Environmental Economics Handout 4: Expected and Present Value Calculations

- A. In environmental economics there are many settings in which decision makers confront problems that involve both uncertainties and long time horizons. This is true at the personal level and at the policy level of analysis and choice.
- B. Not all environmental "facts" and "relationships" are known with certainty.

Many of those relationships are long term and partly random.

For example, smog densities depend on the weather (wind and rain) as much as effluent levels.

C. The policies adopted to address environmental problems may be well conceived or not, and well-enforced or not.

For example, a regulation normally requires enforcement, and enforcement is rarely perfect. So, there is always some probability that some one may violate the law and get away with it. How should such effects affect penalties or the type of regulation adopted?

- D. Together, these imply that the costs and benefits of environmental regulations are uncertain or probabilistic in both the short and long term.
- E. Indeed, as more subtle environmental problems have come to be addressed through public policy, the relevant flows of benefits and costs have grown longer and longer and less and less certain.

The benefits and costs of forest management span decades.

Those associated with global warming span centuries.

F. Thus the effects of time and uncertainty for economic (and political) calculations need to be addressed.

Neither uncertainty nor long time horizons make systematic cost benefit analysis impossible, although they do make it more difficult.

For example, a firm can still choose a pollution levels when environmental regulations are imperfectly enforced and major long term investments need to be considered.

The government (voters) may still analyze the proper level and enforcement of a Pigovian tax, even if the tax will be imperfectly enforced and must be in place for decades to have its desired effect. And, the great meta-questions may also be approached using a cost-benefit framework, adjusted to take account of uncertainty and long time horizes. What is the proper level of regulation itself?

G. This handout provides an overview of the tools used by economists (and most other policy analysts) to assess the costs and benefits of alternative environmental policies in settings where those policy decisions have **long term** consequences that are, at least partly, stochastic.

I. Decision Making under Uncertainty

A. In many environmental policy areas, the benefits of a regulation or damages done by pollution are at least partly the consequence of chance

For example, local air pollution is a much greater problem when the air is stagnant (hot humid Summer days) then when it is windy (a cool breezy day in the Spring).

To engage in policy analysis in such settings requires some way of taking account of the range of possible outcomes that any single policy might generate.

B. DEF: The mathematical expected value of a set of possible outcomes, 1, 2, ... N with values V₁, V₂, ... V_N and probabilities of occurrence P₁, P₂, ... P_N is

$$E(\mathbf{V}) = \sum_{i=1}^{N} \mathbf{P}_{i} \mathbf{V}_{i}$$

The **mathematical expected value** represents the long term *average* value of the distribution of values.

Recall that a probability distribution has the properties: $\Sigma Pi = 1$ (something has to happen) and $Pi \ge 0$ for all i (all possibilities, i, have positive probabilities of occurrence $1 \ge Pi > 0$, and impossibilities, j, have Pj = 0).

C. The *expected benefit* associated with a probabilistic setting is calculated in a similar manner:

$$E(B(V)) = \sum_{i=1}^{N} P_i B(X_i)$$

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where the N "value possibilities" are now measured in benefit terms associated with the affected individuals.

D. Illustrations: roll of a die, lottery game, terrorism

Suppose that a single die is to be rolled. Clearly which face turns up on top is a random event.

Suppose that you will be paid a dollar amount equal to the number on the face that winds up on top.

Since the **probability of a particular face winding up on top is 1/6** and the value of the outcomes are 1, 2, 3, 4, 5, 6, arithmetic implies that the expected value of this game is $3.50 = (1)(1/6) + (2)(3.5) + (3)(1/6) + \dots + (6)(1/6)$.

If you played the game dozens of times, your average payoff per roll would be approximately \$3.50.

E. Appendix: For Economics Students Bound for Graduate School

[In most theoretical work benefits are calculated in "utility" terms rather than in the dollar terms used in most applied work and policy analysis.

Utility functions that can be used to calculate expected utility values that properly rank alternative outcomes (according to expected utility) are called **Von-Neumann Morgenstern utility functions**.]

Von-Neuman Morgenstern utility functions are bounded and continuous.

Von-Neuman Morgenstern utility functions are also "unique" up to a linear transformation (and considered by some to be a form of *cardinal* utility).

