Part II

I. Introduction: What Has Been Learned from Part I

Part I of the book provides an overview of the main neoclassical models and implications. That model provides a nicely connected schematic of how prices and outputs of the goods and services are determined—or perhaps more accurately, tend to be determined or drawn towards—in well-functioning markets. Prices emerge because terms of trade are part of the process of buy and selling goods, whether for money or in exchange for other goods (barter). Such terms of trade, in a sense, determine the prices of all goods and services. What neoclassical theory demonstrates is that in cases where more than one consumer and more than one firm are involved, there are tendencies in these terms of trade that can be deduced if it is assumed that consumers and firms have reasonably stable and consistent aims in life.

To that end, relatively simple models of the aims in life were developed—initially by utilitarians in the late 18th and 19th centuries, but subsequently adopted by social scientists that use "rational choice" models to analyze social phenomena. For economists, this implied that consumers attempt to maximize net benefits or utility. Firm owners and managers attempt to maximize profits insofar as production are means rather than ends—although ultimately, they too are regarded to be utility maximizers. As long as their choice settings are well understood (by themselves and the model-builders relying on such coherent objective functions) their choices could be analyzed as constrained optimization. The mathematical implications of constrained optimization, in turn, implied that consumers had various demands (wants that they could attempt to satisfy using money and other resources at their disposal) and that firms had various opportunities for suppling services that consumers might purchase.

Equilibria in such market networks emerge when prices adjust so that supply equals demand in all of the markets of interest. Indeed, price vectors exist that can simultaneously "clear" all markets. Few neoclassical economists would insist that every market is always in equilibrium (although a few may do so). Rather the models are thought to reveal tendencies that all markets exhibit. Firm owners produce their products at approximately least cost, which requires hiring particular mixes of inputs that are jointly determined by production technology, input prices, and demands for the outputs to be produced. Consumers, likewise, choose combinations of goods that are expected add most to their lifetime satisfaction (utility), given their wealth, market prices, and their long-term objectives (tastes, preferences, etc.).

In the models developed before WWII, both consumers and firms were usually assumed to have complete (perfect) information about all the factors that generated their choices—even though most thoughtful economists recognized that that would rarely if ever be true, except perhaps in markets where the same products had been purchased and sold for many years. In the second half of the twentieth century, far more attention was given to the often-unstated informational assumption being made about both firms and consumers. These generated many extensions of the basic model worked out by neoclassical economists—and in some cases, induced more caveats and more complex models. Nonetheless, the neoclassical models provide the points of departure for most contemporary research. Both theoretical and empirical economic research generally rests on neoclassical foundations.

II. Overview of Part II

Part II focuses on some important implications of time and imperfect information and changes in information that affect the pattern of exchange in societies with significant market networks. In the core models of neoclassical economics, neither firms nor consumers ever make mistakes. There are no agency problems within firms and no disappointments about the products produced and sold—more over nothing new is ever produced. Although it was well known that such things happened, the first geometric and mathematical models abstracted from such problems.

These neglected factors were analyzed in what might be called the second-generation models or extended neoclassical models. In many cases, analysis of the effects of imperfect information undertaken somewhat narrowly and separately, rather than integrated into neoclassical models. Although not thoroughly integrated into the core neoclassical models, the results helped to explain the variation in prices for similar goods that violate some of the conclusion of the perfect competition model. For example Stigler's (1961) model of the effect of limited knowledge of prevailing prices helped to explain why prices of similar goods usually vary. Similar somewhat fewer sharp implications were reached for outputs, profits, and patterns of exchange.

Part III explores extension of the neoclassical models to choice settings other than those that directly involve production, exchange, or innovation. It turns out, however, that micro-economic extensions to such fields and law and economics, political economy, and socioeconomics besides providing insights into the nature of law, politics, and non-market social interactions also have implications about the extent of trade and the rate at which commerce expands (or contracts) through time. Thus, these topics are not truly beyond economics but provide explanations for differences among market equilibria at a point in time and through time, and also shed light on potential sources of disequilibria and adjustments toward new equilibria.

Chapter 6: Intertemporal Choice

I. Introduction

Most neoclassical models are timeless in the sense that "time" is left out of the model. That is not because time is never important, but because for some purposes leaving time out of a model or analysis does not undermine its ability to help us better understand the puzzle or phenomena being modeled and analyzed. If a consumer decides that he or she will spend one month's wages in a particular way, the fact that the actions associated with that decision do not take place for a month (or year or decade) does not necessarily influence the optimization process that led to that decision or its consequences for market prices. The period of analysis is simply assumed to be the one that is relevant for the decision and usually its associated action to take place.

However, there are cases in which time matters. Time cannot be ignored when actions are taken today that affects one's possibilities in the future—if one is rational and forward looking. Indeed, the phrase "forward looking" implies that decisionmakers take account of the consequences of present actions on future possibilities. For example, a consumer's decision to spend a certain amount of money in the future may affect the extent to which he or she works today. Or, planning ahead may affect present consumption decisions reducing present consumption provide additional money for future expenditures. Or, a consumer may engage in the opposite type of behavior. He or she may borrow against future income to pay for capital goods (computer, automobile, house, etc.) or for ordinary consumption today.

The same logic applies to decisions made by economic organizations (firms) who may borrow against future profits to pay for capital goods that will be used in production today or in the near future—or attempt to build a cash reserve today that can be used smooth out predictable fluctuations in a firm's cash flow over the course of time. (Many businesses have sales patterns that are connected with the seasons and business cycles and hold onto their employees because it reduces training and recruiting costs during both sorts of cycles when they are believed to be temporary. They may, for example, create a wages fund, saving some net income during the most profitable times in the year to pay their employees in the less profitable times. Teams and the knowledge of the routines required for efficient team production benefit from stability of the team members. Examples of seasonal demands include the demand for toys, holiday foods and beverages, and the market for housing (because of school year effects).

