Chapter 5: Market Clearing Prices

I. Introduction: The Neoclassical Synthesis

In the previous three chapters, it has been shown how rational choice models and methodological individualism can be used to characterize consumer choices about purchases of goods and services, firm decision about how much to produce, and firm decisions about how to produce the things and services that are intended to be sold in markets. All these types of choices can be modelled using geometry, specific functions, and quite general abstract mathematical characterizations of the relevant choice settings. In this chapter, implications from those models are used to develop the neoclassical theory of prices.

That theory is less a theory of price determination than a theory of market-clearing prices. The models that have long been employed to characterize market equilibria demonstrate that equilibrium prices exist—that is to say that prices exist that induce firms and consumers to, in a sense, coordinate their decisions so that the quantities of goods and services brought to market by firms exactly equals that demanded by consumers—although the decision are all independently made and advance the private interests of those making them.

Models of equilibrium prices that connect up the choices of consumers and firms is sometimes referred to as the neoclassical synthesis. In the period just before and after World War II, various mathematical models of consumer and proprietor behavior had been developed in models that were more or less compatible with one another and so could be integrated into models of markets of price determination in both output and input markets. Although, mathematical modeling was not the most commonplace mode of economic analysis at that time (closely reasoned verbal accounts were more commonplace then), the economists who used mathematical models demonstrated how they could be combined to provide an internally consistent and rigorous characterization of equilibria in both individual markets and in entire commercial networks. Those more integrated models were regarded to be major breakthroughs in economic understanding. They also undercut, at least to some extent, the various critiques of neoclassical economics that had been developed in the past half century or so (including many from Marxist economics).

It bears keeping in mind that the models make a variety of implicit assumptions. For example, for the most part, they implicitly focused on routine choices—that is to say, choices with respect to well-understood goods and services that are produced by methods that are also well-understood. Such choices are in a sense perfectly informed because those making relevant choices have completely accurate expectations about the nature of the goods and services at issue, their normal prices, and their manner of production. Not all choices are routine in this sense, but the models worked out clearly demonstrated that fundamental relationships exist between consumer preferences, production methods (technology), input prices, and the prices of final goods and services.

Also, although the models demonstrate that equilibrium prices exist, they do not demonstrate that they necessarily emerge. Indeed, Arrow and Hurowitz (xxxx) demonstrated that the Walrasian tantamount process often imagined to do so, does not always yield equilibrium prices.

It should also be acknowledged that the neoclassical synthesis predicts a narrower band of prices than actually observed in practice—the so-called law of one price. This is, course, because the models abstract from informational problems that might account for the variation in prices. As a well-integrated and sophisticated models, the neoclassical synthesis does not attempt to explain every possible event in an economy, only the ones that are commonplace or that tend to be commonplace. As models, they necessarily abstract from many details of economic life in order to illuminate the most important relationships might be worked out. Much can be abstracted from without reaching mistaken conclusions about the tendencies of consumer choices, firm decisions, or nature of equilibrium prices—although as will be seen in parts II and III, bringing more factors into the models often improved our understanding of how markets operate.

It should also be acknowledged that lack of realisism—in the sense of departures from realism—is sometimes generated by the assumptions required to make the mathematics tractable—which is to say able to characterize choices and outcomes. The need for assumptions simply to allow mathematics to be applied, may account for the limited acceptance of mathematical models among economists in the first half of the twentieth century—and they remains somewhat controversial among a subset of economists today.

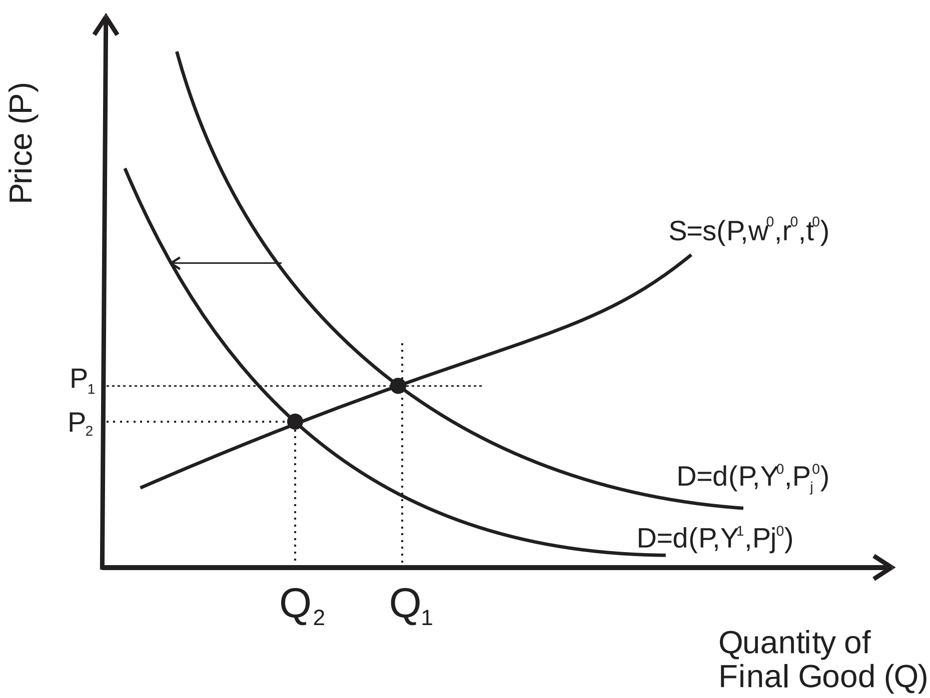
Nonetheless, by rigorously illustrating that equilibrium prices exist and affect (coordinate) the decisions of consumers and producers, neoclassical models demonstrate that an internally consistent theory of prices is possible. Verbal and geometric illustrations had been undertaken before, as in Marshall's textbooks and in others that followed his lead. The mathematical models demonstrated that earlier intuitive and geometric claims about equilibrium were not logically incoherent or products of ideology. A theory of equilibrium prices could be grounded in plausible mathematical characterizations of market-relevant choices that produced logically rigorous and clear implications about how prices are determined in in well-functioning market networks.

