THE CALCULUS OF COMMERCE

Chapter 1

On the Usefulness of Imperfect Models

ECON 701 · Advanced Microeconomics

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Roadmap for Today

- Why economic models are useful even though none of them is perfectly realistic.
- How science in general — not just economics — proceeds through model building.
- Two classic illustrations: the physics of falling objects, and the economics of human volition.
- How models are tested, refined, and occasionally replaced over time.
- Methodological individualism and rationality — the two foundations of microeconomic reasoning.
- How this textbook is organized, and the mathematical toolkit assumed going forward.

Models Are Not the Same Thing as Mathematics

- Pure mathematics is a closed logical system; its theorems are true by the rules of logic alone and make no claims about the real world.
- Economics uses mathematics to build models meant to describe real-world choice settings — that is a scientific claim, not a purely logical one.
- An inference drawn correctly from a model is always “true” logically, whether or not it describes the world accurately.
- Whether a model's economic implications actually hold in the world is a separate, empirical question.

No Model Is Perfectly Realistic — And That's Fine

- If a model were perfectly realistic, every inference drawn from it would show up in the real world; no model meets that standard.
- Realistic-but-imperfect models still capture many important features of the choices they describe.
- Results are typically stated “ceteris paribus” — other things being equal — a caveat that hides a great deal of complexity.
- Some things are left out on purpose to sharpen the analysis; other things are left out because no one yet knows they matter.

Imperfect Models Can Still Be Very Useful

- A model explaining even half the variation in prices or output is genuinely remarkable — and often the best tool available.
- Usefulness is judged partly by fit to known facts, and partly by whether the model reveals genuinely novel relationships.
- A simple model that is easy to teach and reason with can be more valuable to students than a more accurate but unwieldy one.
- Mastering a simple model is usually the fastest path to understanding a more sophisticated one later.

Every Science Is a Model-Building Enterprise

- The use of simplified models is not unique to economics — physics, biology, and every other science relies on them too.
- Specialization means every field leaves out the influence of factors studied in other fields.
- Physicists, biologists, and economists each abstract from the world differently to focus on what matters most to their own questions.
- Because no field is fully independent of the others, every model is, to some degree, incomplete.

Illustration: The Simple Theory of Gravity

Gravity alone explains its path almost perfectly.

Same gravity, but wind and aerodynamics scatter its path.

Volition dominates — the object chooses where to go.

The lesson: a model can be excellent for one case and nearly useless for another, without being "wrong."

A First Model of Volition: The Selfish Gene

- One simple model: organisms behave as if trying to maximize the chance their genes reach the next generation.
- This explains a great deal about biological behavior — even for organisms with no volition at all, such as viruses.
- Applied strictly to humans, it predicts people should try to maximize their number of descendants.
- Small families, celibacy, risk-taking, and war routinely contradict that prediction — economists need a broader model of volition.

Utilitarian Roots: From “Happiness” to “Utility”

> *“Nature has placed mankind under the governance of two sovereign masters, pain and pleasure …*
>
> *They govern us in all we do, in all we say, in all we think.”*
>
> *— Jeremy Bentham, An Introduction to the Principles of Morals and Legislation*

- Utilitarian philosophers argued that all persons share one goal: maximizing happiness.
- Nineteenth-century economists adopted “utility” as their term for this same idea of well-being.
- Because it is left open-ended — happiness from anything — the utility framework can represent an enormous range of behavior.

Samuelson's Fix: Revealed Preference Theory (1948)

- Critics said the utility-maximization model was too vague or unrealistic to be scientific.
- Samuelson showed that if a person's choices are merely internally consistent, numbers can be assigned to bundles that behave exactly like a utility function.
- This lets economists model people “as if” they maximize utility — without claiming they consciously do so.
- What matters for the model is consistency of choice, not conscious pursuit of happiness.

How Models Get Better: A Never-Ending Cycle

- Rival models often explain the same phenomena about equally well, so choosing among them is not automatic.
- A less accurate model that is easier to communicate or estimate may still be preferred for teaching or applied work.
- Anomalies — cases a model fails to explain — usually drive the next round of refinement.
- Model building, empirical testing, and refinement form a continuous cycle sometimes called “bootstrapping.”

Scientific “Truths” Are “Relatively Absolute Absolutes”

- Unlike mathematics, every science treats its accepted “truths” as provisional — true enough for now, subject to later revision.
- Newtonian physics still works for ordinary engineering, but Einstein's theories take over near the speed of light.
- Core neoclassical price theory from the late 1800s is still taught today, even as it keeps being extended and refined.
- Refinements to any well-established model typically take decades, sometimes centuries, to be fully worked out.

Illustration: Why We Still Say “Sunrise”

- We have known since Copernicus and Newton that the Earth rotates and the sun does not move around us.
- Our everyday language — “sunrise,” “sunset” — still reflects the older, more intuitive flat-earth picture.
- Even the “exact” published time of sunrise is really an approximation that ignores mountains, weather, and elevation.
- The simple models in Part I of this book work the same way: useful and mostly right, but not perfectly precise.