The demand for savings and loans creates a market for a variety of products that would not exist in a timeless world—various types of financial firms emerge to service such desires for intertemporal services. In most cases, such firms serve as "intermediaries" between persons desiring to save and those desiring to borrow, with the interest rate or rate of return on investment being the benefit for savers and the cost for borrowers. As a consequence types of loans and savings accounts exist that would not except for their time-dependent interest in saving, long term investments, and borrowing.

This chapter develops some mathematical methods and models that can be used to characterize both intertemporal decision making and markets for savings and loans. Again the focus is on optimization, and again the focus is on circumstances in which buyers and sellers (borrowers and savers) are fully informed about the alternatives being bought and sold. It is the timing of such decisions and actions that are the main focus of attention, rather than being put aside to simplify the analysis.

For the most part, optimal decisionmaking through time rests on the notion of **present discounted value**—the mathematics of which emerges naturally when it is recognized that both borrowing and saving have opportunity costs.

II. Intertemporal Choice: Time Discounting and Present Values

The simplest way to think about "present discounted value" is to think about the amount in the present (PV) that you would be indifferent to having now rather than some other value (F) in in T years.

One way to estimate this, if one thinks in money terms, is to calculate the amount of money that one would have to invest today to have F dollars T years in the future.

- If the interest rate or rate of return is r, one can just apply the compound interest formula. $PV (1+r)^T = F$

- Solving for PV yields PV = $F_T/(1+r)^T$ which is the basic formula for calculating the present value of some value in the future.
- To make the formula concrete, suppose that F is $20,000 that T=2 and R=3% or 0.03. In that case, PV = $(20{,}000)/(1.03)^2$ = $18851.92
- Notice that PV of future among F goes down when the interest increases and when the time period increases.
- The PV of $20,000 in two years at an interest rate of 5% is
  PV = $(20{,}000)/(1.05)^2$ =$18,140.59
- The PV of $20,000 in ten years at an interest rate of 5% is
  PV = $(20{,}000)/(1.05)^{10}$ = $12,278.27

If one thinks purely in financial or money terms, one would be indifferent between $12,278.27 today and $20,000 in 20 years. This assumes that no inflation occurs (or that F_T is in inflation adjusted terms) and that there is no risk involved about whether the future amount will be paid or not. When one takes account of inflation either everything should be in inflation adjusted (real) terms (including the interest rate, where the real interest rate is the nominal rate of interest less the average annual inflation rate over the period of interest)—or everything should be in nominal (ordinary dollar) terms. When there is the risk that amount F will not be paid, then one needs to also take account of the risk using the methods that will be developed in chapter 7.

Let **Ft** be the value of some asset or income flow "**t**" time periods from the present date. Let **r** be the interest rate per time period over this interval. The present discounted value of Ft is

$$P(F_t) = F_t/(1+r)^t \qquad (1)$$

The present value (here P) of a series of future income flows (which may be positive or negative) is simply the sum of the present values of the individual elements of that series. When done over **T** years when the interest rate is **r** (as a fraction) per period the present value is:

$$P = \sum_{t=1}^{T} F_t/(1+r)^t \qquad (2)$$

The present discounted value of any series of values is the sum of the individual present values of each element of the series. This formula always "works" but it is somewhat cumbersome to use as the planning period, T, becomes relatively large.

Many decisions involve long term flows of costs and benefits that need to be evaluated by a decision maker or group of decisionmakers. These flows are easiest to compare if one can construct a common "metric" for the purposes of comparison. The present value of a series of benefits and/or costs through time is the amount, **P**, that one could deposit in a bank at interest rate **r** and used to replicate the entire stream of benefits or costs, F_1, F_2, F_3, ... F_T. That is to say, you could go to the bank in year 1, withdraw the amount (B_1) for that year, return in year 2, pull out the relevant amount for that year (B_2), and so on ... . When thought of in this way, it should be obvious that the present discounted value of a series of future amounts is simply the sum of the present values of each element of the series—which is in equation 2.

Another useful formula is one that characterizes the present discounted value of a steady flow of values on off into the next T years. In cases where a constant value is received through time, e.g. $v = F_1 = F_2 \ldots = F_t \ldots = F_T$, a bit of algebra allows the above present value formula to be reduced to:

$$P = v\ [\ ((1+r)^T - 1)/r\ (1+r)^T] \qquad (3)$$

This formula can be derived as follows:

First multiply $P = \sum_{t=1}^{T} v/(1+r)^t$ by (1+r) which yields

$$(1+r)P = \sum_{t=0}^{T-1} v/(1+r)^t$$

Subtract P from (1+r) P which yields: $rP = v\ [\ 1/(1+r)^0 - 1/(1+r)^T)]$. (Note that all the terms in the two sums are the same except for the first and last one, so they cancel out.) Recall that $1/(1+r)^0 = 1$ so $rP = v\ [\ 1- 1/(1+r)^T)]$. Putting the lefthand term over a common denominator yields $rP = v\ [\ (1+r)^T - 1]\ /\ [(1+r)^T]$. Dividing both sides by r yields

$$P = v\ [(1+r)^T - 1]\ /\ [r\ (1+r)^T] \qquad \text{QED}$$ [1].

