Chapter 4: Production: Making Things to Sell

I. Introduction: Household Production versus Production for Sale

When economists talk about production, they refer to activities undertaken to transform a group of materials and services into others of greater value that the producers plan to sell to other persons, groups, or organizations. Production is not just making stuff or providing services, but making stuff and providing services that are *sold* to others in in their communities or trading networks. Although preexisting things may be sold—"used" cars, books, and cell phones, for example, are often resold—for the most part the economic theory of production involves the processes through which new things or services are created and sold to others.

This is not to say that the term "production" is never used in other ways. For example, that term is also used by economists to describe the activities of persons who raise some their own vegetables in gardens, bake bread, or construct sheds to store tools in their backyards, without any intent to sell them. Such activities are often termed household production (Becker, xxxx). In places and periods before commercial societies emerged, a good deal, perhaps most, of a family's time, knowledge, and materials were devoted to such productive activities. Foodstuffs were homegrown or the product of hunting in nearby forests, cloth was home spun, clothing was homemade, and cooking was conducted over fires fueled by wood harvested from nearby woodlands, and so forth. Although a lot of time was spent producing goods and services, relatively little of it was produced for sale—and thus it would be almost irrelevant for price theory.

Even today, most of what we bring home from a grocery and building stores is used in household production—as the "inputs" of bread, peanut butter, and jelly, together with labor, and capital in the form of knives, plates, and napkins are used to construct a peanut butter and jelly sandwich. Additional equipment and more labor are used to clean up afterwards. Lumber may be transformed into book shelves or tables using labor and capital (saws, measuring tapes, and so on). In cases in which household production takes place in a commercial society, household production indirectly affects prices through demands for inputs and effects on the markets for final products. Conversely, market prices also affect decisions to engage in household production—insofar as household production is often undertaken because the quality of the final product produced at home is deemed superior to that which can be purchased in markets for the same cost.

The difference between household production and production for sale in markets is largely a difference in objectives. In household production, the aim is to increase utility directly either because the production process itself is valued or the outputs are regarded to be more pleasing or less expensive than substitutes that could have been purchased in markets. Production for supply takes place with income or profits as the objective, which, in turn, is used by those receiving to advance their interests—utility. Maximizing utility is the aim for each type of production, but production for sale does so indirectly though effects on personal or household income, rather than directly by producing things and services for a family's own consumption.

Intermediate cases exist in which a person or group of persons produce some things for themselves (as with vegetable gardens, hunters, or carpenters) and sell part of the things produced to their acquaintances or in local markets. These were often the first types of economic organizations. Large commercial organizations became commonplace in the nineteenth century at about the same time that dense and ubiquitous commercial networks emerged. Contemporary organizations devoted to production are often huge, with employees scattered around the world and in number larger than many medium sized cities. Such huge organizations are relatively new—less than a couple of centuries old. Although smaller, often, family-based firms are still commonplace, large economic organizations and governments account for about half of the jobs in the United States.

It is the market relationships of late nineteenth and early twentieth century commercial societies in the West that neoclassical economics emerged to explain. Production within both relatively small and large organizations devoted to selling their output(s) was thus a natural focus of attention. However, the models developed continue to provide insights into the production decisions of contemporary economic organizations throughout the world.

II. Production with One Input

The simplest technologies for production use only a single variable input such as labor. Other inputs may be fixed in some way, either by nature, as with gathering fruit from wildly growing fruit tree, or because the other factor is generally held constant while production increases, as an ax may be used to cut on tree down or twenty. One may need an ax to produce firewood, but after one has an ax, the rest is all labor. The production in such cases is simply a function like $Q = q(L)$ where function q is assumed to be strictly concave—e.g. to exhibit diminishing returns over the full domain of production. And, total variable costs would simply be $C=wL$. Production costs are expenditures on inputs.

However, the cost functions that we used in chapter 3 were all functions of the firm's output, a specification that was important for the firm's decision about how much output to produce and bring to market. To create a cost function in terms of output rather than inputs, we need to determine the relationship between output and labor inputs. In the one input case, this is the inverse of the production function $L = q^{-1}(Q)$ where the "-1" denotes the inverse function. If, for example, $Q = aL^b$, the inverse function—the labor required to produce a given output, is the solution for L as a function of Q, which a bit of algebra finds is: $L = (Q/a)^{1/b}$. Given that relationship we can write the cost function as a function of output by substituting the inverse function that describes how much labor is being used as a function of quantity into the firm's cost function. In the first case, $C = wq^{-1}(Q)$ and in the second case $C = w (Q/a)^{1/b}$.

