Studying for the Exam. The best method of studying for the final exam is to (i) review the class notes and readings, (ii) work through the study guide and (iii) review the lecture problems. Everything covered in the notes and in class may be included on the final, although clearly the exam will focus most of its attention on the fundamental concepts and games.

The hardest questions on the study guide will be more difficult than the hardest questions on the final. So, if you can do everything on the study guide you should be in good shape for the exam. The actual problems and terms on the exam will, of course, be somewhat different than the practice problems included below.

A. Identify and or Define

a. transitive ordering
b. utility function
c. better set
d. convex set
e. prisoner's dilemma
f. pure strategy
g. dominant strategy
h. Nash equilibrium
i. Pareto optimal outcome
j. mixed strategy
k.Cournot duopoly
l. Stackelberg duopoly
m. rational choice
n. "hyper" rationality
o. bounded rationality
p. extended form
q. sub-game perfect equilibrium
r. PDE
s. coordination game
t. Von Neuman Morganstern Utility

B. Use a two person two strategy matrix of payoffs to characterize the following social settings:

i. The Prisoner's Dilemma
ii. The gains from a mutually beneficial exchange
iii. The Tragedy of the Commons: in which output from a common property resource falls if usage is greater than a moderate level, M. (After M is reached, further use reduces total output.)
iv. The Free Rider Problem: in which a pure public good is to be provided. Assume that the subjective opportunity cost of producing a single unit of the public good is $6 and the benefit of producing the good is 5$ for each person. The cost of that unit of the good is shared, if both contribute to the good's production.
v. The Pork Barrel Dilemma: in which two coalitions favoring two negative net benefit projects. Because each coalition includes a bare majority of the parliament, in Nash equilibrium both projects are built, although the average member of each coalition is better off if neither had been built.
vi. The Samaritan's dilemma
vii. Plato's justice dilemma.
viii. The Stag Hunt game.

C. Construct a 2-firm Nash-Cournot model of duopoly for a market in which the demand curve is linear and the *marginal* cost of production is C.

i. Determine the equilibrium output.
ii. Determine the equilibrium price and profit levels for the two firms.
iii. Show that an increase in the cost of production leads to an increase in price and decrease in profits for both firms.

D. Use a game matrix to represent the "paper, rock, scissors" game.

i. Show that a mixed strategy is the only possible Nash equilibrium.
ii. Find a Nash equilibrium mixed strategy triple for your game.
iii. In what sense is your solution an equilibrium?

E. Suppose that Al and Bob have Cobb-Douglas utility functions defined over personal income, Y, and leisure, L, with equal exponents, so $U = Y^{0.5} L^{0.5}$. Assume that team production takes place via a Cobb-Douglas production function with equal exponents defined over Al and Bob's labor, so $Q = L_A^{0.5} L_B^{0.5}$. Al and Bob are each be paid their full marginal product (which is partly determined by the other's effort) and can allocate 16 hours between work and leisure. [This may seem like a very tricky game setting, but it is much easier than it looks.]

i. Find the Nash equilibrium of this team production game.
ii. Compare this equilibrium with the Pareto optimal level of effort, liesure, and team production that maximizes the sum of their utilities.
iii. Draw diagrams of these two solutions in the L^A x L^B plane.
iv. Can these results be represented as a two by two game matrix?
v. What is the normative problem?
vi. What side payment from Al to Bob and/or from Bob to Al can solve the problem?

F. Suppose that Al and Bob are in the running for a new position at their firm. The position brings with it a change in salary in the amount of R dollars. The individuals can spend their time producing unobservable output, Q, for the firm, or in transmitting evidence of their productivity, E, to their boss. The latter does not, itself, increase the firm's output. Initially assume that the employees are indifferent between "signalling" and "working."

i. In case one, the signals, E^A and E^B, are all that the firm's owner bases his decision on with the probability that Al gets the new salary being $P^A = E^A / [E^{A+} E^B]$ and the probability that Bernd gets the new salary is $P^B = E^B / [E^{A+} E^B]$. Characterize the work/signaling allocation decision of each employee on a typical work day if each unit of signaling costs the employee C units of utility.
ii. What happens to the "signaling equilibria" if the salary of the new position increases?
iii. What happens to the "signaling equilibria" if the cost of signaling increases?
iv. What happens if the firm can directly observe the output of each person and gives the promotion to the most productive person?

G. Develop an extended form (two period) game in which a contract between two individuals is self-enforcing.

i. Now represent your game in matrix form, and demonstrate that the Nash equilibrium is the same as in your extended form representation of the game.

ii. Explain the key assumptions of your game that make your contract self enforcing.
iii. How could your game be changed to create an enforcement problem?

H. Analyze the **Centipede Game.** The Centipede game is a finite length game in which each player may choose to stop the game on his turn. The player that stops the game receives payoff P+2t, where t is the number of times that the game has been played. The other player(s) receive 0. At time T the game ends, and the last player recieves P+2T.

i. Characterize the game in extended form using a game tree.
ii. What is the subgame perfect equilibrium of this game.
iii. Can this game be represented in matrix form?

I. *Repeated PD Game.* Analyze a PD game that is repeated two times.

i. First, develop a game tree representation of the problem. Note informational assumptions made at the various nodes, and the equilibrium strategy.
ii. Second, develop a matrix representation of the game in normal form, and determine the equilibrium. (Hint, player strategies are now DD, DC, CD, CC.)
iii. Characterize the Nash equilibrium of the game.
   a. Is this equilibrium sub-game perfect?
   b. Why or why not?
iv. Discuss the advantages and disadvantages of these two representations of this game.
v. Use a game in normal form to analyze a PD game that is repeated 3 times.

