An Introduction to Evolutionary Game Theory

and Myopic Decision Making

I. Introduction: Two Decisionmaking Environments

A. Most mainstream game theory explores a setting in which game participants are fully rational and well informed.

i. They know every player's strategy set.

ii. They know the number of players in the game.

iii. They, therefore, know the full range of game outcomes that may arise.

iv. They know their own payoff for every possible game outcome.

v. **Thus every player is able to choose the best possible strategy.**

a. Although they may not win the prize, they purchase the expected utility maximizing number of lottery tickets.

b. Although they may be trapped in a prisoner's dilemma, no strategic adjustment by a single player can make **that player** better off.

c. The results are the best that can be achieved given the strategy sets available to individual players and their utility payoffs from the anticipated outcomes.

d. (Of course, the strategy sets assumed may be smaller than those we observe in the real world. For example, players in PD game can not form organizations to encourage cooperation.)

vi. **These are reasonable assumptions in relatively familiar and simple social settings.**

B. However, there are also complex and unfamiliar social settings in which players know only a subset of the information that would be useful to know.

i. In such cases, players may not know their complete strategy set, may not know the number of players in the game, may not know all the possible game outcomes.

ii. In such cases, players will not be able to choose the best possible strategies for themselves.

C. Moreover, it may be the case that individuals lack the ability to calculate the best strategies, given what they know.

i. For example, it is well-known that calculating the best strategy for playing a geme of chess is beyond the computational ability of grand chess masters or computers.

ii. Even though its rules can be learned in less than an hour, there are so many moves that are possible, the "decision tree" gets very large very quickly.

iii. In practice, both grand masters and computer s plan just a few moves ahead, and re-assess their strategies as the game emerges.

iv. Thus, it may be very reasonable to assumed that game players may not be able to perfectly optimize in complex settings, even if they have quite complete information sets.

D. Today we take up game theoretic analyses of such complex social settings.

II. A Digression: Does Complete Rationality implies Static Decision Making?

A. The overwhelming majority of game theory is static in the sense that the players make essentially one decision and then simply implement that decision as the game takes place.

i. It is as if, each player wrote down a complete decision tree for a game of chess at the beginning of each game and simply played out his or her preferred strategy as moves in the game take place.

ii. A robot or computer program could do this just as well, once a strategy is written down.

B. There are ways to create multiple decision games, by for example introducing stochastic factors or informational conditions that change through the game in ways that cannot or at least are not anticipated.

i. For example, one can imagine a game of football in a setting where it might rain or not. Clearly the best strategy is affected by the actual weather observed, which can not be known beforehand.

ii. However, even in this case, a forward looking game player can chose conditional strategies that include all the possibilities: e. g. play strategy C is the weather is clear and strategy R if it rains.

C. If a player knows all the possible outcomes that can occur and all the "messages" or "signals" that he or she might receive during the game, then he or she can in principle write down incomplete conditional strategies for playing the game that maximize his or her expected payoff.

i. In this sense, fully informed rational choice games can be represented as "single move" games.

ii. At the beginning of the game, each player makes a single choice of strategy and plays it as the game unfolds.

D. To introduce a bit of genuine dynamics, game theorists and other model builders often move a bit away from the full rationality assumption, by, for example, assuming that the game players have only local knowledge about the game, or can analyze only a few possibilities of their true strategy set.

i. Such players may be said to exhibit "bounded rationality" or "informationally bounded" decision making.

ii. In many cases, such players will behave **myopically**. That is to say, they behave as "local" rather than "global" optimizers.