Given that cost function, the firm's output when price equal P would be its profit maximizing output, which would be found in the usual way. Write down the profit function, $\Pi = PQ - wq^{-1}(Q)$, differentiate with respect to Q, set the result equal to zero (which it will be at Q*) and solve for Q*. In the more abstract case this yields $P – w (d q^{-1}/dQ) = O$, and a supply function based on the implicit function rule, of $Q^* = s(P, w)$. The implicit function differentiation rule could be used to find the effects of price and wage rates on the firm's output (note that the production function could be used as a zero function by subtracting Q from it).

In the case with a concrete functional form, $\Pi = PQ - w (Q/a)^{1/b}$. Differentiating with respect to Q and setting the result equal to zero yields: $d\Pi/dQ = P – w (1/ba) (Q/a)^{(1-b)/b} = 0$ at Q*. Solving this for Q* yields $Q^* = a[baP/w]^{b/(1-b)}$. Note that the intuitively expected signs for the effect of price on a firm's output (positive) and the effect of the wage rate on supply (negative) can be determined simply be inspection of the firm's supply function. P is in the numerator and w is in the denominator of the term taken to the complex power. Q* increases as P increases and falls as w increases. The quantity of labor hired can be determined by substituting Q* into the function that describes labor usage as a function of the firm's output.

In terms of the mathematics, the complexity mostly generated by the necessity of generating a cost function that specifies a firm's total cost as a function of output levels. Note that we could have reversed the order of this process in this case—the case where production involves just one input. Profit could have been written in terms of labor usage as $\Pi = Pq(L) - wL$. The ideal level of labor is that which maximizes profit, which can be determined by differentiating the profit function with respect to L and setting the result equal to zero. This yields: $P (dQ/dL) – w = 0$. The first order condition implies that the firm will hire labor up to the point where its marginal revenue product, P (dQ/dL) equals the wage rate (w). L* can be characterized using the implicit function theorem as L* = g(P, w) and the output associated with that choice characterized using the production function, as with Q* = q(L*). Given the variables in L*, Q* can be written as Q* = s(P, w) as before.

Unfortunately, the very clean and general derivation can only be undertaken for one input functions. Nonetheless, the result that firms hire inputs up to the point where their marginal product times the price of the output equals the price of the input, is quite general and a very useful rule of thumb to keep in one's mind. The longer somewhat more complex calculations undertaken first are similar to those that are taken to derive a firm's total cost function when it has a multiple input production function.

III. The Geometry of Production with More than One Input

The geometry used to model production choices is similar to that used to characterize consumer choices among goods and services. However, instead of attempting to maximize utility given a budget constraint, firms attempt to maximize production for given levels of expenditures on inputs. One difference between consumer and firm choices is the assumption that a firm's expenditures are in a sense unlimited because of access to personal wealth capital markets and so can choose the expenditure level to undertake. That will normally be the one that produces the output that maximizes profits as characterized in chapter 3. However, to determine this output levels requires that the firm's cost function be determined.

Figure 4.1 illustrates a firm's choice of inputs for three possible expenditure levels and desired outputs. The C-shaped curves are referred to as isoquants, each curve represents how a various combinations of inputs that could be used to produce a single output (an iso (single) quantity (quant) of output). The two inputs are usually capital (equipment) and labor, although others may be more relevant for the product of interest. And, as in geometric representations of consumer choices using utility functions, we are limited to production processes with two inputs because pages and computer screens are two dimensional. The straight lines resemble budget constraints for the simple reason that they also represent expenditures on two goods (L and K) at given prices (w and r). In this case, they are inputs that can be used to produce a product or service for sale in markets. Each iso-cost line characterizes the various combinations of inputs that could be purchased with a given expenditure level (total cost). Firms naturally attempt to minimize the cost of producing each possible output level and each output-expenditure combination characterizes a point on its total cost curve or function. This occurs at tangency points between the isocost and isoquant lines, three of which are illustrated. There is a total cost (total expenditure on inputs) and an output level at each point of tangency. Together they describe how the firm's production process and costs change as output expands.