II. The Geometry of Supply and Demand

Perhaps surprising, the mathematical models developed in chapters 2, 3, and 4 deepen our understanding of the geometric models than possible without them. For example, they show clearly why variables other than price affect consumer and firm choices. Consumer income, the prices of other goods and what are sometimes called "tastes" (e.g. the shapes and arguments of of utility functions) all affect the location and shape of individual demand curves and thus also market demand curves. Similarly, the prices of inputs and technology affect the location and shape of a firm's supply curve through effects on decisions about how and what to produce products for market. The same factors are relevant for both short and long run decisions. When demand and supply curves are drawn based on intuition, it is not clear what is determining the location and shape of those curves.

Figure 5.1 illustrates the market equilibrium for a final good taking into account the insights generated by the mathematics in the previous three chapters. The improvements include (i) explicitly labeling the demand and supply curves as functions with other variables being held constant (e.g. taking given values, as denoted with the "0" superscripts). (2) Making them somewhat irregular curves rather than straight lines to reflect the more general shapes implied by the mathematical models, and (3) having the market supply curve slope upwards at a somewhat increasing rate, reflecting diminishing marginal returns (at least in Ricardian markets). Figure 5.1 also illustrates on of the more powerful implications of the supply and demand model of price determination—its ability to be used to explain the effects of changes in the variables being held constant. In the case illustrated, average consumer income in the market of interest falls ($Y^1<Y^0$), with the result that the demand curve shifts back to the left (not "down"), because the quantities purchased by the typical consumer will fall if the good in question is a normal good (and most goods are).

Figure 5.1 Equilibrium Prices In a Competitive Market for a Final Good or Service (Effect of reduced average consumer income)

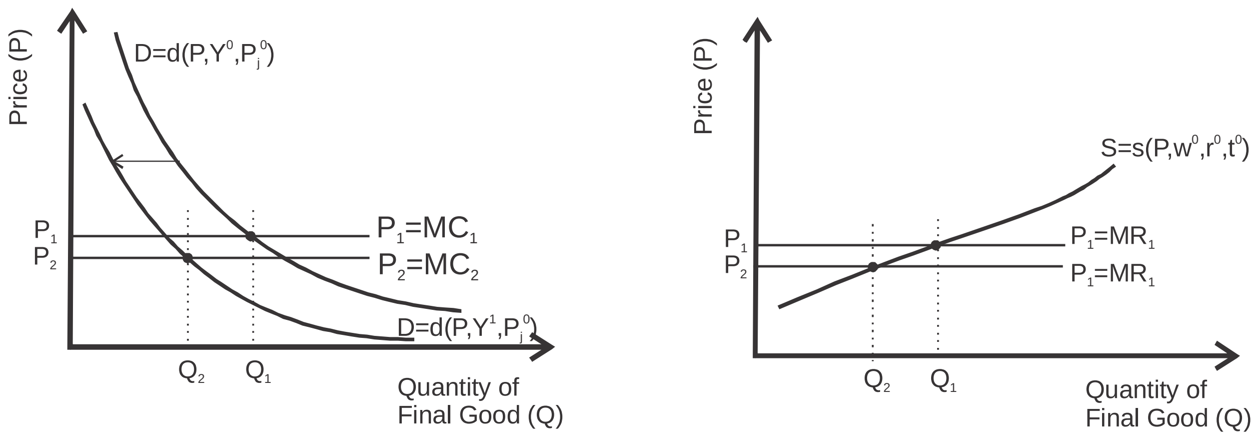


To show how markets affect individual firms and consumers, one simply takes the price information from the market diagram and incorporates them into the individual decision diagrams, which underlie the market demand and supply curves. Such diagrams show that when prices change, individual's change their behavior. They also show how factors being held constant to draw a particular pair of supply and demand curves influence the location of such curves. If one or more of those other factors change, individual consumer and firm behavior changes and thus (if enough of them do) the market demand and supply curves (noticeably) change as well.

Figure 5.2 shows how changes in the price of a final good affect a typical consumer and firm. (The quantity scales have been adjusted to make the diagrams look similar—although an individual firm normally produces far more of the goods sold than a single consumer purchases.) The (P_1,Q_1) equilibria, illustrate the original purchasing and production decisions of the consumer and firm respectively. Each adopts strategies that advance their self interest (as it is understood by the individual and firm or firm owner) as much as possible given the prevailing market price (P_1). Consumers purchase the quantities of goods that maximizing their utility or net benefits given their market price, holding other aspects of their choice setting constant. Firms maximize their profits (here long run profits) given the prevailing price, holding technology and the prices of inputs constant. If average consumer income declines in the local market of interest, a typical person's demand curve shifts back to the left and purchases decline—although because prices fall, not quite as much as would have been induced by the reduction in income alone. Suppliers face lower prices and so produce less and profit less.

Changes in the demand for inputs are not shown, but they would fall with decreases in production. Fewer inputs would be devoted to producing the goods sold in this regional or local market. The assumption that the individual firm's supply curve does not shift implies that its input prices for the relevant firms were not affected by what ever economic shock caused average consumer income in this market to decline. This would be the case, for example, if the products sold were produced in another region that was not affected by the income reducing shock. But, the other case could be modelled as well.

Figure 5.2 Individual Consumer and Firm Adjustments
To changing Market Conditions



The pattern of adjustments by firms and consumers are exactly those necessary (in the ideal theoretical case or case modeled) to set market demand equal to market supply in the new circumstances with lower average consumer income. This is Adam Smith's invisible hand and Hayek's price-coordinating signal operating in a partial equilibrium setting.