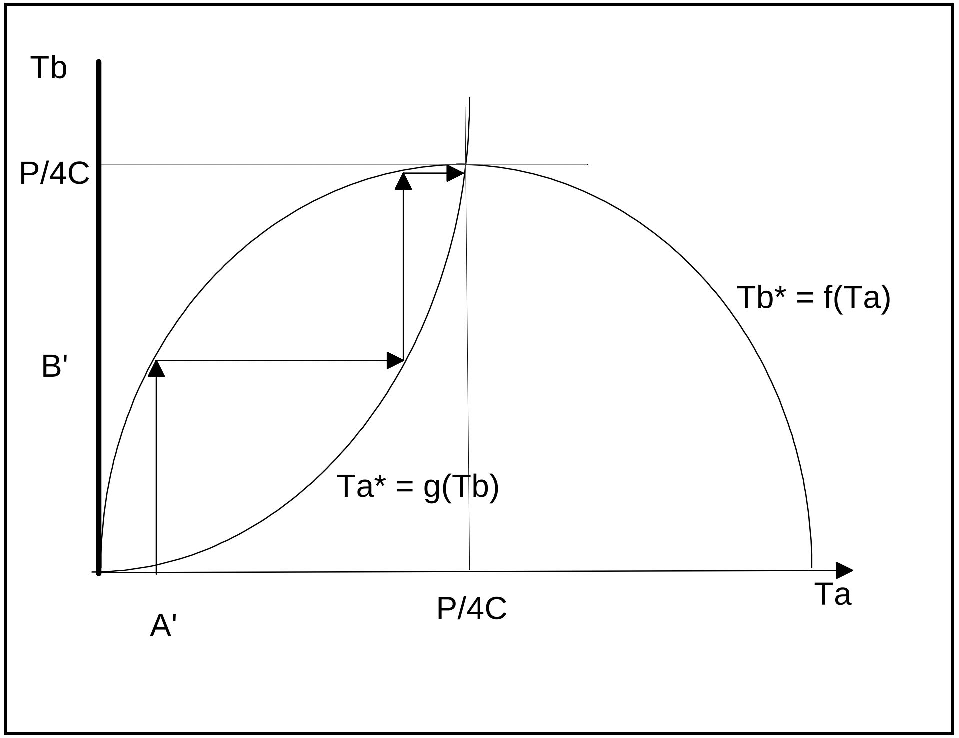
III. Illustration of Myopic Play using Best Reply Functions

A. In some cases, it make little difference whether we assume that players are fully informed and completely rational or poorly informed and myopic decision makers.

i. That is to say, there are many settings in which the same equilibrium emerges whether players are fully rational or myopic.

ii. This can be illustrated with a tool that we have already developed: the "best reply" or "reaction" function.

B. Illustrations of convergence to Nash equilibrium assuming myopic play and the reaction function of the Lottery Game that we analyzed a few weeks ago.



i. If we assume that "Al" plays A', and player "Bernd" plays his ideal response to A', Bernd will choose B' (using his best reply function).

ii. "Al" may then choose her best reply to B' using her best reply function, and so on.

C. In this case, a series of myopic adjustments converges gradually to the Nash equilibrium of the game.

i. (Such games are said to be **dynamically stable.**)

ii. In such cases, strategies converge to the Nash Equilibrium and stay there even if neither player knows that a Nash Equilibrium exists, or has any private knowledge of game theory per se.

D. Notice that our choice of "rational" or "myopic" models does not matter very much in cases in which the game has a unique Nash equilibrium that is dynamically stable.

i. Whether players are fully rational or not, there is a tendency for the outcome of the game to occur at the Nash equilibrium.

ii. The "full rationality" characterization of the Nash equilibrium, simply reduces the mathematical complexity of the calculations necessary to demonstrate that an equilibrium exists and that it has a particular form--whether it is a specific strategy combination, or a specific algebraic representation of the equilibrium.

E. On the other hand, **not every Nash equilibrium is dynamically stable,** and thus there are settings in which the predictions of "fully rational" and "myopic"choice models differ.

i. For example, myopic adjustment will never reach a Nash equilibrium that is not dynamically stable.

ii. If some equilibria are dynamically stable and others are not, then only the dynamically stable equilibria are possible outcomes in myopic (local optimizing) models of behavior. Rational choice game-theoretic models imply that all Nash equilibria are possible

iii. Moreover, myopic adjustments in games without a dynamically stable Nash equilibrium may "explode" (T heads toward infinity) or "disintegrate" (T heads toward zero).

