firms improve the quality of the products sold—in the sense that the goods sold become better adapted to the preferences of the consumers served and their intended uses of the products purchased. Products are not indestructible atoms, but highly malleable because of their multi-attribute natures. Thus an apple is not an apple, nor a cell phone a cell phone, nor an automobile an automobile—rather these terms refer to general categories of goods rather than to particular ones—and there are often substantial differences with in those categories at moment and time and through time as different groups are served and as innovations occur.

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