The math-informed geometry can be extended to characterize different kinds of markets, such as cases in which products are locally produced and an employment shock reduces demand and the cost of labor inputs, or cases in which products are purchased from international vendors rather than local ones, and so forth. Such variations in market type will affect the slope of the supply curve that consumers in a locale can purchase goods and services from.

III. Abstract Characterizations of Market Clearing Prices and Comparative Statics

The implicit function theorem can be used to characterize equilibrium prices in terms of the exogenous parameters of the choice settings faced by the typical consumer and firm. To see this, consider the demand function, D=d(P,Y,Pj) and the supply function S=s(P,w,r, t) used in figure 5.1. At the market clearing price we know that D=S, thus we also know that

$$d(P^*,Y,Pj)-s(P^*,w,r,t) = 0 \equiv H \text{ at } P^*, \qquad (5.1)$$

the market clearing price. Given a "zero equation" and assuming that its differentiable, we can use the implicit function theorem to characterize any of the variables in the zero equation, and use derivatives of the H function to characterize the comparative statics of equilibrium prices in that market.

The implicit function theorem implies that we can write P* as a function of the other variables in the demand and supply functions.

$$P^* = f(Y, Pj, w, r, t) \tag{5.2}$$

In econometric terms, equation 5.2 is the reduced form equation for the equilibrium price in the market being analyzed (that of figure 5.1). Comparative statics can be calculated and signed by applying the implicit function differentiation rule. For example, if we want to determine the effect of an increase in average consumer income, the implicit function differentiation rule implies that it is:

$$\frac{dP^*}{dY} = \frac{\frac{dH}{dY}}{-\left(\frac{dH}{dP^*}\right)} = (\frac{dD}{dY})/-[\left(\frac{dD}{dP^*}\right) - \left(\frac{dS}{dP^*}\right)] \tag{5.3}$$

We have assumed that the good in question is a normal good, so we know (by assumption) that the numerator is positive ($\frac{dD}{dY} > 0$). The denominator is more complicated. The demand curve in figure 5.1 is downward sloping, so we know that $\frac{dD}{dP^*} < 0$. The supply curve is upward sloping, so $\frac{dS}{dP^*} > 0$. Notice that this implies that the term insides the brackets is less than zero. The negative sign in front of that term implies that it has the opposite sign. Thus, the sign of the numerator is positive and the sign of the denominator is also positive, so we know that the predicted effect of an increase in average consumer income on market prices. Whenever demand and supply functions have the usual characteristics (e.g. the ones demonstrated in chapters 2 and 3), an increase in average consumer income will cause higher prices in markets for final goods.[1]

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[1] There are, it should be noted, exceptions to this rule. The models developed have all assumed diminishing marginal returns in production (at the margin) which implied upward sloping market supply curves in both the short and long run. Marshall's characterization of long run supply as generated by the entry of and exist of identical efficient sized firms would imply a horizontal long run supply function, unless variations in supply affected input prices. If they did so, the result above

Similar steps can be undertaken to determine the predicted results for changes in the prices of substitutes (Pj) or in the input prices (w and r) or improvements in technology (T). It turns out that the comparative statics of diagrams grounded in the mathematics of the last three chapters is quite general—but this is only known for sure because of the mathematics just undertaken.

IV. Algebraically Solving for Market Equilibria with Concrete Functions

Given specific forms for a market's demand and supply curves, one can often characterize the equilibrium in ways that provide equations that can be estimated using conventional econometric methods. These include reduced form types of results like those in the previous section and structural equations of the demand and supply variety. In many cases, this is a very straight forward exercise in algebra.

For example, consider linear forms of the supply and demand curves in figure 5.1 such as $D=-aP+bY-cPj$ and $S = eP - fw - gr + ht$, where a, b, c, e, f, g and h are constants. In the case where unbiased estimates of these supply and demand curves has been undertaken, instead of letters, the coefficients would be particular numbers. The algebra is essentially the same in either case. To begin with, P* has the property (by definition) of setting demand equal to supply. This implies that

$$-aP^* + bY - cPj = eP^* - fw - gr + ht \qquad (5.4)$$

To solve this equation for P*, shift the P* terms to the lefthand side of the equation and the other terms to the lefthand side.

$$-aP^* - eP^* = -bY + cPj - fw - gr + ht$$

Add up the terms on the left and then divide by the implied coefficient to find:

$$P^* = (-bY + cPj - fw - gr + ht)/-(a+e) \qquad (5.5)$$

Notice that this reduced form equation for prices is linear in average consumer income, input prices, and prices of other relevant final goods. Comparative statics can be calculated

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would be associated with Marshallian long run supply as well as the more Ricardian approach used here.

by differentiating this equating for equilibrium prices by the variable of interest. For example, the effect of an increase in income is:

$$\frac{dP^*}{dY} = (-\frac{b}{-(a+e)}) = b/(a+e) > 0 \tag{5.6}$$

This is the rate at which prices increase as average consumer income increases.

The disadvantage the two derivations above is that they assume that there is an "average" consumer or that all consumers are essentially identical. We know from personal experience that this is not true. The products carried to the checkout kiosks at department stores vary widely, as do the products placed in shopping baskets in grocery stores. Differences in preferences, income, and in the effects of prices of other goods vary widely. The same variation is evident in industries in which firms of various sizes coexist for decades at a time. Fortunately, the derivation of equilibrium prices is not limited to cases in which consumers and firms are essentially homogeneous. To see this, suppose that every consumer has a linear demand curve similar to that used in the previous example, but that each person's income differs and the intercept and coefficients differ as well, as with:

$$D_i = a_i P + b_i Y + c_i P_o \tag{5.7}$$

In this case the market demand curve is the sum of the N individual demand curves of each individual in this market. No two individuals need be the same.

$$D = \sum_{i=1}^{N}(a_i P + b_i Y + c_i P_o) \tag{5.8}$$

Similarly, suppose that each supplier in this market has a unique supply function because of variations in access to inputs, location, managerial talent, or technology.