F. Of course, further light can be shed on the "rationality" of individual choice of strategies in laboratory and computer experiments.

i. Consequently, there is a fairly large literature on laboratory experiments concerning behavior in a wide range of game settings.

ii. Unfortunately, the experimental literature is not able to guarantee that the payoffs are the same as those written down in mathematical models, because **the true payoffs are subjective utility levels in the minds of the players**, which may reflect norms and theories that experiment designers cannot fully control.

iii. Thus the experimental literature is very useful, if it is not perfectly conclusive.

IV. Population Dynamics and Evolutionary Game Theory

A. Another approach is to model "non-rational" evolution of propensities to adopt strategies.

i. That is to say, suppose the decision makers have so little information or so little computational ability that they simply adopt strategies and apply them without a careful consideration of the games in which they will be used.

ii. Players are not necessarily "irrational" but simply unable to fully optimize in the circumstance of interest.

B. Evolutionary game theorists usually assumed that players are randomly assigned to games with other players in the population (society) of interest.

i. Note that in this setting there may be both short term and long term equilibria.

ii. That is to say, in the short run, there will be stable game outcomes within a given population of players, and, thus, stable cumulative payoffs for the players using the particular strategies.

iii. In the long run, players who are not doing very well (relatively low cumulative payoffs) may choose different strategies, or may die out and be replaced by players using more successful strategies.

a. As the population of strategies in the game changes, the cumulative payoffs of particular strategies will change--because this depends in part on the other players that one plays against or with.

b. In the long run, a stable combination of strategies may emerge from this process of social or biological "replication."

iv. Such "evolutionary stable equilibria" are often made up of "evolutionarily stable strategies" (**ESS**), strategies which either do better than every other possible strategy, (and which do **exactly** as well as every other surviving strategy.)

C. Evolutionary game theory generally assumes that players occasionally "take stock" of the performance of their strategies relative to others being used by other players in the game setting of interest.

i. This may "taking stock" may be represented as a locally rational decision to improve individual strategies or as non-rational process analogous to biological evolution.

a. Some players my decide to switch from their old strategies to another strategy with a higher payoff.

b. Or, alternatively, players with relatively better performance may be more likely to find a mate and replicate themselves, or poorly performing strategies may be less likely to survive.

ii. (It is the latter case that makes evolutionary game theory of interest to biologists as well as social scientists who are skeptical about rational choice models.)

D. An Illustration of conditions for equilibrium populations of strategies

i. Suppose there is a society with a fixed population of size P composed of N players using M strategies, with N > M.

a. The fraction of the population using strategy j can be written as Fj, and the number of such players can be written as Nj = Fj N.

b. Each time that a player is placed in a game with other players (who are chosen at random), he achieves a payoff which varies with the other players in the game. Suppose there are K possible combinations of players, k = 1, 2, ... K. In this case, the payoff that player "j" receives in game "k" can be written as Uik.

c. And j's cumulative payoff is approximately the sum of the payoffs from each possible team times the probability of being in a game with players using particular strategies.

d. For example, in a series of two person games, the expected utility for player j is

$$U_j^e = \Sigma_K F_k U_k$$

ii. Suppose that the fraction of type j players in the next round is determined by their relative payoffs so that $F_j = U_j^e \,/\, \Sigma_N U_i^e = \Sigma_K F_k U_k \,/\, \Sigma_N \Sigma_K F_k U_{ik}$

iii. Stable population shares requires this ratio to remain stable through time for each type of player.

iv. This occurs in two cases:

- (i) when there are no other types of players in the game, and N = 1.
- (ii) It also occurs when the payoffs (U_i^e) are the same for all strategies (player types) that remain in the game.

a. In the latter case, there is no tendency of a strategy to increase it's population share relative to the others.

b. (In all other cases, the above average shares will tend to increase and below average shares tend to decrease.)