Chapter 7: Risk and Market Outcomes

I. Competition and Profits

In the choice settings modeled in the first 6 chapters of the book, all the relevant decision makers were assumed to have clear overarching goals (utility, net benefits, or profit) and complete knowledge about their possibilities and the consequences of their choices. In such settings, no mistakes are ever made by rational decisionmakers. Thus, consumers choices always truly maximize their utility and firms always exactly maximize their profits.

Differences among markets and individuals may still exist, but they are not based on differences in knowledge or risks. In markets where a large number of firms (and potential firms) compete for the purchases of consumers, rates of return across markets tend to converge to the same rate. Any market with above average rates of return attracts entry by both new firms and firms previously selling products in other markets. In markets where firms earn below average rates of return, firms tend to exit and, to the extent possible, shift their resources to ones earning average or above average returns. Thus, in equilibrium rates of return equilibrate across markets. This is sometimes referred to as a "zero profit" equilibrium because firms (and firm owners) are earning just their opportunity cost rates of return on their investments (although their net revenues are all greater than zero).

This process is reinforced by the purchasers of goods and services. They purchase their goods from suppliers with the lowest price. This causes firms with above average prices to lose sales (in the limit all sales) and encourages such firms to lower prices. Such pricing pressures tend to cause all prices to converge to a single price for each good, one that is equal to each firm's marginal cost of production. (In Marshallian markets this will also equal the firm's lowest long run average cost.) As a consequence, price competition, in turn, induces all the firms in the market to be efficient—indeed equally efficient in Marshallian models—and thus all firms use the "best" (least-cost) production methods and are efficiently sized. General equilibrium theory, some decades later, demonstrated that prices exist that can simultaneously clear all markets, including input markets and markets for loans.

The context of such "perfect" competition is, of course, not the only one that can exist in the real world. A number of deviations form Marshallian competition are possible. First, firms may have unique resources—locations, especially talented managers, honest hard-working personnel, or better access to particular inputs. In such cases, what we have termed Ricardian markets tend to emerge. In those circumstances, profits may differ among firms and firms may differ in size even in settings in which there are many suppliers of the same products. Second, the demand for various attributes of products may differ sufficiently among consumers, and the firms that exist markets for particular types of goods may produce similar but not identical goods. In such cases, each firm may face its own downward sloping demand curve. Third, there may be various entry barriers that limit entry and exit possibilities both for new and existing firms. These may exist because of patent protection, regulations such licensing or paperwork that is costly for new entrants to satisfy, aspects of particular locations that limit the number of firms that can be supported, as water in a desert tends to limit the number and size of farms that can be supported even when the soil is fertile. Limits on entry may also occur because of differences with respect to internalized norms regarding innovation and competition. Fourth, there may be sufficient economies of scale (often caused by relatively large fixed costs) that only single firm or a very small number of firms can be supported by market demand in equilibrium. Fifth, consumers and firms may not know enough to make mistake-free decisions. Significant ignorance may exist about technological possibilities, the prices available at rival sellers, and the profits realized by firms selling in different markets. Or, the processes that generate consequences for firms and consumers may include random or unpredictable factors.

The first and fifth deviations explain why profitability varies among firms selling identical products, even if employees, owners, and consumers are essentially similar beings with similar goals. The third variation in circumstances suggests that government policies may be important determinants of the extent and efficiency of markets in a given country. The first and fourth were addressed to some extent in the second part of chapter 3, where firms were price makers rather than price takers. The fourth of these deviations from the circumstances of Marshallian competition provides an explanation for variation in market structure or industrial organization across markets. Industrial organization is a large, specialized field, and complete coverage of that field is beyond the intent of this book, as true of most of the subfields of microeconomics. That the attributes of a single family of goods—such as cell phones or automobiles—may vary in ways that matter to consumers is addressed in Chapter 15. The third ones—the economic effects of differences in legal, regulatory, and cultural environments—are analyzed in Part III of the book.

This chapter analyzes with the last of these deviations from the Marshallian circumstances. Individuals may simply not know enough to truly maximize profits or utility. Limitations on human knowledge occur for many reasons. This chapter focuses on two of these: risks (the existence of stochastic phenomena) and uncertainty (the existence of phenomena that are either currently or ultimately unpredictable). Other types of information problems are analyzed in chapter 9.

All these deviations from the Marshallian context affect the scope and density of market networks of exchange, production, and innovation—and insofar as rational choice models can be used to analyze their effects, they are all parts of the field of microeconomics.

II. Risk and Uncertainty: Two of the Limitations on Human Knowledge

This chapter deals with a two types of incomplete knowledge, namely that associated with phenomena that are produced via complex chains of causality that are not fully understood and others may be generated by well-understood random phenomena. In both cases, the consequences of one's choices are unpredictable in the sense that the exact outcome of an activity or quality of a product cannot be known with certainty before the activity is undertaken or a product is produced and purchased. When one rolls a die (a single dice), one knows that a 1, 2, 3, 4, 5, or 6 will appear on top after it stops rolling, but not which number will be wind up on top after the die stops rolling. The outcome can be regarded as either the outcome of a complex physical process or as a truly random process.

Statistical theory provides us with several ideas that can be used to describe choice settings in which decision makers are less than entirely certain of the consequences of their decisions. In some choice settings, one may be able to describe the consequences with well-known probability functions. A probability function maps possible outcomes into the probability that that outcome will occur. For example, the outcomes of a single roll of a symmetrical cube is distributed "uniformly" with each side of the die being equally likely to be on top after the cube stops rolling. In probabilistic settings, one never knows the exact outcome associated with particular actions, although they know the average or typical result, as well as the range of possible outcomes.

The phenomenon that can be described with a probability function are not always truly random, in the sense that they are not mechanically caused. However, in many cases, the process generating them is sufficiently complex that causal chains are difficult to describe, but nonetheless generate outcomes that have the properties of truly stochastic phenomena. The "random" number generators of computers are examples of such phenomena. Computer programmers may know how such "random" numbers are generated, but not by many other persons. Probabilistic ideas may also be used to describe settings in which causal change are complex, and neither the probabilities nor domain of possibilities are completely understood—as a mode of thought for making choices in uncertain situations (subjective probabilities).