[1] QED is an abbreviation for the Latin phrase *quod erat demonstrandum*, which means "that which was to be demonstrated". It is often used at the end of a mathematical proof

Note that this constant flow of benefits (or costs) formula **has a limit** as T approaches infinity, namely: **$P = v/r$** . This is a very convenient formula. There are many long-term investments and regulatory policies that have very long lives that can be thought of as infinitely lived investments as a "first approximation", because the last few billion terms have little effect on the present value of long term flows of costs or benefits.

Illustrative Applications

These formulae can, for example be used for cost benefit analysis. Suppose that a dam can be built that cost $1,000,000 and will produce $50,000/year in electricity for 40 years. Is the dam worth building if the interest rate is 5%/year?

- Use the PV formula: $P = v [((1+r)^T - 1)/r (1+r)^T]$
- The PV of the future benefits are
  $P = 50{,}000[((1.05)^{40} – 1)/(.05)(1.05)^{40}] = \$857{,}954.31$ answer NO
- What if the interest rate is 2%/year? In this case PV = $1,367,773.96 answer YES
- Discount rates matter. Note that the benefits off in the distant future are worth far less when $r = 0.05$ than when $r = 0.02$
- Note that if the dam would provide electricity forever, then
  $P = v/r = \$50{,}000/0.05 = \$1{,}000{,}000$ in that case the dam project exactly breaks even (ignoring any maintenance expenses) But, also note that the all the years after year 40 add relatively little to the present discounted value of the future benefits.

Suppose that Al can afford to pay $5000/year in car payments for 5 years toward a new automobile. If the bank's opportunity cost rate of return is 7%, what is the largest amount that the bank will loan Al given his budget?

- Use the PV formula: $P = v [((1+r)^T - 1)/r (1+r)^T]$
- $P = 5000 [((1.07)^5 – 1)/r(1.07)^5] = \$20{,}500.99$
- That is the bank's opportunity cost of tying up P dollars during the 5 years the loan will be repaid.

III. Intertemporal Choices Using the Present Value Formulae

These sorts of present value calculations can all be used to determine the net-benefit maximizing decisions whenever the payoffs and costs of functions of a control variable of interest such as an investment in some capital good. In many cases, the time-dimension of the choice is not as central as might have been expected. In others, the time dimension is quite important.

Suppose that a project of interest has upfront costs that vary with the output to be produced, which involves a loan to purchase a capital good. The loans annual payments are rc(K) where r is the prevailing interest rate and c(K) is the cost of the capital. The annual cost of capital in that case is rc(K). The capital, in turn, generate benefits such as revenues (R) that vary with the extent of the capital purchased, as with R=f(K, P) where P is known and expected to be constant during the planning period. If the life of the project is T years, the present value of the net benefits or profits from the project can be written as:

$$\Pi = [f(K,P) - rc(K)][\,((1+\mathrm{r})^T - 1)/\mathrm{r}\,(1+\mathrm{r})^T] \qquad (4)$$

Differentiating with respect to K and setting the result equal to zero yields:

$$\Pi_K = [f_K - rc_K]\left[\frac{(1+\mathrm{r})^T - 1}{\mathrm{r}}(1+\mathrm{r})^T\right] = 0 \text{ at } \mathrm{K}^* \qquad (5)$$

Multiplying both sides of equation 5 by 1 over the second term in bracket yields:

$$[f_K - rc_K] = 0.$$

The first order condition, perhaps surprisingly, is not affected by the planning horizon T nor by the discount factor (the term inside the second set of brackets).

The interest rate matters, but only because this determines the annual cost of the capital good being employed. To maximize the profits from a capital project of this sort, one simply purchases capital so that the annual marginal revenue generated (f_K) equals its annual marginal cost (rc_K).

The same sort of calculation can be undertaken for irregular flows of benefits and costs using the original summation version of the present value formula. For example, suppose that the benefits from the capital purchase varied through time—perhaps systematically, perhaps not. In that case, the objective function would be:

$$\Pi = \sum_{t=1}^{T}[f(K,P)_t - rc(K)]/(1+r)^t \qquad (6)$$

And the associated first order condition, using subscripts to denote partial derivatives with respect to the variable subscripted, is:

$$\Pi_K = \sum_{t=1}^{T} [f_{tK} - rc_K]/(1+r)^t = 0 \; at \; K^* \qquad (7)$$

In this case, one sets the present discounted value of the marginal revenue (or other benefit) generated by the capital project equal to the present discounted value of the cost of the capital project. The irregularity of the marginal benefit flows is important, and as their present discounted value falls, a smaller capital investment becomes optimal. Such a reduction in the present value of marginal revenues, for example, be caused by an increase in the discount rate for a project that has increasing revenues (or other benefits) through time. (That effect would be reinforced by an increase in the marginal cost of capital through its effect on the annual cost of the capital project.)

IV. Intertemporal Utility Maximization

The above model can also be applied to consumer choice models based on the net-benefit model developed in Chapter 2. And in some cases, it may be extended to the the utility maximizing model, if one believes that a plausible lifetime utility function has the form $U = \sum_{t=1}^{T} u_t(X_{1t}, X_{2t}) / (1+r)^t$. Intertemporal utility maximization problems generally express the relevant budget constraints in present discounted value terms, with W equal to the present value of future or lifetime income, as with $U = \sum_{t=1}^{T} Y_t/(1+r)^t$, and expenditures on goods and services also represented as the present discounted value of future expenditures, as with $E = \sum_{t=1}^{T}(P_{1t}X_{1t} + P_{2t}X_{2t}) / (1+r)^t$.