Figure 4.1 Relationship between Output, Cost and Input Mix for a Price-Taking Firm

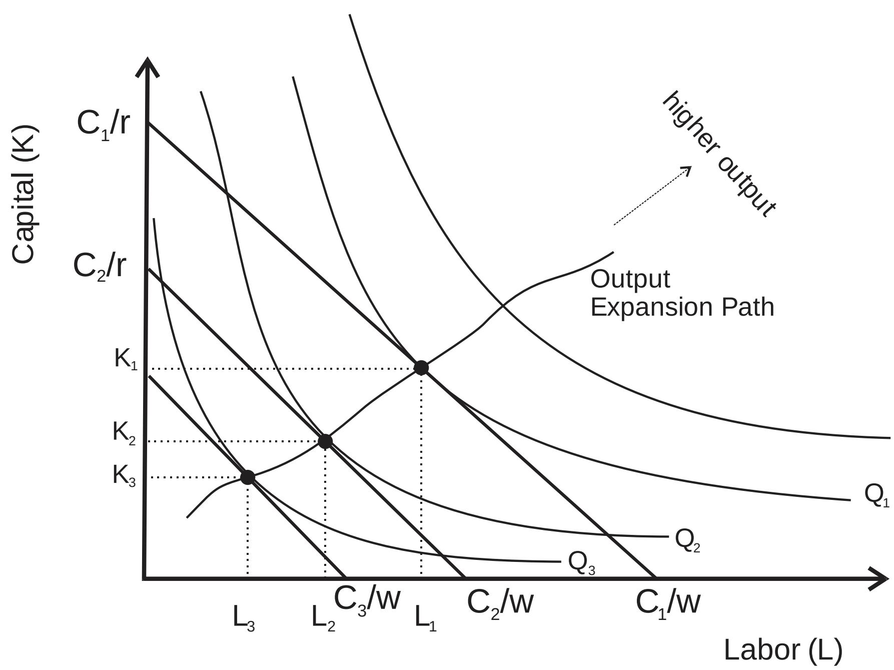

Figure 4.1 thus shows in principle how firms produce their products when they use a particular technology or combination of technologies for production. No innovation is being represented. They face particular input prices (w, wages, and r, the rental cost of capital). If any of these parameters of a firm's choice setting changes, the various combinations of inputs used to produce its output will change and so will its cost curve and cost function. For example, with the assumed C-shaped isoquants, firms will use more capital and less labor when the price of labor increases, and their total and marginal cost for production will tend to increase. The latter is based on intuition rather than anything that can be deduced directly from the diagram, because the iso cost curves, as labelled, use only abstract values for cost (C_1, C_2, C_3) and quantity (Q_1, Q_2, and Q_3) rather than particular numbers (100, 75, 50). If we knew the numbers associated with the iso cost curves and isoquants, we could have more precisely characterized total cost and marginal cost. This would require knowing both the precise production function used and input prices.

Nonetheless, the qualitative results generated by diagrams similar to figure 4.1, provide a number of insights. Profit maximizing firms produce with input mixes that equate marginal rates of technical substitution (the slope of an isoquant) with their relative prices (the slope of an iso cost line). As output expands, the mix of inputs employed may change. More capital goods, for example, might be employed. On the other hand, the general impression of such diagrams is that input use is roughly proportional to output.

The use of more capital-intensive methods requires somewhat counter intuitive shapes and locations for the isoquants—but ones that can be drawn. It turned out that in the nineteenth century, such curves were associated with the production of a number of products—as with the production of steel, many chemicals, and electricity. Thus special cases of this geometry are often relevant if one want's to understand the types of economic organizations that were becoming commonplace in that century and the next. Older small businesses, tended to have more linear output expansion paths, where all inputs increased roughly in proportion to output. Thus the production decisions of both large and small firms could be described with the same profit-maximizing model of firms or firm owners. Both were consequences of the same general types of choice settings.

The distinction between the long run and short run adjustments can also be explained by the same model. If one (or more) of the factors of production is fixed in the short run, the cost of increasing output tends to be higher than when it is not. Normally the factor that is most time-consuming to adjust in the short run is capital. Erecting new factories and equipping then using takes longer than adding a few more people to the production process. Although it should be noted that some types of labor may also take a long time to change—as with some types of firm specific human capital and others that require long training periods, such as doctors, lawyers, and economists. In the usual case, this means that short run supply decisions cannot use capital-labor substitution to economize on rising labor costs in the short run, although they can do so in the long run. The second less common problem of fixed labor can be represented in a similar way. Both problems for firms imply that reaching a higher isoquant will require greater expenditures on inputs in the short than in the long run. Thus, short run marginal cost curves tend to steeper than long run marginal cost firms.