Persons are less than perfectly informed in such circumstances, because they do not know the precise consequences of their choices or all of the relevant characteristics of their future choice settings. However, there is a sense in which they know as much as can be known. If the process is truly stochastic, then all that can be known about it is its probability function or probability density function. The broader the range of random possibilities is, the more important are the stochastic aspects of the choice setting and the less certain individuals can be of the outcomes of their choices. For example, uniform distributions are bounded between outcome L and outcome H, the lowest possible outcome and the highest, whereas normally distributed processes are unbounded. In the later cases, it might be said that "anything" can happen, although one can calculate the domain in which, for example, 95% of the outcomes will fall.

Frank Knight argued that economically relevant settings in which one does not understand a stochastic phenomenon or a complex causal chain sufficiently to describe it with 1:1 causality or with a probability function. In some cases, this might be because of ignorance. In others, it might be because the phenomenon itself is so idiosyncratic that it cannot be described with a probability distribution. Such settings, Knight termed "uncertain," whereas he terms the settings that can be characterized with probability functions to be settings of "risk" rather than uncertainty. Knight argued that risky choices can be completely modelled and have clear implications, whereas uncertain choices cannot be and do not. He also argued that all profits (and losses) realized in Marshallian competitive markets arise because of uncertainty rather than risk for reasons that we'll discuss later in this chapter.

The text is somewhat less careful in our vocabulary in this chapter than Knight was. It uses the terms risk and uncertainty more or less interchangeably, and it uses the term Knightian uncertainty for what Knight refers to as uncertainty.

III. Probability Functions, Expected Values, and Expected Utility

There are many economically relevant choice settings in which the benefits and costs of particular choices (or policies) are at least partly the consequence of random factors that can be represented with a probability function. Fortunately, a modest extension of the rational choice model can be used to characterize decisions in choice settings in which a well-understood probability function generates factors that are relevant for individual choices. In those circumstances, microeconomists normally assume that consumers maximize "expected" utility and firms maximize "expected" profits, rather than utility or profits per se.[1]

The notion of "**expected value**," itself, is an idea taken from statistics and means the average result associated with a large series of "draws" from a stable random process of some kind.

> DEF: Every **probability function** assigns probabilities to discrete events (here events 1, 2, … N) such that the sum of the probabilities is 1.0 and the numbers assigned to particular outcomes characterize the relative frequency or likelihood that that possibility occurs. (The probability that something will actually happen is 1, is completely certain, thus one of the possibilities will always occur.)

$$1 = \sum_{i=1}^{N} P_i \qquad with\ P_i > 0 \tag{1}$$

All possibilities, i, have positive probabilities of occurrence $1 \geq Pi > 0$. All impossibilities, j, have a zero probability of occurring and are not considered parts of a probability function.

The mathematical expected value is the sum of the values of those possibilities (here V_1, V_2 … V_N) times their particular probabilities of occurrence (here P_1, P_2, … P_N). It characterizes the large-sample *average* value of the distribution of the possible values in such samples.

> DEF: The **mathematical expected value** of a set of possible outcomes, 1, 2, ... N with values V_1, V_2, ... V_N and probabilities of occurrence P_1, P_2 , ... P_N is:

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[1] Here, it should be acknowledged that this is a larger assumption than the assumption that individuals maximize utility. If individuals have an over-arching goal, they will necessarily try to advance that goal with all of their "rational" decisions. Or, if they have an internally consistent preference ordering that ranks all relevant possibilities, they will behave in a manner consistent with models that assume that utility is maximized. In contrast, maximizing expected utility is only one of many ways to deal with risky situations. Instead, one might, for example, attempt to minimize risk (e.g. attempt to avoid risky situations and attempt to reduce the worst case outcome to the least damaging ones in the situations they find themselves) rather than attempt to maximize average utility associated with risky choice settings. They might focus on maximum likelihood probabilities when making choices rather than averages, as for example when crossing a street, one may ignore the probability that a "crazy" driver will come down on them and cause catastrophic losses. How one deals with risky settings is itself a choice. The use of average (expected) utility as metric for assessing the relative merits of risky choices is simply one of many strategies they may employ.

$$E(V) = \sum_{i=1}^{N} P_i V_i \quad (2)$$

Expected utility is a special case of expected values, namely it characterizes the average utility realized when "value" is measured in terms of utility (as utils). The *expected* utility associated with a probabilistic setting is calculated is thus in a similar manner:

$$E(U) = \sum_{i=1}^{N} P_i U(v_i) \quad (3)$$

where the *N* possible outcomes (v_1, v_2, … v_N) are associated with utility levels through an individual's utility function. To use this formula for expected utility calculations, one has to assume that the number of outcomes are finite and countable, that the values are finite, and that each outcome has a positive probability associated with it.

Expected values can also be calculated for random phenomena with a continuous domain. In those cases, a probability density function such as f(x) is used for the calculations, rather than a probability function. A probability density function is constructed so that the area under that function equals 1 and the probability that x takes a value between x' and x" is the area under that function between x' and x". Expected utility such cases is determined using an integral, rather than a summation, as with

$$E(U) = \int_{-\infty}^{\infty} \mathrm{f(x)U(x)dx} \quad (4)$$

That a probability function or probability density function is known is not an unreasonable assumption in many circumstances, and it is a reasonable first approximation of many others. The probabilities assigned may be the result of careful empirical work (frequentist probabilities) or based on theoretical reasoning (many natural phenomena are normally distributed, so this one probably is as well). Or, it may reflect cumulative learning about the likelihood of particular events that are continually updated as more evidence is gathered (Bayesian updating). Such probabilities are educated guesses rather than necessarily accurate. This last case is one way to use probabilistic choice models to think about choice settings that Knight would regard to be uncertain. Such models, for example, are used in chapter 8 to characterize a subset of entrepreneurial choices.