$$S_j = e_j P - f_j w - g_j r + h_j t \tag{5.9}$$

Market supply is the sum of all these individual supply functions.

$$S = \sum_{j=1}^{M}(e_j P - f_j w - g_j r + h_j t) \tag{5.10}$$

The market clearing price, P*, sets D equal to S in this market or:

$$\sum_{i=1}^{N}(a_i P^* + b_i Y + c_i P_o) = \sum_{j=1}^{M}(e_j P^* - f_j w - g_j r + h_j t) \tag{5.11}$$

To characterize this price, we have to solve for P*, which requires isolating the terms with P* in them from the others.

$$\sum_{i=1}^{N}(a_i P^*) + \sum_{i=1}^{N}(b_i Y + c_i P_o) = \sum_{j=1}^{M}(e_j P^*) + \sum_{j=1}^{M}(-f_j w - g_j r + h_j t)$$

Shifting the P* terms to the lefthand side and the others to the right yields

$$P^*[\sum_{i=1}^{N}(a_i) - \sum_{j=1}^{M}(e_j)] = \sum_{j=1}^{M}(-f_j w - g_j r + h_j t) - \sum_{i=1}^{N}(b_i Y + c_i P_o)$$

Dividing to solve for P* yields:

$$P^* = \{\sum_{j=1}^{M}(-f_j w - g_j r + h_j t) - \sum_{i=1}^{N}(b_i Y + c_i P_o)\}/[\sum_{i=1}^{N}(a_i) - \sum_{j=1}^{M}(e_j)] \qquad (5.12)$$

Equation 5.12 characterizes the market clearing price for this very heterogeneous market in which consumers and firms may all differ—indeed they may live in different countries or continents. N be a number in the millions and M in the thousands. This single price, P*, affects each their choices. Although it is not the only factor that matters to them, as long as the others are reasonably stable, adjustments in the price of the final good induces all of these market participants to alter their behavior—not because they care about the others (although they may), but because it is in their interest to do so. Such changes in behavior increase their utility or profits. At the market clearing price, all these individuals and organizations have adopted patterns of behavior such that supply is exactly equal to demand—the market, whether a local or world-wide one "clears."

Of course, in the real world, economic factors are not the only ones that affect consumption and output decisions—there may be government policies that inhibit or encourage production. There may be taxes of various kinds and tariffs. But even with all of these factors, there is a tendency for market prices to clear the market of interest—although in this case, as will be demonstrated in part III, the persons involved may face somewhat different prices whose difference from world prices are determined by local government policies of various kinds.

Neoclassical economics tends to focus quite narrowly on purely economic factors, but others can be brought into the analysis.

V. Price Theory Between Monopolistic and Competitive Markets: the Spectrum of Cournot Models of Market Prices

There are three widely used models of duopoly: (1) *Cournot* (based on symmetric quantity competition), (2) *Bertrand* (based on symmetric price competition), and (3) *Stackelberg* (based on asymmetric quantity competition with a first and second mover). The Cournot duopoly model is the most widely used in economics textbooks and is where we'll focus on in this part of chapter 5. Its contribution to the present chapter is that it provides a theory of prices that lie between the price equal marginal cost solutions of perfectly competitive markets and the monopoly pricing model developed in chapter 2..

The Stackelberg model is less widely used, but the Stackelberg modeling structure is often used to model games in which players make decisions in a sequence of some kind, and chose their strategies in part because of anticipated reactions on the part of the other players. It is a useful method of modeling settings where anticipating the future decisions of a rival are an important determinant of a firm's current pricing and output decisions. The Bertrand model is less widely used, but should not be totally neglected, because it yields simple and direct predictions about pricing and output. Namely, there are settings in which duopoly can produce competitive prices.

In the **Cournot** model (sometimes called Cournot/Nash duopoly), two firms produce identical goods and make their *output* decisions independently of one another. Each firm takes the other's output given, and selects its own best output given that assumption given the *downward sloping market demand curve* for the product in question. Total output and market price are represented as equilibria to the "noncooperative" production game between the two firms. Best reply functions are used to characterize each firm's ideal output levels for given outputs of their rival. The equilibrium occurs when both (or all) firms are simultaneously on their best-reply functions—which geometrically can be represented as a point in the strategy space where all the best reply functions intersect one another. (As it turns out, Cournot actually invented this concept of Nash equilibrium well before Nash. In 1838.)

As an illustrating example of a Cournot duopoly setting, suppose that the market in question has a demand curve: $Q = 1000 - 10P$. We have not used many concrete functions in our models, and it is done here, because it facilitates comparison of equilibria prices and outputs and variations of the Cournot model. Assume that both firms are profit maximizers, sell identical products, and that profit is simply revenues from sales less the cost of producing the goods sold. For example, firm "A's" profit can be written as $\Pi^A = PQ^A - C^A$.

In order to know (or estimate) their profits they will have to know the market price. The market price depends on the total amount brought to the market by both firms, not simply that brought by firm "A." Demand curves "slope downward," which means that the more output is "brought to the market" the lower prices tend to be. Given our assumed demand curve, $Q = 1000 - 10P$, and the effect of total market output on market price can be written as: $P = 100 - 0.10^*Q$. (As mentioned before, this way of writing a demand curve is often called an inverse demand curve, because it goes from quantities into prices, rather than from prices into quantities.

If there are just two firms, A and B, then $Q = Q^A + Q^B$ and firm A profits can be written as:

$$\Pi^A = [\ 100 - 0.10^*(Q^A + Q^B)\ ]\ Q^A - C^A \qquad (5.13)$$

To simplify a bit more, let us also assume that the cost function is the same for each firm exhibits constant returns to scale and can be written as:

$$C = 5Q$$

which implies that the profit of firm A is simply $\Pi^A = [\ 100 - 0.10^*(Q^A + Q^B)\ ]\ Q^A - 5Q^A$

or (multiplying),

$$\Pi^A = 100 - 0.10(Q^A)^2 - 0.10Q^AQ^B - 5Q^A \qquad (5.14)$$

Notice that firm A's profit (his payoff in this game) is affected by the other player's output decision, as is typical in a game setting.