Diagrams of this sort can also be used to characterize production functions that are actually quite common, but that do not lend themselves to the tools of calculus. Namely production processes in which discontinuities or impossibilities of substitution occur. One case occurs when only particular mixes of inputs can be used to produce the desired output. Bicycles, for example, always have two wheels (here partly by definition, but also partly because of the physics of riding them). If the price of wheels or hubs changes, no substitution (holding quality constant) of, say, pedals for wheels is possible. Each bicycle needs of each. The isoquants for such products tend to be L-shaped rather than C-shaped and production takes place at the kink (where the vertical of the L intersects with the horizontal part). Others possible uses of inputs are wasteful and would be avoided since they do not increase the output of the final good (bicycles). The discontinuity in the slope of the isoquant at the "corner" of the L limits the extent to which the optimization methods of calculus can be employed to account for such production.

IV. The Calculus of Production Using Concrete Production Functions

Just as the geometric representation of a firm's production decisions is similar to that used for modeling consumer choices. The calculus-based model of a firms production decision is very similar to that of consumer theory. Even the family of functions focused on tend to be similar. Cobb-Douglas and multiplicative exponential functions are among the most common families of functions used. Unfortunately, the results are more somewhat more difficult to derive and more complex for production than they are for consumption. Nonetheless, the results cast useful light on the factors that influence market supply and often produce results that can be estimated using the conventional statistical methods of econometrics.

As in Figure 4.1, we'll again begin by focusing on a two-input production process, where the inputs are labor and capital. The cost of outputs produced in that way in a setting in which firms are price takers in the input market is simply $C = wL + rK$ where w is the cost of labor (L) and r is the cost of capital (K). As in the geometric case, the cost depends on factor prices (w and r) and the production function used to produce the products to be sold. Suppose also that the firm's output from the use of labor and capital is $Q = L^eK^f$, where both e and f are greater than 0, but less than 1.

The "technology" of production, in principle, affects the exponents or functional form of the production function, which for the firm of interest is presently from the family of exponential multiplicative functions, e.g. $Q = L^eK^f$. It, like the prices of inputs and outputs, is regarded to be constant during the period of interest (sometimes called the planning horizon) and outside the control of the firm. L and K are the firm's control factors. The other terms (w, r, t) are parameters of the choice setting.

That the exponents are assumed to be less than 1 and greater than zero implies that the each factor of production contributes to output but also exhibits diminishing marginal returns. If their sum is less than 1, this implies that the overall production process also exhibits diminishing marginal returns and the associated marginal cost function will be upward sloping—as laboriously developed below. If their sum is exactly 1, this would be a **Cobb-Douglas** production functions, and the overall production process would exhibits constant returns to scale.

The firm's cost function can be derived either by minimizing the cost of a given output or by maximizing the output achieved for a given cost. Each of these approaches can be regarded as the "**dual**" of the other.

We'll first use the maximizing output for a given cost approach, because this is similar to that used for deriving consumer demand and is, in some sense, the "natural" way to characterize a firm's cost function. Unfortunately, the results are more difficult to derive in this way and more difficult to interpret than the less intuitive solution to the dual problem—as we will see. Both solutions obtained are similar to consumer problem above, in that they characterized a firm's demand for inputs for a given overall expenditure on inputs, although the objective function that we are modelling at this point is output or production (Q), rather than utility.

Maximizing Output for a Given Cost

The Lagrange approach is a bit easier than the substitution method in this case, because the assumed objective function is a multiplicative-exponential function. (However, as usual, the individual equations are more difficult to interpret than those obtained using the substitution method.) Since L is begin used to characterize the quantity of labor used in production, we'll use a "script L" for the symbol representing the Lagrange function ($\mathscr{L}$).