Most economists are quite willing to assume that circumstances exist where all the possible outcomes are known, probabilities can be assigned to them, and that the possibilities are countable or be modelled as countable. However, as noted above, there are also statistical tools for dealing with probabilistic phenomena whose outcomes are not countable. Nonetheless, in the probabilistic choice settings that are relevant for economics, the probabilities are themselves are usually individual estimates that are updated as research, policies, or persuasive campaigns take place, rather than precisely known by the individuals and organizations.

The assumption that probability or probability density functions are known allows models to be constructed that provide numerous useful insights into how probabilities affect the choices made.

Illustration of the Difference Between Expected Values and Expected Utility

To illustrate the difference between expected values and expected utility consider, the expected roll of a die (a single dice). Suppose that a single die is to be rolled. The face that turns up on top is a random event. Suppose that you will be paid a dollar amount equal to the number on the face that winds up on top. Since the probability of a particular face winding up on top is 1/6 and the value of the outcomes are 1, 2, 3, 4, 5, 6, arithmetic implies that the expected value of this game in money terms is \$3.50 = (1)(1/6) + (2)(3.5) + (3)(1/6) + ...(6)(1/6). If you played the game dozens of times, your average payoff per roll would be approximately\$3.50.

Note that the expected value of a single roll of a die is 3.5, a number that actually is impossible, rather than "expected" in the usual sense in ordinary English. The values are all whole numbers. This is not always the case, but this example illustrates that the meaning of "expected value" is a technical one: namely the large sample average result, rather than the result that you would most commonly observe. There are many probability distributions in which the average value is also the mode, as with the normal distribution, in which case the expected value is both the average result and also the most likely value to be observed,

Next, we'll repeat the exercise for a concrete utility function, namely $U = V^{.5}$, where V is the winnings from a particular roll of the die. In this case

$$E(U(V)) = \sum_{i=1}^{N} (1/6)\,(i)^{.5} \qquad (5)$$

Or $U^e = (0.1667) + (0.2357) + (0.2887) + (0.3333) + (0.3726) + (4082) = 1.8053$

Note also that if the intermediate cases were for some reason impossible—or simply ignored as far as prizes are concerned, there would be just two possibilities, each with a probability of .5. In that case the expected utility is:

$$U^e = .5\,(1)^{.5} + .5(6)^{.5} = (.5) + (1.2247) = 1.7247 < 1.8053 \qquad (6)$$

Expected utility falls because the stochastic event becomes "riskier" when only the extremes outcomes are possible. Note that the expected value of a single roll of the die is 3.5 in each case. Al, as we shall see later in the chapter, is quite risk averse.

Utility functions that can be used to calculate expected utility values that consistantely rank alternative outcomes (according to expected utility) are called **Von-Neumann Morgenstern utility functions**. Von-Neuman Morgenstern utility functions are all complete, transitive, continuous, and exhibit monotonicity. In addition, they have the property of what is sometimes called substitutability which is a form of internal consistency with respect to stochastic circumstances. If one is indifferent between outcomes x and y, then one is also indifferent between px and (1-p)z and py and (1-p)z, where p is the probability of event x. And, if z is regarded to be better than x, than $pu(x) + (1-p)u(y) < pu(x) + (1-p)u(z)$.

Experiments have been undertaken to use various gambles to create Von Neuman – Morganstern Utility functions—which, as it turns out, do not perfectly explain individual behavior under uncertainty in laboratories, but do so reasonably well. Von-Neuman Morgenstern utility functions for particular individuals are also "unique" up to a linear transformation (and considered by some to be a form of cardinal utility), because one can do arithmetic with them.[2]

Expected Utility with Continuous Probability Functions

Many economic choice settings concern variables that exist in a continuum, rather than being discrete. It is such choices that lent themselves to analysis using calculus-based models. Similar models can be developed to characterize choices where the outcomes are at least a bit uncertain, as the quality of an individual piece of fruit, bottle of wine, or automobile may not be known beforehand, because quality is itself a random variable. To see how uncertainty about quality affects consumer choices, consider the following choice setting.

Suppose that Al has a two-good strictly concave utility function, $U=u(A,B)$ where the prices of goods A and B are P_A and P_B respectively. Al has W dollars to spend in the period of interest. The quality of good A is not known at the point of purchase, whereas that of good B is known with

[2] There are many possible explanations for the departures from the prediction associated with maximizing expected utility in experimental settings. One is simply that individuals are not very good at statistical theory, in which case, individual choices tend to be error prone. Another is that there are many other plausible strategies that individuals might adopt for coping with risky choice settings.

certainty. Given f(q), the density distribution of the quality of some good that an individual may purchase, the expected utility for Q units of good A can be written as an integral of the following sort. The density function is distributed between the lowest quality, L, and the highest quality, H, possible for the good of interest. (Note that the substitution method has been used to characterize good B as a function of purchases of good A.)

$$\mathrm{U}^e = \int_L^H \mathrm{U}(\mathrm{Q}(\mathrm{q}), (W - P_A Q)/P_B)\mathrm{f}(\mathrm{q})\ d\mathrm{q} \tag{7}$$

The integral written above is the expected (or average) utility associated with purchase of Q units of the good with stochastic quality. Note that the quality is not entirely unknown, but always lies between L and H and is distributed between those to levels in a manner that is well-understood and can be represented with a probability density function, f(q), that can be integrated.