To find firm A's profit maximizing output, we need to find the "top" of the profit function, which can be found where the slope of the profit function is zero. Differentiating with respect to Q^a and setting the result equal to zero yields:

$$100 - 0.20\, Q^A - 0.10 Q^B - 5 = 0 \qquad (5.15)$$

Solving this "first order condition" for Q^A allows us to characterize the output level that maximizes firm A's profit for each output level that B might choose:

$$100 - 10 Q^B - 5 = 0.20\, Q^A$$

$$\text{or } Q^A = 475 - 0.50 Q^B \qquad (5.16)$$

This equation is **the best reply function** of firm A. It tells firm A how much to produce to maximize firm A's profits for every possible output level of firm B. There is no dominant pure strategy. The optimal response varies with the particular among brought to market by B.) A similar best reply function can be found for firm B (Do this as an exercise!)

$$Q^B = 475 - 0.50 Q^A \qquad (5.17)$$

At the Nash equilibrium, both firms will be on their best reply function--that is to say both firms will be maximizing their profits given the other firm's output. This implies that

$$Q^B = 475 - 0.50 Q^A \quad \text{and}$$

$$Q^A = 475 - 0.50 Q^B$$

are both simultaneously satisfied. Substituting for Q^B into Q^A allows us to find the Nash equilibrium levels of output for firm A.

$$Q^A = 475 - 0.50\,(475 - 0.50 Q^A)$$

Gathering the Q^A terms and a bit of arithmetic yields $0.75\, Q^A = 475/2$. Thus $Q^{A**} = (4/3)(475/2) = (950/3) = 316.6$

By symmetry, we also know that $Q^{B**} = 316.6$, which implies that total output, $Q^* = 633.3$, namely, the sum of the outputs of the two firms.[2]

Substituting back into the profit function allows us now to determine each firm's payoff:

The profit of firm A is simply $\Pi^A = [\, 100 - 0.10*(Q^{A*} + Q^{B*})\,]\, Q^{A*} - 5Q^{A*}$

or, substituting for the "Qs" we have $\Pi^A = [\, 100 - 0.10*(633.3)\,]\, 316.6 - 5(316.6)$

A bit of algebraic arithmetic yields:

$\Pi^A = (95)(316.6) - 63.3\ (316.6)$ or $(31.66)\ 316.6 = 10{,}023.556$

In equilibrium both firms are simultaneously maximizing profits and, because of the symmetry assumed, both will earn the same profits from their sales of identical quantities.

Geometrically the equilibrium occurs at the intersection of the two firm's best reply curves.

As a practice exercise, show that total output increases relative to the monopoly market and prices fall.

Recall that a monopolist will simply maximize profit, which in this case is:

$\Pi = [\, 100 - 0.10*(Q)\,]\, Q - 5Q$

Find the profit maximizing output for the monopolist by differentiating its profit function with respect to Q, setting the result equal to zero, and solving for Q. The optimal output for a monopolist facing the same demand curve is $Q^*=475$ which, of course is less than 633.3. So, we know that prices are lower in the Cournot duopoly case than in the monopolistic case.

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[2] Symmetry could also have been used to simplify the math a bit, by using the symmetry "trick" on either firm's best reply function. We could have simply replaced both quantities with Q^{**} and solved the resulting equation for Q^{**}. The longer derivation is done above as an illustration of how to do find the equilibrium if the firms are not effectively identical.

Cournot-Based Models of Industries with More Firms

The Cournot model can be generalized in a number of ways. For example, the inverse demand curve and cost function can be made more abstract and/or general. One can also extend the Cournot approach to characterize markets in which more than two firms interact. This extension shows that a continuum between monopoly and competitive markets may exist in a rather neat way. We'll use a bit more abstract model to characterize N-firm variations of Cournot competition.

Consider the case in which the inverse demand curve is linear: $P = a - bQ$, $C_i = cQ_i$, and there are *N* identical firms participating in the market. Each firm makes its decision independently of one another. (That is to say, there are no cartel-like meetings or coordination of output strategies). We'll focus on firm 1 and regard the output of the other firms to be Z to reduce the amount of notation we need to deal with.

Firm 1's profit in this case is:

$$\Pi_1 = a - b(Q_1+Z)\ ]\ Q_1 - c\ Q_1 \tag{5.17}$$

Differentiating with respect Q_1 yields:

$$a - 2bQ_1 - bZ - c = 0$$

This can be solved for Q_1 as a function of parameters of the demand function, the outputs of the other firms and firm 1's cost function. Shifting the Q_1 term to the right, allows a bit of algebraic arithmetic to yield:

$$Q_1^* = (a-bZ-c)/2b \tag{5.18}$$

This is firm 1's best reply function and similar functions can be derived for all the other firms as well.

The easiest way to find the Nash equilibrium is to assume that there is a symmetric equilibrium in which case all the firm outputs are the same. Simplifying $Q_1^* = Q_2^* = \ldots Q_N^*$ into the above yields:

$$Q_1^* = (a-b(n-1)Q_1^*-c)/2b$$

Adding b(n-1)Q*/2b to each side yields:

$$Q_1^* + b(n\text{-}1)Q_1^*/2b = (a\text{-}c)/2b \rightarrow 2bQ_1^* + b(n\text{-}1)Q_1^* = a\text{-}c$$

which implies that at the **symmetric Nash Equilibrium**:

$$Q_1^{**} = (a\text{-}c)/b(n+1) = [(a\text{-}c)/b]\ [1/(n+1)] \qquad (5/19)$$

Total market output is $n\ Q_1^{**} = [(a\text{-}c)/b]\ [n/(n+1)]$ (5.20)

Substituting this into the inverse demand curve, gives us the market price:

$$P^* = a - bQ = a - b\{[(a\text{-}c)/b]\ [n/(n+1)]\} = a - [(a - c)][n/(n+1)]$$

(Note that this formula works for the 2-firm case worked out above.)