The Lagrangian equation associated with maximizing the output achieved at a given cost is

$$\mathscr{L} = L^e K^f + \lambda(C - wL + rK) \quad (4.1)$$

As in the consumer choice model, there are 3 first order conditions, two with respect to the control variables (L and K) and one with respect to the Lagrangian multiplier. (There would be more first order conditions if there were more control variables as with 2 kinds of labor or more constraints.)

$$d\mathscr{L}/dL = eL^{e-1}K^f - \lambda(w) = 0 \quad (4.2)$$

$$d\mathscr{L}/dL = fL^e K^{f-1} - \lambda(r) = 0 \quad (4.3)$$

$d\mathscr{L}/d\lambda = (C - wL + rK) = 0$ (4.4)

To find the input demands of the firm, we follow the same steps as in the previous consumer constrained optimization problems: shift the lambda terms to the righthand side of the equations and divide one equation by the other to generate:

$eL^{e-1}K^f/fL^eK^{f-1} = w/r$

Which simplifies to:

$eK/fL = w/r$

If we want the firm's demand for labor, we solve this equation for K and then substitute that result into the constraint (the derivative of the lambda term, equation 4.4)

$K = (w/r)\ (f/e)\ L$

Thus, $C = wL + r\ (w/r)\ (f/e)\ L$

Reversing the sides and factoring yields:

$L\ (w + w(f/e)) = Lw(1+f/e) = C$

Solving for L yields an expression for the firm's demand for labor:

$L^* = (C/w)(1/(1+f/e)) = [e/(f+e)]\ [C/w]$ (4.5)

Notice that this expression looks just like the expression that we found for the consumer demand function, but in this case, it characterizes this firm's demand for labor for a given expenditure on inputs (C). A similar result can be obtained for the firms demand for capital.

$K^* = [f/(f+e)]\ [C/r]$ (4.6)

Notice also that the pattern of input demand is determined by the relative productivity of the inputs (as reflected in their respective exponents), the amount that the firm plans to spend on all of its inputs (C), and input prices (the wage rate (w) and the cost of capital (r)).

However, the cost function we need describes costs in terms of outputs. What we have at this point is the ability to describe outputs in terms of expenditures on inputs. If we know how much money is spent on all inputs, we also know how many of each of the inputs are employed. This allows us to determine how much output is produced using the production function. This can be determined by substituting the ideal input quantities into the production function. Recall that the firm's output is $Q = L^eK^f$

Our two input demand functions allow the firms output to written as a function of production costs by substitute the two input demand functions into the production function:

$$Q = \{[e/(f+e)]\ [C/w]\}^e\ \{[f/(f+e)]\ [C/r]\}^f$$

This characterizes output in terms of overall expenditures on inputs, their productivity (as characterized by the exponents) and input prices. This expression can be solved for C (the cost or expenditures on inputs). Begin by factoring it out of the righthand side expression.

$$Q = C^{e+f}\ \{[e/(f+e)]\ [1/w]\}^e\ \{[f/(f+e)]\ [1/r]\}^f$$

Next solve for C as a function Q and the other parameters of the choice setting. This allows Cost (C) to be written as a function of output (Q), which is what we need for a total cost of production function.

$$C^{e+f} = Q\ /\{[e/(f+e)]\ [1/w]\}^e\ \{[f/(f+e)]\ [1/r]\}^f$$

Now take he e+f root of both sides to characterize total cost as a function of output levels, technology (represented here as the exponents), and input prices.

$$C^* = \{Q\ /\{[e/(f+e)]\ [1/w]\}^e\ \{[f/(f+e)]\ [1/r]\}^f\ \}^{1/(e+f)} \qquad (4.7)$$

This is one of the possible characterizations of C*, and the firm's cost function.

An Alternative Derivation: Minimizing the Cost of Given Outputs

Another way to derive a firm's cost function is to use the "dual" of the firm's optimization problem. In some cases, this yields a cleaner and more direct result. The "dual" choice problem requires us to minimize cost (expenditures on inputs) subject to producing a given output Q. Essentially, the dual just reverses the objective function and constraint. The new Lagrangian function is:

$$\mathscr{L} = \ - wL + rK + \lambda(Q - L^eK^f) \qquad (4.8)$$

(I've again used a script L ($\mathscr{L}$) for the Lagrangian equation, because L is being used for the quantity of labor employed producing the good of interest.)

Again there are 3 first order conditions (first derivatives begin set equal to zero), two with respect to the control variables (L and K), and one with respect to the Lagrangian multiplier. The first two are very similar to those we derived before, but the last is quite different.

$$d\mathscr{L}/dL = w - \lambda(eL^{e-1}K^{f}) = 0 \quad (4.9)$$

$$d\mathscr{L}/dL = r - \lambda(fL^{e}K^{f-1}) = 0 \quad (4.10)$$

$$d\mathscr{L}/d\lambda = (Q - L^{e}K^{f}) = 0 \quad (4.11)$$

Notice that the only major difference in the first order conditions is the derivative with respect to the Lagrangian multiplier, λ.