The quantity that maximizes expected utility can be found by differentiating expected utility with respect to Q and setting the result equal to zero. The first order condition in this case takes the form:

$$\mathrm{U}_Q^e = \int_L^H [U_A f(q) - U_B(\frac{P_A}{P_B})]d\mathrm{q} = 0 \ \text{ at Q*} \tag{8}$$

where A and B subscripts indicate derivatives with respect to the variables subscripted. Notice that the partial derivatives are results obtained by differentiating the integrand. The integral domains are carried forward and the function being integrated (the integrand) is replaced with its relevant first derivatives. Also notice that what one obtains are terms for the expected marginal benefit (the integral of the first term, in terms of utils) and for the expected marginal cost of units of A (the integral of the second term, again in terms of utils).[3]

The implicit function theorem implies that the Al's demand for good A, here Q*, can be written as Q* = g(P_A, P_B, W, H, L). The density function of quality uncertainty affects the shape of this function. However, as written, the demand function does not include a variable that characterizes that effect, but does include the end points of the domain (L and H) of that function.

[3] If one characterizes a utility function with a concrete function, then these integrals can often be evaluated. This is also possible for single-variable abstract functions—integrals of which simply return the initial integrand, which are then evaluated at the high end of the range of possibilities (here H) and then the low end of the possibilities (here L), which is subtracted from the high end value, which in turn gives one the (net) marginal utility of a change in Q.

If the utility function or the probability density function had included a "conditioning" variable, such as weather, that variable would have been included in the demand function.

The domain of the integral is determined by the probability density function. In the case used above, there is presumed to be a lowest (L) and a highest (H) quality. In other cases, such as the normal distribution, the limits would minus infinity and plus infinity. Some density functions are "full domain." Anything may happen, but some events more likely than others and some are extremely unlikely.

One uses the term "probability density function" (pdf) rather than "probability function" here, because probabilities are associated with integrals of (areas under) the density function, rather than by the function itself. Thus, the total area under both a conditional and unconditional probability function is 1 (by definition).

IV. Risk Aversion and the Demand for Insurance

> **DEF**: An individual is said to be *risk averse* if the expected utility of some gamble or risk is less than the utility that would be generated at the expected value (mean) of the variable that determines utility.

A **risk averse** person is one for whom the expected utility of a gamble (risky situation) is less than utility of the expected (mean) outcome when obtained with certainty. In mathematical terms, a person is risk averse if and only if $U(x)^e < U(x^e)$ where X is a binary random event, with one possibility, x', occurring with probability P and the other occurring with probability (1-P). $x^e = Px' + (1-P)x''$. This property is true of every possible pair of possible outcomes for a risk averse person.

This property also implies that any net benefit or utility function that is strictly concave with respect to income, exhibits risk aversion with respect to income or wealth types of variables. Why? Because expected utilities are convex combinations of utilities. Recall that a function is strictly concave only if af(x') + (1-a)f(x'') < f(ax' + (1-a)x'') for any x' and x'' and any value of a with 0<a<1. If one substitutes a probability for the term a, you can see that the two definitions are essentially identical.

A ***risk neutral*** individual is one for whom the expected utility of a gamble (risky situation) and utility of the expected (mean) outcome are the same. $U(x)^e = U(x^e)$. A ***risk preferring*** individual is one for whom the expected utility of a gamble is greater than the utility of the expected (mean) outcome. $U(x)^e > U(x^e)$.

The degree of risk aversion is often measured using the *Arrow-Pratt* measure of (absolute) risk aversion: $r(Y) = -(d^2U/dY^2)/(\frac{dU}{dY})$ which is a measure of how steeply downward sloping the marginal utility of income is at a particular point. In general, this implies that the more steeply downward sloping the marginal utility of income curve is, the more risk averse an individual is.

In the illustrating example above where $U = V^{.5}$, the marginal utility function is quite steeply downward sloping $\frac{dU}{dY} =.5V^{-0.5}$ and $\frac{d^2U}{dY^2} = -.25V^{-1.5}$, so

$$r(V) = -\frac{[-.25V^{-1.5}]}{.5V^{-0.5}} = (.25V^{-1.5})(.5V^{0.5}) = .125/V \qquad (9)$$

Note that the degree of risk aversion for this function varies with V, decreasing as V increases.

The utility Functions that imply risk-averse behavior are all strictly concave, as illustrated below. If the above individual, Al had been risk neutral, the Arrow-Pratt measure would have been zero. If $U = V$, then $\frac{dU}{dY} = 1$ and $\frac{d^2U}{dY^2} = 0$, which implies that $r(V) = \frac{0}{1} = 0$.

The Geometry of Risk Aversion and Risk Premia

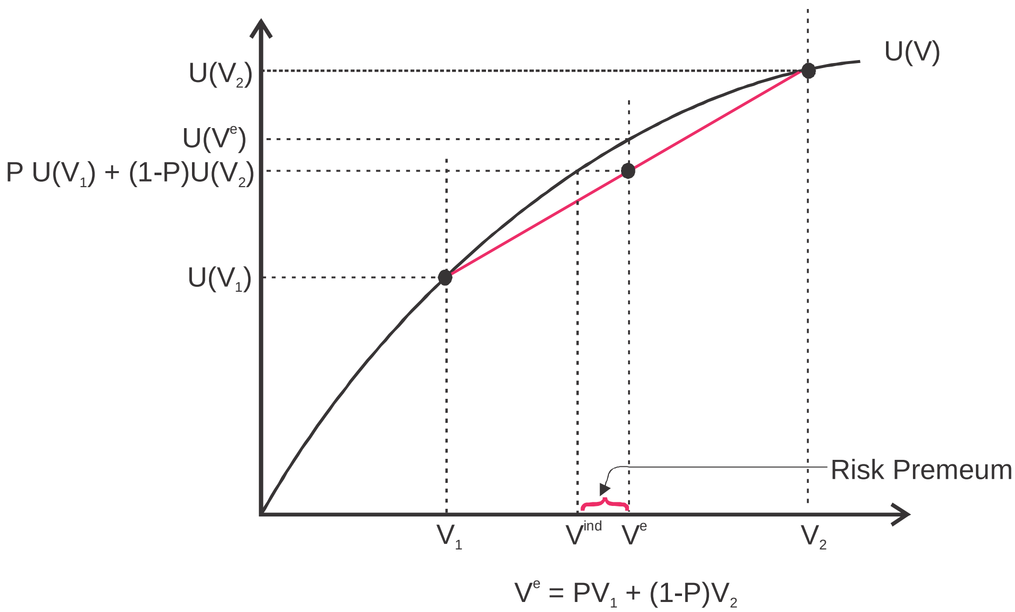
The figure below illustrates a choice setting in which an individual is risk averse and facing a risky environment in which either an outcome with the value V_1 or another outcome with the value V_2 will occur. The individual cannot influence which outcome it will occur, but knows that the probability of V_1 is P, which implies that the probability that V_2 is (1-P) (Recall that that the probabilities for the only two possible events have to add up to one.)