J. Use backward induction to demonstrate that the "rational" method of playing a finite-repeated prisoners dilemma game (say 2, 3, or T times) is defection.

i. How would an uncertain ending point affect your answer?
ii. How would the Folk Theorem change your conclusion for an infinitely repeated PD game?

K. *Fraud*. Suppose that Al uses George's garage for all car repairs.

i. Suppose that after paying his bill, cheating by George can always be determined by Al.
   a. George gains $25.00 each time he honestly services Al's car and $35.00 if he cheats.
   b. If Al leaves before service is obtained George's payoff is 0.
   c. Al receives $15.00 of consumer surplus if he uses George and $10.00 if he uses another garage (known to be honest, but a bit further away and more expensive). However, if George cheats, Al loses $15.00 (of surplus).
ii. Al tells George that if he ever believes that he has been cheated by George that he will never return to George's shop.
   - Is this a credible threat?
iii. Analyze this agency problem as a one shot game.
   - Should George cheat Al and/or should Al use George? Explain.
iv. Now consider the setting in which the garage game is to be repeated *two periods*.
   - Is the game now subgame perfect in non-cheating by George and use of George's by Al? Explain.
v. Now consider the setting in which the garage game is to be repeated *three periods*.
   - Is the game now subgame perfect in non-cheating by George and use of George's by Al? Demonstrate and explain.

L. Philosophical Applications: Hobbesian Anarchy

i. Use a 3 by 3 game matrix to characterize the "Hobbesian Jungle."
   a. Is there a Nash equilibrium?
   b. Is the Nash equilibrium Pareto Optimal?
   c. Note incentives to form a peace treaty of some kind.
ii. Now create a game that allows you to analyze individuals incentives to abide by a completely voluntary peace treaty.
   a. If not, how a might Hobbes' Leviathan might solve the problem.
   b. Is it possible for Leviathan to make the situation worse than in your model of anarchy?
      - If not, you have "proven" Hobbes' argument in favor of anarchy.
      - If so, you have proven Locke and Rousseau's critique of Hobbes. Explain.

M. Philosophical Application: Rawlsian Justice. Suppose that citizens chose "constitutional rules" from four different two-person or two-class games (representing four different sets of rules for the post contract game society).

i. Write down four two person team production problems with different assumptions about wages or productivity.
ii. Suppose that the probability of being in class "A" equals that of being in class "B."
iii. Which set of rules will a citizen prefer (a) if he/she is risk neutral, (b) is he/she is risk averse ($U = Ya$ with $0 < a < 1$), and (c) if he/she is very risk averse and so chooses on a maximin basis.
   a. In what sense are all of these possible results consistant with Rawls' characterization of justice as agreement from behind the veil of ignorance?
   b. Can you construct a fifth game that would be preferred to the four that you have characterized above?
   c. What constrains the feasibility of alternative games in a Rawlsian setting?
iv. Can a third "class" of citizen-voters, say the middle class, be added to your games?
   - Would this modification any of your above conclusions? (Show this.)

N. Develop a game theoretic representation of some other philosophical problem not already used in class.

O. Provide an overview of Axelrod's tournament-based analysis of repeated PD-games and Congleton-Vanberg's analysis of repeated PDE games.

i. In what sense, if any, do the results of these tournaments conflict with the usual rational choice model of games?
ii. Explain why tit-for-tat works so well in Axelrod's two tournaments.
   a. How important are the game payoffs in determining his results?

b. What role does cardinal utility (as opposed to ordinal utility) play in his method of ranking strategies?

c. Could the Axelrod tournament be done with only ordinal payoffs? Why or why not?

iii. Congleton and Vanberg use a version of the "replicator dynamic" to conduct their evolutionary simulations.

iv. Is cooperation rational or evolutionary in these simulations studies? Explain.

P. Philosophical issues:

i. Game theorists use a mathematical representation of utility which can be interpreted in various ways: it can simply be a mathematical representation of observe patterns of decision making (revealed preference) or it can be regarded as a mathematical representaion of the human "pleasure-pain" machine.

a. Are their significant methodolcotical differences between these two interpretations?

b. Does the analytical representation generate testable hypothesis about human nature, or merely provide a convenient representation of observed behavior?

ii. Utilitarian social welfare theory requires cardinal utility whereas the Pareto criteria only require ordinal utility. Discuss the advantages and dissadvantages of these two philosophical theories of the "good society," given your answers to question i.

iii. Expected utility calculations are one of many ways to represent decision making under uncertainty.

a. What assumptions about human nature are necessary, if any, for this to be a good representation of human decision making?

b. Are their other rational (consistent) methods of making decisions in probablistic settings?

c. Are there evolutionary advantages to either of these methods of making decisions when the risks faced are grave (say falling to one's death).

iv. Laboratory experiments suggest that a good deal of human behavior is inconsistant with the predictions of game theory. For example, experiments normally find a good deal of cooperation in PD games, which is far more than the "zero" predicted. On the other hand, there is significant defection.

a. What do these experiments tell you about the limits of game theory as a model of human decision making?

b. What do these experiments tell you about the limits of game theory as social science?

c. Does a theory have to be "perfect" to be correct or useful?