A Short History: From Aristotle to the AER

- Aristotle developed early theories of money and of “equilibrium” fair prices, tied to his theory of proportional justice.
- Adam Smith's Wealth of Nations (1776) used careful prose and evidence — not mathematics — to explain markets and specialization.
- Condorcet and Borda applied mathematical, rational-choice models to voting rules just before the French Revolution — decades ahead of most economists.
- Formal mathematical modeling of markets began in the late nineteenth century and became dominant across economics by the 1970s–80s.

Methodological Individualism

- All social phenomena — markets, politics, norms, organizations — are treated as the consequences of individual decisions.
- Prices and quantities in a competitive market emerge from the independent choices of thousands or millions of buyers and sellers.
- “Organizations,” “groups,” and “societies” have no properties beyond what emerges from the individuals composing them.
- Simple (“lean”) models assume purely self-interested choice; richer models add sympathy, hostility, norms, and internalized rules.

What Economists Mean by “Rational”

- In everyday English, “rational” suggests dispassionate, well-informed, sane reasoning.
- Economists mean something narrower: preferences that are transitive — if A is preferred to B and B to C, then A is preferred to C.
- Even a myopic or eccentric person can be “rational” in this sense, so long as choices are internally consistent.
- Local internal consistency is all that is needed for the choice-theoretic models used throughout this book.

How This Book Is Organized

Part I

Core Neoclassical Price Theory

- Consumer choice, firm output decisions, and production.
- Builds the behavioral foundations for price determination.

Part II

Time, Risk & Uncertainty

- Entrepreneurship, product variety, multi-product firms.
- An introduction to elementary game theory.

Part III

Beyond the Market

- Extends rational choice to law, politics, and norms.
- Effects on markets and on firm decision-making.

Throughout: general, teachable results rather than a catalog of special cases.

The Toolkit: What Chapter 1's Appendix Covers

- Preference relations — reflexive, symmetric, and transitive — the building blocks of “rational” choice.
- The formal definition of a utility function, and why its existence is equivalent to transitive preferences.
- Basic set-theory concepts used throughout the book: limits, closed sets, bounded sets, and compact sets.
- Convexity — why “better sets” and budget sets are assumed convex, and what that assumption buys us mathematically.

Appendix: Properties of Preference Relations

Reflexive: aRa for every a in X

Symmetric: aRb implies bRa

Transitive: aRb and bRc implies aRc

- Weak preference R means “at least as good as”; strong preference P means “better than.”
- Indifference I is built from R: if a is at least as good as c, and c as good as a, the two are indifferent.
- Rationality in microeconomics is usually defined as a transitive weak preference ordering.
- If every bundle can be ranked against every other, preferences are said to be complete.

Appendix: From Preferences to Utility Functions

$$U : X \to \mathbb{R} \text{ such that}$$

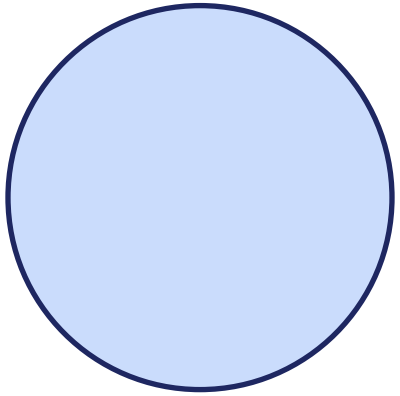
$$\text{if } a \, P \, b \text{ then } U(a) > U(b), \text{ and if } a \, I \, b \text{ then } U(a) = U(b)$$

- A utility function exists if and only if preferences over the relevant domain are transitive.
- Having a utility function does not require preferences to represent "satisfaction" — only that they are consistent and complete.
- Indifference curves are simply the graph of all bundles that generate the same utility level.
- Historical curiosity: economists graph price on the vertical axis, quantity on the horizontal — a convention traced to Alfred Marshall.

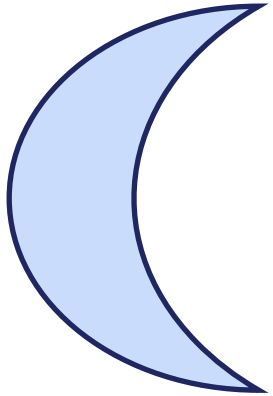
Appendix: Set-Theory Concepts You'll Need

- A set is closed if it contains all of its limit points, and bounded if no point lies infinitely far from the others.
- A set that is both closed and bounded is called compact — most opportunity sets in economics are assumed to be compact.
- A set is convex if the line segment joining any two of its points lies entirely within the set.

Circles, squares, and triangles are convex; doughnut and star shapes are not.



Convex



Not Convex

Why Convexity and Compactness Matter

- “Better than” sets — all bundles preferred to a given bundle — are standardly assumed to be convex.
- The budget set (all bundles affordable at given prices and wealth) is convex by construction.
- Convexity and compactness are what make differentiable functions — and calculus-based optimization — possible.
- These two assumptions underlie essentially every graphical and calculus-based model used later in this book.

Key Takeaways

- Every model — in economics or any other science — is imperfect; usefulness, not perfection, is the right standard.
- Good models fit known facts reasonably well and reveal at least one genuinely novel relationship.
- Microeconomics rests on two pillars: methodological individualism and rational (internally consistent) choice.
- The mathematical tools in the appendix — relations, utility functions, convex and compact sets — are the language the rest of this book is written in.

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