As with our models of consumer choice and a firm's production decisions, , a good deal about the nature of intertemporal choices can be generated from simple two or three period models of choice. This greatly reduces the mathematical complexity of such models, without much loss of generality.

Suppose that Al's utility function is $U = u(C_1, C_2)$ and her intertemporal budget constraint is $Y_1 + Y_2/(1+r) = C_1 + C_2/(1+r)$, where Y_1 and Y_2 are incomes in period 1 and 2, r is the interest rate or opportunity cost rate of return, and C_1 and C_2 are consumption levels in the two periods. Note that Al's person's wealth, W, is the present value of current and future income, and r is the relevant interest rate. Note also that the effect of time discounting is left implicit in this characterization and might be based on the present value formula or might not. Let's also assume that either there is no inflation or that the income and consumption flows and the interest rates are in "real" or inflation adjusted terms.

Both the Lagrangian and substitution methods can be used to characterize Al's optimal consumption expenditure in each period. Concrete functional forms such as the Cobb-Douglas and its variations with exponents that do not sum to one allow consumption in both periods to be characterized as a function of wealth, interest rates and prices in the two periods.

An Illustrating Example with a Concrete Functional Form

Let $U = C_1^a C_2^b$ and let $Y_1 + Y_2/(1+r) - C_1 - C_2/(1+r) = 0$. Form a Lagrange equation and then differentiate with respect to C_1, C_2, and λ.

$$L = C_1^a C_2^b + \lambda(Y_1 + Y_2/(1+r) - C_1 - C_2/(1+r)) \quad (8)$$

For the purpose of this model, we'll denote partial derivatives of the Lagrange function with subscripts.

$$L_{C1} = aC_1^{a-1}C_2^b - \lambda = 0$$

$$L_{C2} = bC_1^a C_2^{b-1} - \lambda(1/(1+r) = 0$$

$$L_\lambda = Y_1 + Y_2/(1+r) - C_1 - C_2/(1+r) = 0$$

Shift the lambda terms in the first to equation to the right, divide the first equation by the second, and simplify (as usual for this type of function using the Lagrange method).

$$aC_1^{a-1}C_2^b \,/\, bC_1^a C_2^{b-1} = \lambda/(\,\lambda(1/(1+r))$$

$$aC_2/bC_1 = 1+r$$

The ratio on the left can be interpreted as the marginal rate of substitution between current and future consumption. Note that at the utility maximizing levels of C_1 and C_2, the marginal rate of intertemporal substitution is equal to 1 plus the interest rate. The marginal rate of substitution between future and current consumption is sometimes called the *subjective rate of time* discount.

Solve for C_2 as a function of C_1 and then substitute that into the constraint (L_λ).

$$C_2 = [b(1+r)C_1/a] \quad (9)$$

Substituting yields: $Y_1 + Y_2/(1+r) - C_1 - [b(1+r)C_1/a]/(1+r) = 0$. Shift the C1 terms to the right (e.g. add the negative of their values to each side) and factor.

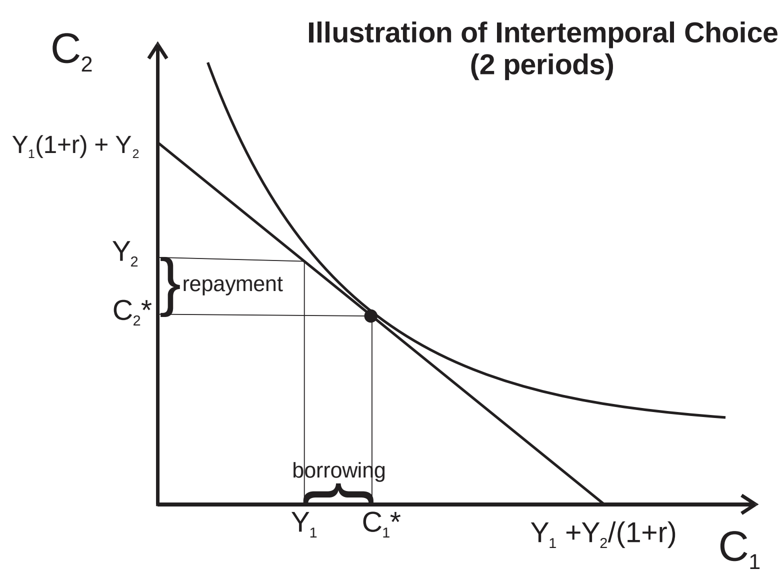
$$Y_1 + Y_2/(1+r) = C_1 + [b(1+r)C_1/a]/(1+r) = C_1 (1+b/a) = C_1 [(a+b)/a]$$

Divide and reverse to find Al's demand curve for present consumption:

$$C_1^* = [a/(a+b)][Y_1 + Y_2/(1+r)] \qquad (10)$$

Note that this is analogous to the usual $C_1^* = [a/(a+b)]W/P$ of the usual non-intertemporal demand functions derived from this family of functions, but here $W = [Y_1 + Y_2/(1+r)]$ and $P = 1+r$, since we measured current consumption in dollars rather than in goods and services. (In effect we are holding the prices of current and future consumption constant.) Note also that C_1^* falls as r increases and increases as income in either period increases.[2]

The geometry of a typical 2-period intertemporal choice is depicted below. The extent of Al's savings is the differences between current income and current consumption, $Y_1 - C_1^*$, which will be negative if he or she borrows against future income to increase his/her current consumption.