To derive the firm's total cost function, very similar steps are undertaken to those in the previous derivation, but in this case, solutions will be in terms of output (Q) rather than expenditures on inputs (C).

Shifting the lambda terms in the first equations to the right and dividing yields:

$$w/r = eK/fL$$

which geometrically can be interpreted as the tangency condition(s) of figure 4.1. If we again focus on labor initially, we want to specify capital in terms of labor, which is

$$K^{*} = (fw/er)\ L$$

Substituting this into the production function and solving for L, again takes a few steps:

$$Q = L^{e}K^{f} = L^{e}[(fw/er)\ L]^{f}$$

L can be factored out of the righthand expression:

$$Q = L^{e+f}(fw/er)^{f}$$

We can then solve for L* in terms of Q:

$$L^{*} = [Q\ (er/fw)^{f}\ ]^{1/e+f} \quad (4.12)$$

Recall that $(x/y)^{-e} = (y/x)^{e}$, thus the ratio inside the brackets "flips" as one derives L*. This characterizes the **demand for labor as a function of output**, productivity (again indicated by the exponents) and the price of labor and capital (w and r).

We can solve for K* in a similar way. Isolating the L (instead of K) yields:

$w/r = eK/fL$

which yields

$L = (e/f)K\ (r/w)$

Substituting this into the constraint yields:

$Q = L^eK^f = [(e/f)K\ (r/w)]^e K^f$

The K can be factored out:

$Q = K^{f+e}\ [(er/fw)]^e$

Solving for K yields

$$K^* = [Q\ (fw/er)^e\ ]^{1/f+e} \qquad (4.13)$$

This characterization of K* is the **firm's demand for capital as a function of output**, input prices, and their productivities. The cost function can now be written in terms of the optimal quantity of labor and capital for various quantities of output:

$$C = wL^* + rK^* = w\ [Q\ (er/fw)^f\ ]^{1/e+f} + r\ [Q\ (fw/er)^e\ ]^{1/f+e} \qquad (4.14)$$

This is another, somewhat more intuitive expression for the firm's total cost of production. Note that the first term is the firm's expenditure on labor and the second is the firm's expenditure on capital used in production in the **optimal amounts** for the output quantity of interest. Note also that it is a simpler expression than the first way, although they should be mathematically equivalent as long as we've made no algebraic errors. In both cases, production costs vary with technology (the size of the exponents) and input prices. Costs clearly rise with input prices (recall that the exponents are less than 1), and costs tend to fall as the sum of the exponents fall.

Both derivations of the firms total cost function have characterized **long run total costs**, because the firm has been assumed to be able to vary all of its inputs.

Connecting Up the Theory of Production with the Theory of Supply

Equation 4.14 can be used this firm's supply curve. Before doing so, it will be useful to simplify the notation a bit by grouping terms and naming the groups. (This reduces the chance that a term will be dropped during the derivation.) Define, term m^L as $m^L = (er/fw)^f$, $m^K = (fw/er)^e$, and define term α as $\alpha = 1/(f+e)$. These two groupings allow the cost function (equation 4.14) to be written as

$$C = w\,(Qm^{L})^{\alpha} + r\,(Qm^{K})^{\alpha}$$

These "new" variables do not change when we calculate profit maximizing outputs, since they do not include quantity as a variable—but they would change if wages, capital rental costs or technology change. The simpler notation reduces the likelihood of algebraic mistakes in deriving the supply curve. After our derivation of the supply curve is complete we can substitute the "real" expressions behind the three new terms back into the equation worked out to see how these variables affect the firm's supply decision.

The firm's profit maximizing output is calculated in the same manner as in chapter 3. Profit is total revenue (PQ) less total cost, now written as $C = w\,(Qm^{L})^{\alpha} + r\,(Qm^{K})^{\alpha}$.

$$\Pi = PQ - w\,(Qm^{L})^{\alpha} - r\,(Qm^{K})^{\alpha} = PQ - wQ^{\alpha}(m^{L})^{\alpha} - rQ^{\alpha}(m^{K})^{\alpha} \qquad (4.15)$$

Differentiating with respect to Q yields:

$$P - \alpha\, wQ^{\alpha-1}(m^{L})^{\alpha} - \alpha\, rQ^{\alpha-1}(m^{K})^{\alpha} = 0 \quad \text{at } Q^{*}$$

The first term (P) is marginal revenue, the others are the firm's marginal cost. The individual terms show the part of marginal cost attributable to labor costs and to capital costs. Keep in mind that we **have derived long run total cost** rather than short run marginal cost, because we are assuming that both labor and capital can be varied in the period of interest. So, this first order condition characterizes the firm's long run profit maximizing output. Short run cost and supply would be derived by holding the quantity of capital or some other input(s) constant, which would be quite a bit simpler in the two-input production case.