DEF: An individual is said to be *risk averse* if the expected benefit of some gamble or risk is less than the utility generated at the expected value (mean) of the variable being evaluated.

Note that this implies that any benefit (or utility) function that is strictly concave with respect to income, exhibits risk aversion. (Why?)

A *risk neutral* individual is one for whom the expected benefit of a gamble (risky situation) and utility of the expected (mean) outcome are the same.

A risk preferring individual is one for whom the expected utility of a gamble is greater than the utility of the expected (mean) outcome.

[More formally, the degree of risk aversion can be measured using the *Arrow-Pratt* measure of (absolute) risk aversion: r(Y) = -U''(Y)/U'(Y)]





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- II. Application: Expected Values and the Logic of Crime and Crime Enforcement
- A. The economic analysis of crime derives from a fine paper written by Gary Becker, who subsequently won a Nobel prize in economics. In that paper, and in many others published since then, a criminal is modeled as a rational agent interested in maximizing his EXPECTED income or utility, given some probability of punishment.
- B. When applied to environmental regulations, this model implies that "potential polluters" will take account of their overall net benefits from pollution including both cost savings and anticipated criminal sanctions.
- C. In the absence of fines or fees for pollution and in the absence of enforcement of fines greater than 0, firms will *choose production methods to minimize their production costs*.

(This does not necessarily mean that firms will pay no attention to air or water pollution, but will do so only insofar as it affects the firm itself.

That is to say, air or water quality that *effects the productivity of the firms workforce* will be taken account of, but not spillovers on others outside the firm.)

D. In the real world, laws are only imperfectly enforced, and firms know this.

Consequently, it is not simply the magnitude of the fine or penalty schedule that affects a firm's decision to "pollute illegally or not,"

but also the probability that it will be caught, convicted and punished.

E. Analyzing enforcement on a firm's choice of production method or output level requires taking account of the "expected cost" and "expected marginal cost" of any fines or penalties that might be associated with those decisions.

In a regulatory environment with fines, the firm's expected profits equal its Revenues less its Production Costs less its Expected Fines. $\Pi = R - C - F^*$ where $F^* = PF$

To the extent that extra output increases Revenues, MR > 0. To the extent that extra output increases production costs MC>0, and to the extent that extra output over the legally allowed amount increases fines or the probability of being punished, $MF^e > 0$.

F. Illustration: "fixed fine schedules." (These do

G. **not** affect marginal costs)

- H. Illustration: "damage-based fines schedules." (A fine schedule such as $F = (Q-Q_{legal})f$, where f is the fine on each unit of output or emission over the allowed level, **do**es affect expected marginal costs associated with outputs greater than Q_{legal})
- I. Note the similarities between Pigovian taxes and optimal enforcement. If the regulation attempts to achieve Pareto efficiency, Q**,

then the smallest fine sufficient to induce the target Q^{**} has the same expected value as a Pigovian tax at Q^{**} .

J. Note also that there is a policy-tradeoff between the probability of conviction and the optimal level of punishment. [Recall that the expected fine is $F^e = PF$]

The larger the fine, the smaller the probability of capture can be to generate the same effect on individuals.

The larger is the probability the smaller the fine can be and still have the same effect.

The effect is determined by the expected fine, PF, in this case.

In more sophisticated analysis it is determined by the marginal expected fine, as drawn above.

III. Inter-temporal Choice: Time Discounting and Present Values

A. To calculate and compare streams of benefits or costs that flow through time, most economists use a concept called "present discounted value."

The **discounted present value** of a series of benefits and/or costs through time is the amount, **P**, that you could deposit in a bank at interest rate **r** and used to replicate the entire stream of benefits or costs, B_1 , B_2 , B_3 , ... B_T .

That is to say, you could go to the bank in year 1, and withdraw the amount (B_1) for that year, return in year 2, pull out the relevant amount for that year (B_2) and so on...until year T, when you would withdraw the final amoung B_{T} ..

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B. DEF: Let **Vt** be the value of some asset or income flow "t" time periods from the present date. Let **r** be the interest rate per time period over this interval.