[2] To find C_2^*, substitute C_1^* into equation 9. $C_2^* = [b(1+r)\{[a/(a+b)][Y_1 + Y_2/(1+r)]\}/a]$ which simplifies to $C_2^* = [b(1+r)/(a+b)][Y_1 + Y_2/(1+r)]$. Note the extra (1+r) in the formula for ideal future consumption relative to the equivalent atemporal calculation of demand using functions from this family. Without that term, one would have characterized the present discounted value of future consumption rather than its actual level.

Geometrically, the above model of intertemporal choice looks like an ordinary consumer choice problem except the axes represent present and future consumption.

Note that in the circumstances modeled, the subjective rate of time discount will be set equal to the interest rate plus 1. The indifference curve tangency implies that the slope the highest indifference curve that can be reached is equal to the slope of the intertemporal budget line. (This is one interpretation of the first steps of solutions to a Lagrangian representation of the choice, as noted above, and it can be derived from the substitution method as well.)

Intertemporal Choices with Abstract Utility Functions

Given an abstract function form for an individual's utility function, calculus can be used to characterize the effect of changes in interest rates on a person's maximal utility levels and to characterize C_1^* and C_2^*. The first-order condition(s) will again imply that the marginal rate of substitution between future and current consumption is equal to one plus the interest rate, $(1+r)$.

Let $U=u(C_1, C_2)$ be an individual's strictly concave utility function and $W = Y_1 + Y_2/(1+r) = C_1 + C_2/(1+r)$ be his or her intertemporal budget constraint. Income levels in the two periods are Y_1 and Y_2 , and the relevant interest rate is r. These three variables are assumed to be parameters of the individual's choice problem, which is to say they are assumed to be exogenously determined as they would be if they were determined by market forces. The individual's choice is assumed to be over the timing of consumption. (In other cases, decisions that affect future income may also possible, as with ones investment in a college education.)

Since there are just two control variables and one constraint, we can solve the constraint for one of the two control variables in terms of the other and substitute it into the individual's utility function. For example, we can solve for C_2 as:

$$C_2 = (1+r)Y_1 + Y_2 - (1+r) C_1. \quad (11)$$

Substituting this into the utility function yields:

$$U = u(C_1, (1+r)Y_1 + Y_2 - (1+r) C_1). \quad (12)$$

Differentiating with respect to C_1 yields:

$dU/dC_1 = du/dC_1 - (1+r)du/dC_2 = 0 \equiv H$ at C_1^* (13)

Note that the first term, du/dC_1, is the marginal benefit from current consumption and the second is its marginal cost, $(1+r)\, du/dC2$, a forward-looking consumer consumes in the current period (today) at the rate where the marginal benefit today equals its marginal cost in terms of reduced utility from future consumption.

A bit of algebra also allows one to use the first order condition to characterize the tangency condition of an indifference curve diagram:

$$\frac{\left(\frac{du}{dC_1}\right)}{\left(\frac{du}{dC_2}\right)} = (1+r)/1 \qquad (14)$$

The implicit function theorem allows consumption in period 1 to be characterized as a function of the parameters of the individual's intertemporal choice setting:

$C_1^* = c(Y_1, Y_2, r)$ (15)

with $C_2^* = (1+r)Y_1 + Y_2 - (1+r)\, C_1^*$

An individual's intertemporal pattern of consumption is a function of his or her present and future income and the interest rate (here it should be acknowledged that more than one interest rate may be relevant for reasons developed towards the end of this chapter.).

The comparative statics of the individual's choice can be characterized using the implicit function differentiation rule. For example, the effect of an anticipated increase in the interest rate on current consumption is:

$dC_1^*/dr = dH/dr \;/\; -dH/dC_1 =$

$$[(Y_1-C_1)(du^2/dC_1dC_2) - du/dC_2 - (1+r)(Y-C_1)du^2/dC_2^2\,] \;/\; -dHdC_1 \qquad (16)$$

where $dH/dC_1 = [du^2/dC_1^2 - 2\,(1+r)du^2/dC_1dC_2 + (1+r)^2du^2/dC_2^2] < 0$.

Note that the numerator can be greater or less than zero depending on whether the individual is a borrower or a saver in period 1.

If he or she borrows, then $Y-C_1< 0$ and the numerator is greater than zero. The denominator is greater than zero so the overall effect of an increase in the interest rate on savers is to increase current consumption. Intuitively, this is because an increase in interest rates reduces lifetime income for borrowers, and that decrease, together with an increase in the cost of borrowing induces them to decrease current consumption. On the other hand if the individual is a save in period 1, the effect on consumption in period 1 is ambiguous. The first and last terms in the numerator are positive in that case and the middle term is negative. If the first and last term dominate the middle term, then the effect of an increase in the interest rate on savings is positive. Intuitively, the higher return on savings implies that he or she has more lifetime income to spend and so uses some of the increase for the present consumption by saving a bit less.

The effect of an increase in future income can be developed in a similar way:

$dC_1/dY_1 = (dH/dY_1)/-dH/dC_1) =$

$$([(du^2/dC_1dC_2) - (1+r)(C_1)du^2/dC_2^2] / -dH/dC_1 \qquad (17)$$

The numerator in this case is positive and the denominator (which is the same as in the derivation for the effect of an increase on interest rates on current consumption) is positive. An increase in future income, thus, tends to increase current consumption.

These quite general findings imply that interest rates and expectations about future income are both important determinants of current consumption.