Note also that as N approaches infinity n/(n+1) approaches 1 and P* approaches c, the marginal cost of the output in this market. Thus, marginal cost pricing emerges from Cournot competition as the number of firms grows large. It can also be shown that firm profits converge to zero as N gets large. Perfect Competition, thus, is a limiting case that can be generated by entry in a Cournot-Nash type model of competition among homogeneous firms.

I. On the Existence and Nature of General Equilibria

Market Clearing Price Vectors

It is one thing to say that a single market may reach equilibrium, it is quite another to say (or prove) that any finite number of markets can clear simultaneously, even given relatively general assumptions about individual tastes and production functions. One of the most impressive contributions of neoclassical economics in the twentieth century is general equilibrium analysis.

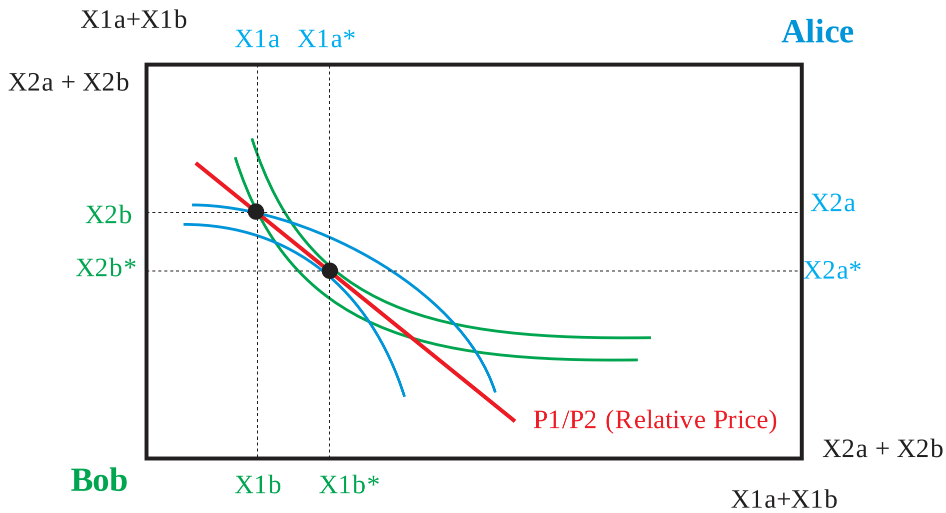
General equilibrium analysis attempts to develop sufficient conditions for the existence of a vector of prices that simultaneously clears all markets by setting demand equal to supply in every market. Léon Walras, one of the founders of neoclassical economics, developed the initial intuition and some of the first mathematical models of such an equilibrium in the late nineteenth century. By now, there are a broad range of general equilibrium models which vary in the extent to which they depart from the assumptions of models of perfect competition, and with respect to the restrictiveness of the mathematical assumptions relied upon. Many of the classic works were developed in the 1950s and 1960s, as part of the great neoclassical synthesis taking place in that period. Gerard Debreu won a Nobel prize in economics (in 1983) for his model and proof the existence of a market clearing price vector in that model, which was published in a short book called *A Theory of Value* published in 1959.

It is surprising that fairly modest assumptions about human behavior, production technology, and budget constraints are sufficient to assure the existence of a market clearing price vector. In this part of the chapter, we'll provide a basic overview of some of the core ideas and mathematics that demonstrate the existence of a market-wide vector of market prices. Debreu's book is recommended to those interest in a more general treatment.

The Edgeworth Box: a Simple Geometric GE Model

The Edgeworth box provides a two-person two-good illustration of a general equilibrium. In an Edgeworth box, two consumers are assumed to have well defined transitive preferences over both goods within the domain of the "box" and an initial endowment of each of the two goods. Prices are "called out" by a "Walrasian auctioneer" until a price is found at which the "excess demand" for both goods is zero. (The excess demand for good i is the sum of desired consumptions of good i less the sum of the original endowment of good i.) At the market clearing price, the quantity that each person wants to sell is exactly the amount that the other wants to buy.

Figure 5.3 The Edgeworth Box



Notice that the equilibrium in the 2-person 2-good case occurs at a tangency point where both person's indifference curves are tangent at the same point on the price line. The area that characterized mutual gains from trade is "football" shaped and generated by the indifference curves (Alice's and Bob's) that run through the endowment point. Note that the endowment point simultaneously characterizes both Al and Bob's endowments. Any move into the football shaped area is Pareto improving. Both players benefit and no one loses.

At the market clearing price, each person wants to sell exactly what the other wants to buy at the price called out by the imaginary Walrasian auctioneer. In the above Edgeworth box, Bob sells good X_2b - X_2b^* units of X_2 at price P_2 in order to purchase good X_1b^* - X_1b units of X_1. Alice does the reverse and consequently the P_1/P_2 price vector is an equilibrium price vector.

In a more informal process of trade, any point inside the mutual gains to trade region (the region between the two indifference curves passing through the original endowment) improve both person's utility level and so make both better off. And, any points inside the lens shaped area where the two sets of indifference curves are tangent to one another can be a trading equilibrium, because at such points there are no further gains to trade. The collection of such points within the box is generally call the “contract curve.” However, in a GE model, its is the prices rather than one-on-one bargaining that determines the final outcome.

General Equilibrium Model of a Large Barter Society

To demonstrate how the logic of the Edgeworth box can be generalized, we’ll develop a model of a large “barter economy” similar to that of the Edgeworth box, but with many traders and products. The model and mathematics are based on an existance proof developed in Varian (1992). To simplify a bit, we’ll assume that no production takes place. Including production is not much more difficult, but such models require the introduction of more mathematical notation and assumptions, and do not provide much additional intuition about how existence proofs are developed. **Typical assumptions** of a calculus-based general equilibrium model (without production) include:

(i.) **Goods**: Goods are characterized by time location and state of the world. There are assumed to be a finite number of goods, k. Agent i's consumption bundle is denoted xi and is a k-dimensioned vector of the goods possessed by i. The amount of the jth good possessed by individual i is denoted x_{ij} .