The present value of V_t is $P(V_t) = V_t/(1+r)^t$

It is the amount, **P**, that you could invest today at interest rate **r** which would yield exactly amount **Vt** after **t** years. (Note that **r** is entered into the formula as a fraction, e. g. 4%=.04)

More generally, the present value of a series of income flows (which may be positive or negative) over \mathbf{T} years when the interest rate is \mathbf{r} (as a fraction) per period is:

$$\mathbf{P} = \sum_{t=0}^{T} (Vt/(1+r)^{t})$$

That is to say the present discounted value of any series of values is the sum of the individual present values of each element of the series.

(This is an important point, and allows a great many complicated streams of benefits and costs to be broken down into parts that are easy to handle and then added up to get the overall present value.)

C. In cases where a constant value is received through time, e.g. $V_t = V_{t+1}$ = v, a bit of algebra allows the above formula to be reduced to:

$$P = v [((1+r)^{T} - 1)/r (1+r)^{T}]$$

This formula has many uses in ordinary personal finance and environmental economics.

- For example the present value of a lottery in which one wins \$50,000/year for twenty years is
- $(50,000) [(1.05)^{20} 1) / (.05 (1.05)^{20})] = (50,000)(12.4622) = $623,110.52$ when the current interest rate is 5%/year.

This is, of course, **much less than** the \$1,000,000 value that lottery sponsors often claim for such contests.

Another use of the formula is to determine one's monthly mortgage or car payments.

A mortgage payment is just the reverse of a present value.

With a loan, you know P (the amount borrowed) and need to solve for "v" given your monthly interest rate r and the number of months over which the payments will be made, T.

- What you are doing with a bank loan is providing the bank with a cash flow approximately equal to the present value of the loan.
- (The bank profits by charging you somewhat more than its "own" market rate of interest.)
- D. Intertemporal utility maximization problems generally express the relevant budget constraints in present discounted value terms.

For example, suppose that an individual can allocate his or her current wealth over two periods (the present and the future).

Geometrically, this problem looks like an ordinary consumer choice problem except the axes represent consumption now and consumption in the future.

• The marginal rate of substitution between future and current consumption is sometimes called the *subjective rate of time* discount.

Optional, for advanced students: Mathematically, if U = u(C1, C2) and W = C1 + C2/(1+r), where C1 and C2 are consumption levels in the two periods, W is the present value of current wealth, and r is the relevant interest rate, either the Lagrangian or substitution methods may be used to characterize the optimal consumption expenditures in each year.

Note that first order condition(s) imply that the marginal rate of substitution between future and current consumption is equal to one plus the interest rate, (1+r).

The implicit function theorem allows consumption in both periods to be characterized as a function of interest rates.

The envelop theorem allows the effect of interest rates on maximal utility levels to be characterized.

IV. An Over View of Benefit-Cost Analysis

A. One of the most important tools of policy analysis is benefit-cost analysis. In principle, benefit/cost analysis attempts to determine whether a given policy or project will yield benefits sufficient to more than offset its costs.

Most systematic normative policy analysis in governments (and firms) is based on (net) benefit-cost analysis.

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Cost-benefit analysis, ideally, attempts to find policies that maximize social net benefits measured in dollars.

We have used this approach to find characterize environmental problems and policies that maximize net benefits.

The geometry of those diagrams can, in principle, be based on present discounted values and expected present discounted values.

(Unfortunately the data do not always exist for this calculation to be made.)

B. Cost benefit analysis often simply attempts to determine whether the benefits of a particular policy exceed its costs, rather than to find the best possible policy.



C. A policy improves a situation if it generates Benefits greater then its Costs.

- Explain why
- Discuss why "opportunity" costs matter in such calculations.
- To what extent is this consistent with the maximize social net benefit norm?.

Cost-benefit analysts carefully estimate the benefits, costs, and risks (probabilities) associated with of alternative policies through time.

- D. If several policies are possible, cost-benefit analysis allows one to pick the policy that adds most to social net benefits (in expected value and present value terms) or that has the highest social rate of return.
 - If only a limited number of projects can be built or policies adopted, then one should invest government resources in the projects or regulationsthat generate the most net benefits.
 - For example, cost-benefit analysis might be used to determine whether a particular dam yields sufficient benefits (electricity generation, recreation use of the lake, etc.) to more than offset its cost (materials used to construct dam, lost farmland and output, habitat destruction, homes relocated, etc.).