Extensions

Both the explicit functional form models and the abstract functional forms can be extended to characterize multiple periods. A multi-period utility function can be generated for the multiplicative form or abstract forms for utility function used above, by adding additional "C" terms for periods, 3, 4…and T with associated exponents in the multiplicative exponential case. The budget constraint in either case would set the present value of lifetime expenditures on the goods under consideration to the present value of income flows during the same planning period—which could be a lifetime.

Another extension is to consider the possibility of continuous flows of utility and income rather than discrete flows. The mathematics of these characterizations is a bit different than for the discrete cases, as shown in the appendix of this chapter.

V. The Market for Savings and Loans in a Setting without Risk

The intertemporal consumption models imply that there may be gains to trade between persons who wish to borrow in the present because their income is less than their desired consumption ($Y_1<C_1*$). Borrowing is possible when future income is sufficient to pay back the loan and provide an acceptable level of personal consumption as it is in this case. Those who wish to save instead of borrowing do so because their present income is greater than their desire for current consumption ($Y_1>C_1*$) and they prefer to shift spending to the period 2.

In simple forms of financial markets, such persons might simply meet up with each other (as still occasionally happens) and the person seeking a loan (the borrower) would receive one from the person willing to make a loan (the saver). In the risk-free environment assumed to this point in the book, such agreements would be relatively easy to consummate, but they might still take significant time to work out if the borrower wants a larger loan than any single saver is willing to make—as would likely be the case for loans to purchase a house or condominium.

As financial markets emerged, this matching process was undertaken by various "middleman" firms (sometimes referred to as financial intermediaries), who would pay savers (r^s) for the temporary use of their savings and charge a somewhat more than that amount (r^b) for individuals who sought to borrow some money. Banks are one example of such firms—but there are many others that vary partly because of differences in the riskiness of the returns for savers and the riskiness of those receiving loans. We'll ignore the effects of risk for now; analyzing those effects are taken up in chapter 7.

For now, we'll continue to assume that all is known, and so there is no risk for the intermediaries nor for those making loans to them. The borrowers are all trustworthy and have sufficient funds to repay the loans in the future, and the intermediaries are honest or sufficiently fearful of penalties for fraud to behave as if they were honest.

The difference between the amounts paid to those depositing funds in the bank (those loaning the bank their money) and those borrowing the money from the bank reflects the bank's cost of doing business as well as the value added (reduced transactions costs) by their services.

In a world of full information and certainty, banks would resemble Marshallian firms. They would tend to be use very similar technologies and inputs. In such cases, competition would induce what might be called their middleman fees (r^b-r^s) to converge toward ones that are equal to the average cost of collecting deposits, assuring their safety, assessing the trustworthiness and future income of the persons taking out loans, and keeping accurate records plus the "normal" return on capital (e.g. a financial intermediary's investments in vaults, buildings, computers, and so forth).

Competition does not, however, reduce the markup to zero, because of the various costs that financial intermediaries bear to provide their services. The interest rates for borrowers are necessarily higher than that paid to savers ($r^b > r^s$), because the services provided by banks and other similar organizations are costly to produce. In cases in which a bank has some monopoly power, as might be the case in towns with only a few banks, there will be a markup beyond that required to cover its costs and provide an "ordinary" rate of return on the capital used to provide the intermediary (banking) services.

Except for the name for the price paid for some inputs (interest paid to depositors) and price of (interest) an intermediary's output (loans to borrowers), the decision of intermediaries in a risk-free environment is basically another straight-forward application of the theory of the firm worked out in chapter 3, with some intertemporal aspects because of the nature of some of its inputs and of the demands for its services.

Ignoring risks and opportunities for fraud by both banks and the persons taking out loans (borrowers), makes banking a very simple and largely risk-free business. The steady state size of a bank (supply of loans, Q) attempts to maximize the present value of profits from providing those services over its planning horizon T:

$$\Pi = \sum_{t=1}^{T}[(r^b - r^s)Q_t - c(Q,w,r)]/(1+r)^t \tag{18}$$

The associated first order condition is:

$$\Pi_Q = \sum_{t=1}^{T} \left[(r^b - r^s) - c_Q\right]/(1+r)^t = 0 \; at \; Q^* \tag{19}$$

In a steady state $[(r^b - r^s) - c_Q]$ is a constant and can be factored out to yield:

$$[(r^b - r^s) - c_Q]\left[\sum_{t=1}^{T}\frac{1}{(1+r)^t}\right] = 0$$

Which reduces to:

$$\left[(r^b - r^s) - c_Q\right] = 0 \; at \; Q^* \tag{20}$$

The bank will have a portfolio of loans the equates its marginal revenue from loans to its marginal costs of providing and servicing those loans and their associate deposits.

Note that in this case, the result is more or less the same as would have been the case for firms in the time-less models worked out in part 1. This is not always the case, but provides the basis for abstracting from time used in that part of the text.

The Demand and Supply of Credit

The demand and supply of credit in a setting where both suppliers and demanders are price takers requires two markets to clear, the market for loans (populated by borrowers) and the market for savings. The demand side of the loan market is populated by borrowers. The supply side of this market is populated by savers. Firms are input purchasers in the savings market and final producers of the loan market.

The results of the previous two sections imply that that the market supply of savings is an increasing function of the interest rate (r^s), whereas the demand for that input to the intermediating firms is a decreasing function of that rate. Conversely, the supply of loans from the intermediary is an increasing function of the borrowing rate, whereas it is demand is a decreasing function of the interest rate (r^b). The market clearing interest rates in those two markets determine the equilibrium difference between the saving interest rate and the borrowing interest rate. Intermediaries adapt to that difference and operate at the scale implied by equation 20.