Let's refer to the individual as Al. Al's utility function is strictly concave, which means that a cord connecting any two points on it lies below the utility function (except for the two points used as end points—which, by definition, are not part of the cord). Assume that Al confronts an uncertain environment in which V_1 occurs with probability P and V_2 occurs with probability (1-P) Al's expected utility in that case is:

$$U^e = PU(V_1) + (1-P)U(V_2) \qquad (10)$$

As P increases from 0 to 1, the expected utilities trace out the cord between $U(V_1)$ and $U(V_2)$ and so will be below the utility function if it is strictly concave.[4]

Expected Utility, Risk Aversion, and Risk Premiums



This geometry is illustrated in the diagram above for a probability, P, that is approximately equal to 0.5, but it would be true for all probabilities $0< P < 1$ and all strictly concave utility functions.

This diagram can also be used to determine how much an individual would be willing to pay to have a certain payoff rather than face a risky or uncertain future. This is done by looking at the certain outcome that a person would be equivalent in their mind to the risky event. If we go to the left from the expected utility associated the two probabilistic outcomes over to the utility function and then down to the horizontal axis, we find the value (labeled V^{ind}) that Al would find equivalent to the risky one faced. (V^{ind} is the certain outcome that generates the same expected utility as the

[4] We have until this point used a "sufficient condition" for strict concavity, namely that a utility function is strictly concave if it has a positive first derivative for V and a negative second derivative for V. In other words, Al's utility function is concave if it exhibits diminishing marginal returns from V. However, however at this point the formal definition becomes a nice bridge between risk aversion and strict concavity. The expected value of V is $V^e = PV_1 + (1\text{-}P)V_2$. Note that if U is strictly concave then $U(V^e) > PU(V_1) + (1\text{-}P)U(V_2)$. As mentioned above, this looks exactly like the definition for concavity except that we've substitute "P" for "α".

risky one faced. "ind", stands for indifferent.) The difference in values, $V^e - V^{ind}$, (assuming that the values along the horizontal axis are in money terms) is the highest price that Al would pay to avoid the risk.

It is also characterizes the lowest expected value that Al would accept to bear the risky environment shown rather than have outcome V^{ind} with certainty. That difference is called Al's **risk premium** for this choice setting or "gamble." Note that Al would accept the gamble (risky environment) rather than V^{ind} only if the expected value of the risky payoff is greater than V^{ind}. How much greater would vary with Al's degree of risk aversion. The more risk averse Al is, the greater the risk premium would have to be.

The latter has implications for businesses in risky circumstances. Risk averse firm owners will demand a risk premium to bear the risks associated with their businesses. In such cases, Marshallian competition would generate different equilibrium profit-rates (returns) in different industries, ones that vary according to the riskiness of the business environment and the risk aversion of firm owners.

Risk premia also have implications for an individual's demand for insurance. An individual's risk premium also characterizes the highest amount that Al is willing to pay for insurance that eliminates the risk confronted. Note that the expected loss can be represented as $P(V_2-V_1)$ which is the distance from V_2 to $V^e=PX_1+(1-P)V_2 = P(X_1 - V_2) + V_2 = V_2 - P(V_2-V_1)$. This last expression characterizes the expected value of the risky setting in terms of the loss that occurs when the unfortunate event occurs—possibly a fire, accident, or a disease. The risk premium is the amount above the objective risk that Al is willing to pay to avoid the risk.

V. A Few Applications

1) Selling Fire Insurance

The existence of risk premiums plus the effects of sample size on sample means implies that selling insurance can be profitable. In our example, fire insurance transfers risk from homeowners to insurance companies. However, if the probability function is well known and the insurance company has many customers, the insurance company has only a very small risk. The average payout from selling insurance would be approximately $P(V_2-V_1)$ per customer, per year, and the price for the insurance can be **up to $P(V_2-V_1)$ + the risk premium** from the above figure. This implies that selling insurance can be profitable—although it does not guarantee it.

If a large number of purchasers for an insurance product exists, firms will have a quite predictable flow of expenses that are approximately equal to the expected value of the average loss, while customers are willing to pay more than that to avoid the risk of such losses. If the risk premium customers are willing to pay is more than enough to cover the cost of sales and administration of the insurance products sold, and insurance companies may be profitable investments. However, competition among insurance providers, in turn, tends to bring profit rate down to the "ordinary" rate of return that firm owners make from their other investments. But the prices for insurance in equilibrium will be sufficient to keep them in business.

Note that both the buyer choices and the insurance provider choices involve the mathematics of expected values rather than utility or profit maximization under certainty. Insurance is a market that would not exist without measurable risks and the ability to shift risk from one group to another. The ability to moderate risks through risk pooling—which is a property of the variance of sample averages, which tend to fall as sample size increases—is another key feature of the risks for which insurance is possible. (Recall that the variance of a sample mean is var(sample mean) = σ^2/n, where σ^2 is the variance of the variable being sampled and n is the sample size.)

2) Application: Expected Benefits Maximization and Uncertain Product Quality

Another case in which probabilistic thinking is likely to take place is regarding products of uncertain quality at the point of sale. For example, suppose that Al is considering purchasing some produce from a farm and knows that some of the produce will be of high quality (H) and some will be of low quality (L) but simply can't tell the difference between the two types or produce at the time of purchase, as is true of many types of produce (corn, potatoes, tomatoes, squash, etc). However, suppose that Al has sufficient experience with the farm or produce shop to know what the probability of a defective product is.