One can solve for Q* (the profit maximizing output) by shifting P to the righthand side, multiply both by negative 1 and factoring.

$$\alpha\, wQ_{\alpha-1}(m_{L})_{\alpha} + \alpha\, rQ_{\alpha-1}(m_{K})_{\alpha} = P$$

$$Q^{\alpha-1}\,[\alpha\, w\,(m^{L})^{\alpha} + \alpha\, r\,(m^{K})^{\alpha}] = P$$

$$Q^{\alpha-1} = P\,/[\alpha\, w\,(m^{L})^{\alpha} + \alpha\, r\,(m^{K})^{\alpha}]$$

$$Q^{*} = \{\, P\,/[\alpha\, w\,(m^{L})^{\alpha} + \alpha\, r\,(m^{K})^{\alpha}]\}^{1/(\alpha-1)} \qquad (4.16)$$

Equation 4.16 is the firm's long run supply function. Notice that this **firm's long run supply curve** is upward sloping in price and the quantity supplied at every price tends to fall as input prices rise (e.g. if either w or r increase—although fully determining this requires checking the derivatives of m^L and m^K to know for sure.

If there are M firms in the market with similar cost functions, then the market supply function (or curve) is simply M times that of the typical or average firm, which is

$$Q^S = MQ^* = M \{ P / [\alpha w (m^L)^\alpha + \alpha r (m^K)^\alpha]\}^{1/(\alpha-1)} \quad (4.17)$$

As before, if firms are not identical or very similar, market supply requires adding up the supply functions of each firm, rather than simply multiplying one of the supply curves by the number of firms in the market. The assumption that suppliers have identical cost functions is sometimes called the Marshallian assumption about competitive markets, as previously mentioned.

V. Production Models with More General Families of Functions

As true of many areas of economics, using more abstract families of functions to ground one's model often makes deriving implications of a particular type of choice setting easier and the results more general. This is true of the theory of production. The easiest applications of these methods are often two- or three-dimensional problems in which there is only a single "degree of freedom" because of the effects of constraints. But, the general approach can be used for any number of variables—its just that in those cases, matrix methods for derivatives need to be employed and these are rarely used in economic research because the results are often very complex and signs obtained are ambiguous—not that ambiguity is never of interest.

When applied to production, one can begin with the two-input case. A general family of production functions is Q=q(L, K) with positive first derivatives, negative second derivatives and positive cross partials. Given this, we can create a "zero function":

$$Q = q(L, (C-wL)/r) \quad (4.18)$$

Differentiating with respect to L to characterize the firm's ideal use of labor gives us:

$$\frac{dQ}{dL} = \frac{dQ}{dL} - \frac{dQ}{dK}\left(-\frac{wL}{r}\right) = 0 \quad (4.19)$$

We can apply the implicit function differentiation rule to solve for any variable in terms of every other variable in the equation. Each of the partial derivatives of function q includes the same variables as the parent function, so this is a case where h(L*, C, w, r) = 0. We'll solve for L*.

$$L^* = l(C, w, r) \qquad (4.20)$$

Once we know the ideal quantity of labor for a given expenditure (C) on inputs, we can substitute that back into the production function to determine the maximum output produced for a given expenditure and input prices.

$$Q^* = q(L^*, (C - wL^*)/r) \qquad (4.21)$$

Notice that if we subtract either side from the other we get another zero function.

$$Q^* - q(L^*, (C - wL^*)/r) = 0 \qquad (4.22)$$

We can call on the implicit function theorem again. In this case our zero function is of the form h(Q,C,r, w) = 0, which we can solve for C:

$$C = c(Q, w, r) \qquad (4.23)$$

This is the firm of interest's long run total cost function. Notice that the cost functions for the multiplicative exponential family of production functions took this form, but included indicators for technology. To include technology using this approach, t would simply be included as an exogenous variable in the production function.

VI. References

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