One can also use cost-benefit analysis to evaluate broad environmental policies.

- Here the question is:
- Does the policy generate sufficient benefits (improved air quality, health benefits, habitat improvements etc.) to more than offset the cost of the policy (the additional production costs borne by those regulated plus any dead weight losses and the administrative cost of implementing the policy)?
- E. The *net-benefit maximizing* norm implies that both good projects, and good regulations, should have **benefit-cost ratios** that exceed one, B/C > 1. That is to say, the benefits of a project should exceed its costs if it is worth undertaking.
- F. The most widely used methods for dealing with uncertainty and time in Benefit-Cost analysis are "Expected Value" and "Present Value" calculations as discussed above.
- G. There are, however, many practical *difficulties* in implementing benefit-cost analysis.

Ordinary economic costs and benefits can often be estimated fairly easily (at the margin) using market prices in cases in which markets are reasonably efficient and competitive.

Many environmental costs cannot be directly observed.

In cases in which the benefits and costs associated with a program continue into the future, all the future values have to be estimated.

In cases where the benefits and costs are not entirely predictable, the probability of benefits and costs also have to be estimated.

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H. Many of the goods and services generated by environmental regulations *are not sold in markets* and so *do not have prices* that can be used to approximate benefits or costs at the margin.

These "implicit prices" can be estimated, but the estimates may not be very accurate.

A good deal of the policy controversy that exists among environmental economists is over the proper method of estimating non-market benefits and costs.

- For example, the recreational benefits of a national forest may be estimated using data on travel time. However, this estimate is biased downward. We know that the benefit must be somewhat greater than the opportunity cost of driving to the forest!
- Survey data can also be used, but people have no particular reason to answer truthfully (or carefully) to such questions as how much would you be willing to pay to access "this national forest," "to protect this wetland," or to "preserve this species."
- I. In spite of these difficulties, benefit-cost analysis has several advantages as method of policy analysis:

It forces the consequences of policies to be systematically examined.

It provides "ball park" estimates of the relevant costs and benefits of regulations for everyone who is affected by a new regulation or program.

It can rule out obviously bad policies, eg ones that cost far more then they produce in benefits.

J. Illustration of Cost-Benefit Analysis: suppose that Acme produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Acme saves \$5,000,000/year by using this disposal method, rather than one which does not endanger the ground water.

What is the present discounted value of Acme's savings (much of which is passed on to consumers) if the interest rate is 10% and Acme expects to use this method for 30 years?

- The easiest method is to use the formula $P = v [((1+r)^T 1)/r (1+r)^T]$
- although the additive formula, $P = \Sigma (Vt/(1+r)^{t})$, can also be used,
- Here: P = $(5,000,000) [((1+.10)^{30} 1)/(.10) (1+.10)^{30}] = $49,574,072.44$

- One could also approximate the present value of Acme's cost savings using the present value of an infinite series formula (P=F/r) which yields (5,000,000/0.1 = \$50,000,000.00
- Note that this simpler calculation produces nearly the same answer, and so is often a good way to check one's math.

Suppose that an environmental law is passed which requires firms like Amex to adopt the more costly but safer technology. If the fine assessed is \$10,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate (interest rate) is 10%?

- The expected fine in a given year has to be greater than the expected cost savings,
- so P*10,000,000 > 5,000,000 for the fine to affect Acme's choice.
- (In this case the interest rate is essential for finding the solution, although we could also use present values for both the penalties and cost savings.)
- The smallest probability of punishment that "works" is 0.5, because this makes the expected fine equal to the expected cost savings.

Suppose that administering the enforcement regime costs \$1.000,000/year that produces a 0.75 probability of punishment. What

- is the smallest annual external damage that can justify the program?
- Given ii, we know that this program will induce Acme to clean up, so the only important question is when the present value of the damages (net of administration costs) are greater than the present value of the cost savings realized by Acme.
- Intuitively, we can see that if the damage per year (D) less the administrative costs (\$1,000,000/year) are greater than the cost savings then the program is worthwhile in cost-benefit terms. (D \$1,000,000 > \$5,000,000)
- This implies that the damages must be greater than \$6,000,000 per year.
- •
- If the damages vary a bit through time, then we would need to use present values to figure this out.
- In that case the present value of the damages avoided minus the present value of the administrative costs would have to be greater than the present value of the cost increase imposed on Acme (and its consumers).