Suppose that there are just two levels of quality that tend to turn up, high quality and low quality. Suppose also that the probability of high quality is F and that price per unit is simply P. Suppose that the benefits of high-quality units is B(Q, H) and the benefits from quality units is B(Q, L) where B(Q,H) > B(Q,L) for every Q.

How many units will Al purchase? Al's expected net benefit from purchasing produce is expected benefits less expected costs:

$$N^e = FB(Q,H) + (1-F)B(Q,L) - PQ \qquad (11)$$

To find Q*, differentiate Ne with respect to Q and set the result equal to zero.

$$F\,(dB^H/dQ + (1\text{-}F)(dB^L/dQ) - P = 0 \qquad (12)$$

The first two terms of the expected marginal benefit of the produce and the last is its marginal cost. To find a specific value we would need to use concrete functional forms for the two benefit functions, as with $B^H = HQ^{.5}$. and $B^L = LQ^{.5}$, with H>L, in which case our first order condition would be:

$$.5FH/Q^{.5} + .5(1\text{-}F)L/Q^{.5} = P \qquad (13)$$

Multiplying both sides by $2Q^{.5}$ yields $HF + L(1\text{-}F) = 2PQ^{.5}$, which implies that $Q^* = [HF + L(1\text{-}F)]^2/4P^2$

In either case, the quantity Al purchases rises with F (the probability of the high-quality type) and with the benefit of the high-quality product, H (an indication of the quality of the high-quality type) and falls as low quality version of the produce decreases or price increases.

This risk is potentially insurable, but it may be too difficult to organize transaction by transaction and so remains an unprofitable type of insurance. (Notice the money-back-guarantees may reduce the buyer's risk, shifting it to the seller—who may then increase his or her price by more than the expected loss per customer. In effect, such guarantees make the seller an insurance company, and they can charge a premium for that service that their risk-averse customers are very willing to pay (at least up to their risk premium).

3) Applications: Quality Control—A Role for Management and Monitoring

Of course, quality variation is not only associated with agricultural products. All good and services have some variation in quality. Within mechanized construction processes, wear and tear, and product failures generate variation in quality affecting the usability of the products produce for buyers of the product. Such variation may affect the durability of the product sold as well as the benefits that it provides to buyers.

Insofar as quality variability can be estimated by consumers, and purchase decision are essentially independent of one another, we can use the net benefit maximizing model to characterize the demand for such products. $N^e = B^e(Q) - C(Q)$ which can be represented as $N^e = Fb^L(Q) + (1\text{-}F)b^H(Q) - PQ$ for the two-quality case, where F is the relative frequency of low-quality units and (-F) is the relative frequency of high value units and P is the price of the units purchased. The benefit functions are both assumed to be strictly concave.

The quantity that a purchaser would acquire would satisfy the first order condition: $dN^c/dQ = Fdb^L/dQ + (1-F)db^H/dQ – P = 0 \equiv H$. The implicit function theorem implies that $Q^* = f(P, F)$ with $dQ^*/dP = dH/dP/-dH/dQ = (-1) / -[Fd^2b^L/dQ^2 + (1-F)d^2b^H/dQ^2] < 0$ given the strict concavity of the benefit functions. The demand function is downward sloping in price. Similarly, $dQ^*/dF = dH/dF/-dH/dQ = [db^L/dQ-db^H/dQ] / -[Fd^2b^L/dQ^2 + (1-F)d^2b^H/dQ^2] < 0$. As quality diminishes, demand falls as well.

Notice that a firm selling this product and facing a downward sloping demand curve can influence the extent of demand though decision that affect the frequency of low-quality units. For example, a firm's monitoring expenditures, M, may reduce F, with F = h(M). In effect firms have two controls in their efforts to maximize profits—monitoring (M) and output levels (Q), rather than simply one as usually assumed. Moreover, in this case, the demand curve faced is partly determined by the firm's decision about quality.

Let P = g(Q, h(M)) characterize its inverse demand function. The firm's profits in this case can be characterized as $\Pi = g(Q,h(M))Q - c(Q,M)$ (assuming that input prices are constant in the period of interest). There will be two first order conditions for its profit maximizing efforts:

$$(dg/dQ)(Q+1) – dc/dQ = 0 \quad (14)$$

$$(dg/dF)(dF/dM)Q - dc/dM = 0 \quad (15)$$

Both first order conditions are simultaneously satisfied at the firms profit-maximizing output. This makes them a bit difficult to characterize in words. The monitoring decision will affect output decision by shifting the demand curve. And the output decision will affect monitoring by affecting the extent to which prices are affected by monitoring (dg/dF). But in each case, the ideal levels occur where marginal revenue generated by changing Q or M equals the marginal cost of Q or M.

Notice that this relationship makes perfect sense in any market where firms face downward sloping demand curves, but is makes less sense in settings where the firms face are price takers and face horizontal demand curves. An increase in quality in that case, might cause all consumers to want to purchase the product from the sole firm that has successfully improved its quality, which it cannot do. Or, it may cause the market to at least temporarily split in half, with one group of firms proving lower quality outputs than the other.

VI. Some Additional Applications of expected utility and expected profit maximizing choices

The 1960s was a period in which rational choice models were applied to fields generally regarded by most economists to lie outside of economics. These new areas of research gradually gained sway inside economics and expended the field to areas of the economics of regulation, socio-economics, law and economics, and political economy. Several of these areas made use of the expected utility maximizing model and expected net benefit maximizing model. Several Nobel prizes were awarded to the pioneers in these new areas of research, several of which are taken up in part III of this book.

4) Applications: Expected Values and the Effects of Regulation

i. One can also use this type of model to model the effects of economic regulation.

For example, in the area of environmental regulations, firms will take account of their overall net benefits from pollution including both cost savings and anticipated regulatory fines when choosing their production methods. In the absence of fines or fees for pollution and in the absence of enforcement of fines greater than 0, firms will choose their production methods to minimize their production costs—as in the models developed in the first part of the course (prior to the midterm).