VI. Intertemporal Aspects of Normative Policy Analysis: Benefit-Cost Analysis

One of the most widely used tools of policy analysis is benefit-cost analysis. In principle, benefit-cost analysis attempts to determine whether a given policy or project will yield benefits sufficient to more than offset its costs.

Cost-benefit analysis, ideally, attempts to find policies that maximize social net benefits measured in dollars. (Every diagram that includes a dead weight loss triangle is implicitly using cost benefit analysis.) Economists use this approach to characterize externality and monopoly problems. It is also used to criticize ideal and less than ideal public policies and taxes. Unfortunately, the data do not always exist for these calculations to be made. The most widely used methods for dealing with uncertainty and time in Benefit-Cost analysis is to use various combinations of "Expected Value" and "Present Value" calculations as developed in the next chapter.

Cost-benefit analyses carefully estimate the benefits, costs, and risks (probabilities) associated with of alternative policies through time. If several policies are possible, cost-benefit analysis allows one to pick the policy that adds most to social net benefits (in expected value and present value terms) or that has the highest social rate of return. If only a limited number of projects can be built or policies adopted, then one should invest government resources in the projects or regulations that generate the most net benefits (the highest rates of return in terms of social net benefits). One can also use cost-benefit analysis to evaluate alternative environmental policies.

When many projects can be adopted, the policy question is essentially a yes or no question is: Does the policy of interest generate sufficient benefits (improved air quality, health benefits, habitat improvements etc.) to more than offset the cost of the policy (the additional production costs borne by those regulated plus any dead weight losses and the administrative cost of implementing the policy)?

The *net-benefit maximizing* norm implies that both good projects, and good regulations, should have **benefit-cost ratios** that exceed one, B/C > 1. That is to say, the benefits of a project should exceed its costs if it is worth undertaking. However, many of the goods and services generated by environmental regulations *are not sold in markets* and so *do not have prices* that can be used to approximate benefits or costs at the margin. These "implicit prices" can be estimated, but the estimates may not be very accurate. Thus, a good deal of the policy controversy that exists among environmental economists is over the proper method of estimating non-market benefits and costs.

For example, the recreational benefits of a national forest may be estimated using data on travel time. However, this estimate is biased downward. We know that the benefit must be somewhat greater than the opportunity cost of driving to the forest! Survey data can also be used, but people have no particular reason to answer truthfully (or carefully) to such questions as how much would you be willing to pay to access "this national forest," "to protect this wetland," or to "preserve this species." In cases where the benefits and costs are not entirely predictable, the probability of benefits and costs also have to be estimated. In cases in which the benefits or costs are largely subjective and concern things that are not sold in markets, these benefits and costs also have to be estimated (but without very reliable data). The probabilities assigned to the various outcomes also are often difficult to estimate.

Thus, although arguably better than nothing, benefit-cost analysis tends to be quite inaccurate. So instead of attempting to find the best (social net benefit maximizing) policies, cost benefit analysis often simply attempts to determine whether the benefits of a policy exceed its costs. A policy is said to improves a situation if it generates Benefits greater than its Costs. This is, of course, a normative statement—one based loosely on the utilitarian school of philosophy.

In spite of all these difficulties, benefit-cost analysis has several advantages as method of policy analysis. It forces the consequences of policies to be systematically examined. It provides "ballpark" estimates of the relevant costs and benefits of regulations for everyone who is affected by a new regulation or program.

A Relatively Simple Illustration of an Environmental Cost-Benefit Analysis

Suppose that Acme produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Acme saves $5,000,000/year by using this disposal method, rather than one which does not endanger the ground water. What is the present discounted value of Acme's savings (much of which is passed on to consumers) if the interest rate is 10% and Acme expects to use this method for 30 years?

The easiest method is to use the formula $\mathbf{P = v\ [(1+r)^T - 1]\ /\ [r\ (1+r)^T]}$

although the additive formula, $P = \Sigma\ (Vt/(1+r)^t)$, can also be used. Here:

$$P = (5{,}000{,}000)\ [\ ((1+.10)^{30} - 1)]/[(.10)\ (1+.10)^{30}] = \$49{,}574{,}072.44$$

One could also approximate the present value of Acme's cost savings using the present value of an infinite series formula (P=F/r) which yields (5,000,000/0.1 = $50,000,000.00. Note that this simpler calculation produces nearly the same answer, and so is often a good way to check one's math.

Suppose that an environmental law is passed which requires firms like Amex to adopt the more costly but safer technology. If the fine assessed is $10,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate (interest rate) is 10% ? The expected fine in a given year has to be greater than the expected cost savings, Thus, P*10,000,000 > 5,000,000 in order for the fine to affect Acme's choice. (In this case the interest rate is not necessary for finding the solution because it is assumed that violations would be detected and fines paid annually. Although, we could also use present values for both the penalties and cost savings.) The smallest probability of punishment that "works" is 0.5, because this makes the expected fine equal to the expected cost savings.

Suppose that administering the enforcement regime costs $1.000,000/year that produces a 0.75 probability of punishment. What is the smallest annual external damage that can justify the program? Given the fine and probability of being caught and punished, we know that this program will induce Acme to clean up, so the only important question is when the present value of the damages (net of administration costs) avoided are greater than the present value of the extra costs borne by Acme (and its consumers).

Intuitively, we can see that if the damage per year (D) less the administrative costs ($1,000,000/year) are greater than the cost imposed then the program is worthwhile in cost-benefit terms. (D - $1,000,000 > $5,000,000). This implies that the damages must be greater than $6,000,000 per year. If the damages vary a bit through time, then we would need to use present and expected values to figure this out.