If the damages were random, perhaps because rainfall is random, then we would have to compare the expected damage reductions (net of administrative costs) with the cost of "cleaning up."

• For example, suppose that on rainy days the "dirty" waste disposal system causes \$20,000,000 of damages and that on dry days, the "dirty" waste

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disposal causes no damages to the local ground water supply. Suppose that it rains one third of the time.

- In this case the expected damages from the "dirty" waste disposal system has expected damages, D^e = (.33) (\$20,000,000) + (.67) (0) = \$6,666,666 per year.
- In this case the cost of eliminating the damage is the cost of the clean up (more expensive waste disposal system) plus the administrative costs (\$5,000,000 +\$1,000,000) while the benefits are the expected reduction in damages: (\$6,666,666 per year).
- The expected present value of the social net benefits from the program over a thirty years can be calculated with formula $P^e = v [((1+r)^T 1)/r]$

 $(1+r)^{T}$ given a planning horizon (T) and discount rate (r). Let T= 30 and r = 10% again.

- $P^{e} = (\$666,666) [((1+0.1)^{30} 1)/(0.10) (1+0.1)^{30}] = (\$666,666) (9.4269)$
- so $P^e = $6,284,603.40$

This program will produce a bit more than 6.28 million dollars of expected net benefits over a thirty year period (in present value terms).

V. A Few Practice Exercises

A. Suppose that Al wins the lottery and will receive \$100,000/year for the next twenty five years.

What is the present value of his winnings if the interest rate is 6%/year?

- How much more would a prize that promised \$100,000/year forever be worth? (Hint: refer to class notes or find the limit of formula IIB as T approaches infinity.)
- B. Suppose that Al can purchase lottery tickets for \$5.00 each and that the probability of winning the lottery is P. If Al wins, he will receive \$50,000 dollars per year for 20 years. The twenty year interest rate is 3%/year.

What is the highest price that Al will pay for a ticket if he is risk neutral?

Determine how Al's willingness to pay for the ticket increases as P, the probability of winning, increases and as the interest rate diminishes.

C. Suppose that Amex produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Amex saves \$1,000,000/year by using this disposal method rather than one which does not endanger the ground water.

What is the present discounted value of this waste disposal technology to Amex if the interest rate is 8%? if it is 5%?

Suppose that an environmental law is passed which requires firms like Amex to adopt the more costly but safer technology. If the fine assessed is \$2,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate is 5%? if it is 10% ?

- D. Suppose that global warming is caused (at the margin) by CO₂ emissions and that to reduce CO₂ emissions enough to affect future temperatures requires policies that will reduce economic output by 5% per year. U. S. GNP is currently about 15 trillion dollars and is expected to grow by about 2.5% per year in the future. How large do expected damages have to be to justify such an aggressive environmental policy?
 - Hint 1: in this case, the future value of GNP is Yt = 15*(1+.025)^t, because of economic growth, which works like compound interest. The reduction in non-environmental income in year t is thus Vt = (.05)15*(1+.025)^t
 - Hint 2: This implies that present values can be calculated using the summation formula $P = \Sigma (Vt/(1+r)^{t})$ by substituting for $Vt = (.05) 15^{*}(1+.025)^{t}$
 - That is to say, $P = \Sigma$ ((.05) (15 trillion) (1+0.025)^t/(1+0.05)^t
 - Hint 3: more generally one can write this expression as $P = \Sigma$ (Vo

 $(1+g)^t/(1+r)^t$ where g is the economic growth rate, r is the discount rate (interest rate), and Vo is the initial value of the "thing" that is growing at rate g.

- Hint 4: It turns out that in a present value problem with an infinite planning horizon, one can use a relatively simple formula to calculate the present values of a series of values that grow by a constant percentage each year:
 - **P** = Vo / (**r**-**g**) where Vo is the initial value, **r** is the discount rate (or interest rate) and g is the long term growth rate.)
- [Now you can easily calculate the present discounted value of the cost of reducing CO₂ emissions in this way, which is approximately 30 trillion dollars.]