(ii.) **Endowments**: An individual's initial endowment of goods is his "pre trade" consumption bundle, w_i , $w_i = [\ w_{i1} , w_{i2} , \ldots w_{ik}\ ]$

(iii.) which is a kx1 vector. An individual’s demand for goods at a particular vector of prices is also a kx1 vector describing his or her ideal consumption bundle, given prices and his or her endowment

$$x_i = [\ x_{i1} , x_{i2} , \ldots x_{ik}\ ]$$

(iv.) A **feasible allocation for the economy** is one that is possible. In the pure exchange case of interest here, it is one where Σxi = Σwi . (A feasible allocation is one in which the total demand for each good equals the economy wide initial endowment of that good.)

(v.) **Agents**: Each consumer i is described by a complete transitive preference ordering >i (which is used to derive a utility function Ui) and an initial endowment w_i .

Each consumer is a utility maximizing price taker. Thus, each consumer maximizes Ui (xi) s. t. Pxi = Pwi (P is a 1xk vector, and w_i and x_i are also kx1 vectors. Note that this describes a budget set or multi-dimensional budget constraint for the case where endowments are in goods rather than "wealth.")

(vi.) (In a model with production, there will also be k production functions which describe how "inputs" can be transformed into "final consumption goods" and individual endowments include inputs as well as final goods.)

(vii.) Individual i's excess demand for good j, z_{ij} , is simply his ordinary demand for good j (his desired consumption) less his initial endowment,

$z_{ij}(P) = x_{ij} (P, Pw_{ij}) - w_{ij}$.

His or her excess demand, z_i, is thus a vector of his or her excess demand for all k goods.

(viii.) **Equilibrium Prices.** A Walrasian equilibrium in a barter economy exists if a price vector P* exists such that $\Sigma\, x_i (P^*, P^*wi) - \Sigma w_i = 0$, which is to say when excess demand in every market is simultaneously equal to zero. Demand correspondence xi is vector representing the utility maximizing levels of all goods for individual i with initial endowment w_i facing price P*.

An equilibrium price vector sets aggregate demand equal to aggregate supply, or equivalently, excess demand equal to zero for each person in every market.

(Notice that a lot of the cleverness of a GE model is writing down a complete model of an economy in very few equations with relatively few behavioral assumptions in a way that will be mathematically useful. Note also that very similar assumptions were used in our partial equilibrium models of consumer choice, but with a money endowment rather than a goods endowment.)

(ix.) **Some Properties of the Model** The budget set is homogeneous of degree 0 in prices.

If you multiply all prices by any constant C, there is no change in any individual's budget constraint. This implies that the demand correspondence x_i is also homogeneous of degree 0 in "all" prices. E.g. there is no money illusion.

The excess demand function (x_i (P, Pw_i) - w_i) is also homogeneous of degree zero in "all prices" for the same reason. Moreover, since the sum of homogeneous functions of degree k is also homogeneous of degree k, the aggregate excess demand function is also homogeneous of degree 0 in all prices.

An Existence Proof can be developed as follows.

(i) Each individual i's vector of desired consumption is determined in the usual way -- by maximizing individual i's utility subject to his budget constraint. The vector of aggregate excess demand is written as $z(P) = \Sigma(x_i (P, Pw_i) - w_i)$

(ii) **Walras Law.** (Varian's version) For any P in S_k (remember there are k goods) excess aggregate demand (in dollars) is zero. Thus, P z(P) = 0. (Sk is the price space in a k dimensioned commodity space.)

Proof: recall that $z(P) = \Sigma(x_i (P, Pw_i) - w_i)$, and also that each person's demand correspondence (vector x_i) is derived by maximizing utility given a budget constraint. Consequently, $Pw_i = Px_i$ for each individual.

This implies that the sum of all the Pw_i vectors has to equal the sum of all Px_i vectors. That is to say, **excess demand is always zero in the aggregate** (measured by the numeraire good, here dollars) **because demand is effective demand**, and so is backed by one's endowment.

(iii) Note that the above implies that if P* is a Walrasian equilibrium and excess demand for commodity j is less than zero, then its price must be zero, e.g. P*j

= 0. (More intuitively, if there is an excess supply of good j , then its price has to be zero.)

(iv) **Proof:** Since P* is a Walrasian Equilibrium the excess demand for all goods is less than or equal to zero, it satisfies z(P*) = 0. If P*j were greater than zero then P*j z(P*) < 0, violating Walras' law.

(v) But Walras' law always holds, so P*j has to be zero. E. Similarly, if all goods are desirable at the margin, prices are greater than zero. Thus, in either case, if P* is a Walrasian equilibrium, then z(P*) = 0.

In order for the money-based excess demand to be zero in this case, excess demand has to be zero in every market. (Supply equals demand in all markets.) The proof is left as an exercise. It is very similar to the previous case.

(vi) Moreover, if K-1 markets have cleared, then the excess demand in the remaining market must be zero. (Recall that the aggregate excess demand in money terms is always zero.) This is the usual version of Walras' Law.

(vii) **Summary**: The aggregate value of excess demand is always zero. If there is an excess supply of a good (an undesirable good) its price will be zero. In all other cases, demand equals supply for all goods in Walrasian equilibrium.

Proof of the Existence of a Walrasian Equilibrium Vector

(i) The proof begins with **Browers Fixed Point Theorem**. If f:Sk-1→Sk-1 is continuous function from the unit simplex to itself, there exists some x in Sk-1 such that x = f(x). Such a point is called a fixed point. In a one \-dimensional case, the unit simplex is just the 0-1 closed interval. (In the two dimensional case it is a 1x1 square, in the three dimensional case it is a 1x1x1 cube, etc.)