In that case the present value of the damages avoided minus the present value of the administrative costs would have to be greater than the present value of the cost increase imposed on Acme (and its consumers). If the damages were random, perhaps because rainfall is random, then we would have to compare the expected damage reductions (net of administrative costs) with the cost of "cleaning up."

For example, suppose that on rainy days the "dirty" waste disposal system causes $20,000,000of damages and that on dry days, the "dirty" waste disposal causes no damages to the local ground water supply. Suppose that it rains one third of the time. In this case, the expected damages from the "dirty" waste disposal system are: D^e = (.33) ($20,000,000) + (.67) (0) = $6,666,666 per year.

In this case the cost of eliminating the damage is the cost of the cleanup (more expensive waste disposal system) plus the administrative costs ($5,000,000 +$1,000,000) while the benefits are the expected reduction in damages: ($6,666,666 per year). The **expected present value** of the social net benefits from the program over thirty years can be calculated with formula $P^e = v\ [\ ((1+r)^T - 1)/r\ (1+r)^T]$ given a planning horizon (T) and discount rate (r). Let T= 30 and r = 10% again.

P^e = ($666,666) $[((1+0.1)^{30} - 1)/(0.10)\ (1+0.1)^{30}]$ = ($666,666) (9.4269)

Thus, P^e = $6,284,603.40

Given all these details, this program will produce a bit more than 6.28 million dollars of expected net benefits over a thirty-year period (in present value terms).

VII. Conclusions

Incorporating time into the models allows one to think systematically about long term plans and how they are affected by the discount rates used. Intertemporal choices have implications not considered in the timeless neoclassical models, namely the possibility of saving and borrowing. Such decisions may be used to smooth out one's lifetime consumption in order to increase lifetime utility.

This possibility at least partly accounts for the existence of financial interemediaries and their various methods for shifting the resources of savers to borrowers in the present and future income from present day borrowers to present day savers in the future. The entire financial sector of an economy would not exist without individual interests in shifting purchasing power from the future to the present or from the present to the future.

In settings where income is subject to random shocks, one may also save for reasons unrelated to consumption smoothing, such as to maintain reserves to deal with risks faced the ordinary course of life. The effects of risks and uncertainty on individuals and thereby on markets is taken up in the next chapter. Another possible reason to save is to provide future transfers of various kinds to one's children, friends, family, or foundations. That topic is taken up, albeit briefly, in chapter 17.

This chapter shows that an interest in income smoothing are a natural implication of diminishing marginal utility and is a sufficient interest to generate at least a modest financial sector. More complete analysis of individual interests in saving simply imply that the financial sector. Of course, the existence of a financial sector is, itself, of interest, but it is also a significant factor in the founding of new firms and for innovation, both of which are important determinants of the extent of both financial and non-financial markets.

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Appendix 13.1: Continuous Time, Dynamic Programing, and Intertemporal Choice

To be written

Appendix 13.2: Some Practice Exercises

1. Suppose that Al wins the lottery and will receive $100,000/year for the next twenty five years. What is the present value of his winnings if the interest rate is 6%/year?, 5%/year, 3%/year? How much more would a prize that promised $100,000/year forever be worth?
2. Suppose that Al can purchase lottery tickets for $5.00 each and that the probability of winning the lottery is P. If Al wins, he will receive $100,000 dollars per year for 20 years. The twenty year interest rate is 3%/year.

   What is the highest price that Al will pay for a ticket if he is risk neutral? Determine how Al's willingness to pay for the ticket increases as P, the probability of winning, increases and as the interest rate diminishes.
3. Suppose that Amex produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Amex saves $1,000,000/year by using this disposal method rather than one which does not endanger the ground water. What is the present discounted value of this waste disposal technology to Amex if the interest rate is 6%? if it is 4%?
4. Suppose that an environmental law is passed which requires firms like Amex to adopt the more costly but safer technology. If the fine assessed is $2,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate is 5%? if it is 10% ?
5. Suppose that global warming is caused (at the margin) by CO_2 emissions and that to reduce CO_2 emissions enough to affect future temperatures requires policies that will reduce economic output by 5% per year. U. S. GNP is currently about 15 trillion dollars and is expected to grow by about 2.5% per year in the future. How large do expected damages have to be to justify such an aggressive environmental policy?

   Hint 1: in this case, the future value of GNP is $Yt = 15*(1+.025)^t$, because of economic growth, which works like compound interest. The reduction in non-environmental income in year t is thus $Vt = (.05)15*(1+.025)^t$

Hint 2: This implies that present values can be calculated using the summation formula $P = \sum^{T} (V_t/(1+r)^t$ by substituting for $V_t = (.05)\ 15*(1+.025)^t$

{ That is to say, $P = \sum^{T} ((.05)\ (15 \text{ trillion})\ (1+0.025)^t/(1+0.05)^t$

Hint 3: more generally one can write this expression as $P = \sum^{T} (V_o\ (1+g)^t/(1+r)^t$ where g is the economic growth rate, r is the discount rate (interest rate), and Vo is the initial value of the "thing" that is growing at rate g.

Hint 4: It turns out that in a present value problem with an infinite planning horizon, one can use a relatively simple formula to calculate the present values of a series of values that grow by a constant percentage each year:

P = Vo / (r-g) where Vo is the initial value, r is the discount rate (or interest rate), and g is the long-term growth rate.)

[**Now you can easily calculate** the present discounted value of the cost of reducing CO_2 emissions in this way, which is approximately 30 trillion dollars, given all the assumptions made.]