(This does not necessarily mean that firms will pay no attention to air or water pollution, but they will do so only insofar as it affects the firm's expected profit through productivity and cost effects. Air or water quality that *affects the productivity of the firm's workforce* will be taken account of, but not spillovers on others outside the firm.)

ii. In the real world, regulations are only imperfectly enforced, and firms know this.

Consequently, it is not simply the magnitude of the fine or penalty schedule that affects a firm's decision to "pollute illegally or not," but also the probability that a person that violates the law will be caught, convicted and punished. Analyzing regulatory law and its enforcement on a firm's choice of production method and output level requires taking account of both the "expected cost" and "expected marginal cost" of any fines or penalties that might be associated with its production and output decisions.

(In addition, firms might face a loss of reputation and therefore reduced demand for their products if they are found guilty of violating regulatory law, but that effect will be ignored or assumed to be part of the fine.)

iii. Consider a case in which production methods are fixed and output is regulated—which is the easiest case to model.

In a regulatory environment with fines, a pragmatic firm's expected profits equal its total revenues less its production costs less its expected fines: $\Pi = R - C - F^e$ where $F^e = PF$. Suppose that Acme's output is sold in a competitive market, its cost function is $C=cQ^2wr$ and that its expected fine is the probability of being caught and convicted, which increases with output in excess of the regulatory limit, $p(Q-Q^R)$ and a fine schedule that increases with the extent of the violation $f(Q-Q^R)$ for $Q> Q^R$.

$$\Pi^e = PQ - cQ^2wr - p(Q-Q^R)f(Q-Q^R) \qquad (16)$$

To make the functional form a bit more concrete, let us assume that $P(Q-Q^R) = a(Q-Q^R)$ and $f(Q-Q^R) = b(Q-Q^R)$. In this case, Acme's expected profits are:

$$\Pi^e = PQ - cQ^2wr - a(Q^R-Q)\, b(Q^R-Q) = PQ - cQ^2wr - ab(Q-Q^R)^2 \qquad (17)$$

Assume that the regulatory constraint is binding on Acme, and so it will take the expected fine schedule into account when making its output decision. Its expected profit maximizing output can be characterized by differentiating the above function with respect to Q, which is a bit more complex than usual because of the "$Q-Q^R$" terms.

$$\Pi^e_Q = P - 2cQwr - 2ab(Q-Q^R) = 0 \qquad (18)$$

iv. This can be solved for Q* . First, shift the Q terms to the left side of the equal sign:

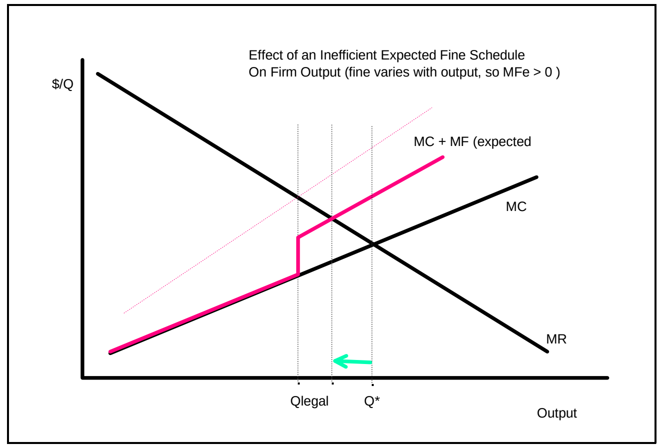
- $P = 2cQwr + 2ab(Q-Q^R) = Q(2cwr + 2ab) - 2abQ^R$
- Adding $2abQ^R$ and dividing yields:
- **$Q^* = (P+2abQ^R)/(2cwr + 2ab)$ (19)**

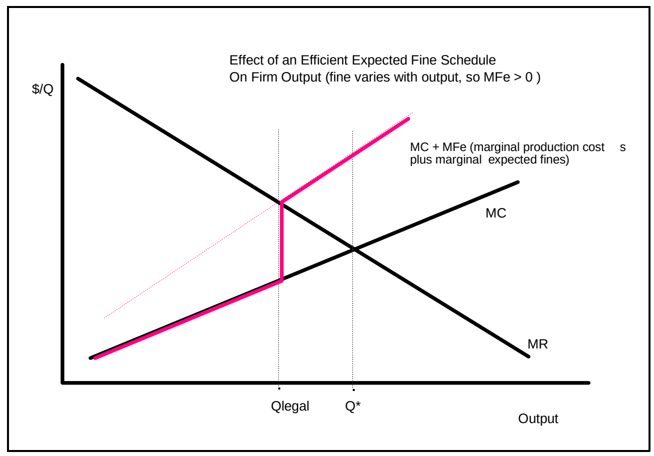
This is Acme's supply function in the regulatory environment modeled.

v. Note that its output now varies with the regulatory standard (Q^R) its input costs (w and r) and parameters of the probability of being fined and fine schedules (a and b).

Acme's output declines as input prices and the expected fines increase (w, r, a, or b increase) and increases as the regulatory threshold (Q^R) increases. (Another possible output is simply Q^R, but this cannot be modeled with calculus because of a discontinuity in the expected cost function at that quantity. See below.)

A. The diagram to the left illustrates Acme's decision in this type of setting (with somewhat simpler probability and fine schedules).





B. For students that have had public economics, note the similarities between Pigovian taxes and optimal enforcement with fines.

If the regulation attempts solve an externality problem and achieve Pareto efficiency, Q**, then the smallest fine sufficient to induce the target Q** has the **same expected value** as a Pigovian tax at Q** (with $Q^R \leq Q^{**}$). The expected fine should equal the expected marginal damages done by the Q**th unit of output.