(ii) To see that a function from this interval to all or part of itself has a fixed point, draw diagram of a function, Y = f(x). Let Y be the vertical axis, X be the horizontal axis. A continuous function goes from [0-1] on the horizontal axis to some part of [0-1] on the vertical axis. Because of continuity, at some point the function will intersect the 45^{o} line from (0,0) to (1,1), at which point

$x^* = f(x^*)$. Such a point, x^*, is said to be a fixed point. (There may be more than one fixed point for a given function.)

(iii) **The ingenious trick in most existence proofs of the existence of an economically relevant equilibrium** is to construct a transformation based on the choice setting that is a continuous function mapping of the variables of interest into themselves. In this case, this will be done with respect to prices.

(iv) One example of such a mapping is the following: First, define the elements of vector g as g_j (Pj) = [Pj + max (0, z_j (P)] / [1 + Σj max (0, Zj (P)] where the prices have been normalized as: Pj = Pj /ΣP (This of course will not affect aggregate demand as we have already established above.) This mapping is continuous because both z and max (0, z_j(p)) are continuous.

(v) This mapping lies in the unit simplex because Σ_j $g_j(P_j) = 1$ for each Pj. Thus, by Browers fixed point theorem there is a P* such that P*j = gj (P*) for all j. (That is to say a k-dimensional fixed point exists.)

(vi) At this fixed point, Pj * = [Pj * + max (0, zj (P*)] / [1 + Σj max (0, zj (P*)]

(vii) **P* turns out to be a Walrasian equilibrium price vector**.

(viii) To see this, cross multiply, which yields Pj * [1 + Σj max (0, z_j (P*)]= [Pj * + max (0, zj (P*)], Next Multiply both sides by the excess demand function, z_j (P*), which yields z_j(P*)Pj * [1 + Σj max (0, z_j (P*)]= zj (P*)[Pj * + max (0, z_j(P*)]

(ix) Add both sides up across all goods: [Σj z_j(P*)P_j*][1 + Σj max (0, zn (P*)]= Σj z_j(P*)[1 + max (0, zj (P*)]

(x) Note that the first term is not the money form of the excess demand function, which we know will be zero because each persons excess demand in money terms has to be zero. (Each consumer has to be able to pay for what they want.) Thus, the left-hand side equals zero. (The first term in brackets terms has to be zero.)

(xi) If the left hand side equals zero, then the righthand side must as well. Thus, Σj z_j(P*)[1 + max (0, zj (P*)] = 0. This can only be true if the first time is zero for each market's excess demand at P*, z_j(P*). Otherwise, the product of zj (P*)[Pj * + max (0, zj (P*)] would exceed zero. Q. E. D.

The economic meaning of this existence proof is that a market clearing price vector exists for any pattern of demand and wealth. That is to say, given the usual assumptions about preferences (and in a more general model, production correspondences) a price vector exists that simultaneously clears all markets. At this price vector, (a) the excess demand for all goods (all things with P>0) is zero, and (b) all potentially tradable "things" with negative excess demand have zero prices.

In a model that includes production, the same results would hold for both output prices and input prices.

Of course, few economists believe that every market has cleared at every moment in time, but rather they tend to believe that the GE price vector is an "attractor," which informal bargaining tends to move the economy towards.

VI. Conclusions: The Power and Weaknesses of Neoclassical Price Theory

Over the course of roughly a century, neoclassical economists developed a very well-integrated theory of market prices grounded in methodological individual. Market prices are a social phenomenon, they are generated by the independent decisions of millions or even billions of persons making decisions in choice settings that affect the demand and supply of product sold in commercial networks. What neoclassical economics shows is that the results of billions of independent decisions in market is not chaotic "in the large" although it may appear so in "the small."

If people are generally forward looking and attempt to achieve the "best" that they can with internally consistent ideas about "best"—all of which can be different for each individual—the result is not chaos, because the price system tends to coordinate decisions so that supply equals demand in every market—or at least produces a tendency for market outcomes to move toward such an equilibrium. In prosperous societies, these predictions are largely validated by experience. Supermarkets and other retail outlets rarely run out of things that their customer's want to purchase. Nor do they routinely have piles of unsold goods sitting around their display areas. Similar outcomes are commonplace in most input markets.

This is not to claim that markets are always in equilibrium or that shortages and surpluses never occur, but the models do imply that such "disequilibrium" outcomes are rare and may be caused by failures of prices to "adjust." The latter may be induced by mistakes by firms in their pricing, a desire for firms to maintain reputations for "fair pricing," time and energy necessary to renegotiate contracts, or by government regulations that make it more difficult—if not impossible—for market-driven price adjustments to take place. The models have "abstracted" from such "transactions costs" in order to demonstrate what equilibrium prices look like and also to demonstrate that they are possible—not just grounded in the pro-market intuitions of persons favoring more open markets for ideological reasons.

The neoclassical synthesis provides a grand and important source of insights about how markets operate and how prices affect all the billions of decisions that create and sustain contemporary commercial networks. However, as a model, the neoclassical synthesis clearly abstracts from many aspects of life in a market economy that may alter some or all of the conclusion reached. For example, there are no information problems and thus no opportunities for fraud or intra-firm shirking or mistakes. There is no legal-regulatory environment, except the implicit one of civil law with its contracts and property rights. And there are no public policies beyond those implicitly assumed—e.g. those required to assure that contracts are kept, fraud and theft is minimized, and no extortion takes place. The core models also provide no model of entrepreneurship—no model of innovation, price speculation, or risk-allocation.

Also, there are limits to its model of equilibrium prices. The demonstration that a market equilibrium exists does not include a plausible mechanism through which such prices might emerge in a competitive market. The real world does not include a Walrasian Auctioneer.

The second and third parts of this book investigate how such implicit assumptions affect how markets operate.

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