C. Note that there is always a policy-tradeoff between the probability of conviction and the optimal level of punishment. [Recall that the expected fine is $F^e = PF$]

D. Some General Implications

i. The larger the fine, the smaller the probability of capture can be to generate the same effect on individuals.

ii. The larger is the probability the smaller the fine can be and still have the same effect.

a. The effect is determined by the expected fine, PF, in this case.

b. The probability that an illegal activity is detected and punished varies with the resources used to enforce the law and the flagrancy of the violation, so the probability of being caught and punished tends to vary with law enforcement budgets and the size of the violation.

c. The politics of enforcement and penalties are partly determined by error rates in detecting criminal activities--sometimes the wrong person is singled out for punishment.

- Puzzle. Given this, how would you pick the appropriate punishment for speeding? for theft? For murder? etc.
- Puzzle. How would the relative importance of the probability of detection and the expected fine be affected by the process of a jury trial and a long delay between being detected and being fined? (Some ideas for doing so are provided in the next chapter.)

- Puzzle: Write down an expected profit function for a firm facing a fine schedule that is imperfectly enforced, but where the fine increases as Q exceeds Q legal. Find the first order conditions and compare them to the above diagrams.
- Puzzle: draw examples of a perfectly enforced and imperfectly enforced "fixed fine schedule." (Such fines do **not** affect expected marginal costs.) Compare your graph with the mathematics of expected profit maximization in this case. Are such fines always irrelevant?

5) Applications: Expected Values and the Logic of Crime and Punishment

i. The economic analysis of crime derives from a classic paper written by Gary Becker (1968), who subsequently won a Nobel prize in economics, only partly for that contribution. In that paper, and in many others published since then, a criminal is modeled as a rational agent interested in maximizing his expected income or utility, given some probability of punishment.

ii. This type of model can be used to model theft and violations of other laws.

In the real world, criminal laws are only imperfectly enforced, and both criminals and ordinary persons who occasionally think about violating a law or two know this. For example, a net income maximizing criminal would maximize an expected function like

$$\Pi^e = PQ - cQ^2 - p(Q)F \qquad (20)$$

where Q is the number of crimes (thefts), price is the average price received by "fencing" the stolen goods, p(Q) is a probability function describing the way that that the probability of being caught and convicted varies with the number of crimes and F is the financial penalty assessed (or if jail time is spent, the opportunity cost of the time spent in jail and any subsequent losses in earnings).

The rational theft chooses Q* such that $\Pi^e_Q = 0$, which in this case requires Q* to satisfy $P-2cQ - p_QF = 0$ or $P = 2cQ + p_QF$ (set the marginal revenue from theft equal to its **expected marginal cost**, which is not known with certainty). Let's give the probability function a concrete form as with: $p = aQ_2$ then $p_Q = 2aQ$ and the above first order condition becomes $P-2cQ - 2aQF = 0$ or $P= 2cQ + 2aQF$, which can be solved for Q.

$$P = Q(2c + 2aF) \rightarrow Q^* = P/(2c + 2aF) \qquad (21)$$

Note that this implies that the rational criminal responds to incentives, his or her crime rate falls as the probability of being caught and convicted rises (e.g., with 2a), as the fine increases, and as the marginal cost of theft increases. Note also that there are tradeoffs between the size of the fine and the probability that a criminal is caught in terms of their overall effect on the criminal.

(This model provides a short form of Gary Becker's classic 1968 paper on crime and punishment.) Addition economic implications of crime and punishment are developed in chapter 15.

Many other examples from law and economics can also similarly modelled. One does not have to be a more or less professional criminal for this logic to apply. One can think of choices to drive faster than the speed limit on a highway or to park without putting money in a parking meter, or to trespass on a neighbor's property, fail to report some income on one's taxes, and so on in much the same manner.

VII. Some General Conclusions about Risky Choices

The main implication of this chapter is that neoclassical economics and its associated models can be easily extended to take account of risk—that is to say, choice settings in which outcomes are uncertain, but the probability of various outcomes can be accurately (or exactly) estimated. In such case, the logic of optimizing choice applies and the choices can be modelled in the usual way using diagrams in calculus that take account of risk.

There are several new implications.

First, that individuals with similar tastes may nonetheless differ in their degree of risk aversion. Individuals may all prefer more income or wealth to less income or wealth, but their utility functions (to the extent that these can be worked out or estimated) may differ in their curvature, in their degree of strict concavity. The Arrow-Pratt measure of risk aversion is one way to measure such differences. Some people are more risk averse than others, and so willing to pay more for insurance than others.

Second, risk aversion creates markets for insurance. If everyone were risk neutral, insurance like products might still exist, but there would be no sales of such products, because consumers would not be willing to pay a premium for those products—and that premium is necessary to cover the cost of administering insurance products.

Third, insurance products and similar products not always considered to be insurance imply that risks can be shifted from one individual to another and from one group to another. Moreover, they also demonstrate how risks can be pooled by insurance companies in a way that actually reduces overall risks, because of the statistics of sample means. As the sample size increases, the variance of the sample mean falls, and thus risk also falls. Insurance companies with accurate estimates of the probability of an insurable event, know almost precisely the amount that they will pay out from year to year.

Moreover, competition among insurers—as noted by Frank Knight—tends to reduce insurance company profits (for honest companies) to "ordinary" rates of return—the so-called zero-profit equilibrium of Marshallian perfect competition. It may not do so entirely, because of differences in size and organization, but the Marshallian implications can serve as a first approximation when there are large numbers of firms that provide insurance.

Fourth, there are many situations in which risks are not pooled and in which consumers, firms, or entrepreneurs have to make decisions in risky choice settings—shall I bring an umbrella or not? In these cases, one can use expected utility or expected profit maximization to model the decisions reached and their comparative statics. Although experimental evidence suggests that individuals do not perfectly behave as predicted, in most cases, the models provide a good first approximation for the average behavior and for how changes in circumstances affect that behavior.[5]

[5] See, for example, for an overview of prospect theory see KD Edwards (1996) or Tversky and Kahneman (1992, 2013)—who won a Nobel prize for their work in this area and others, that show some limits of the rational choice model.