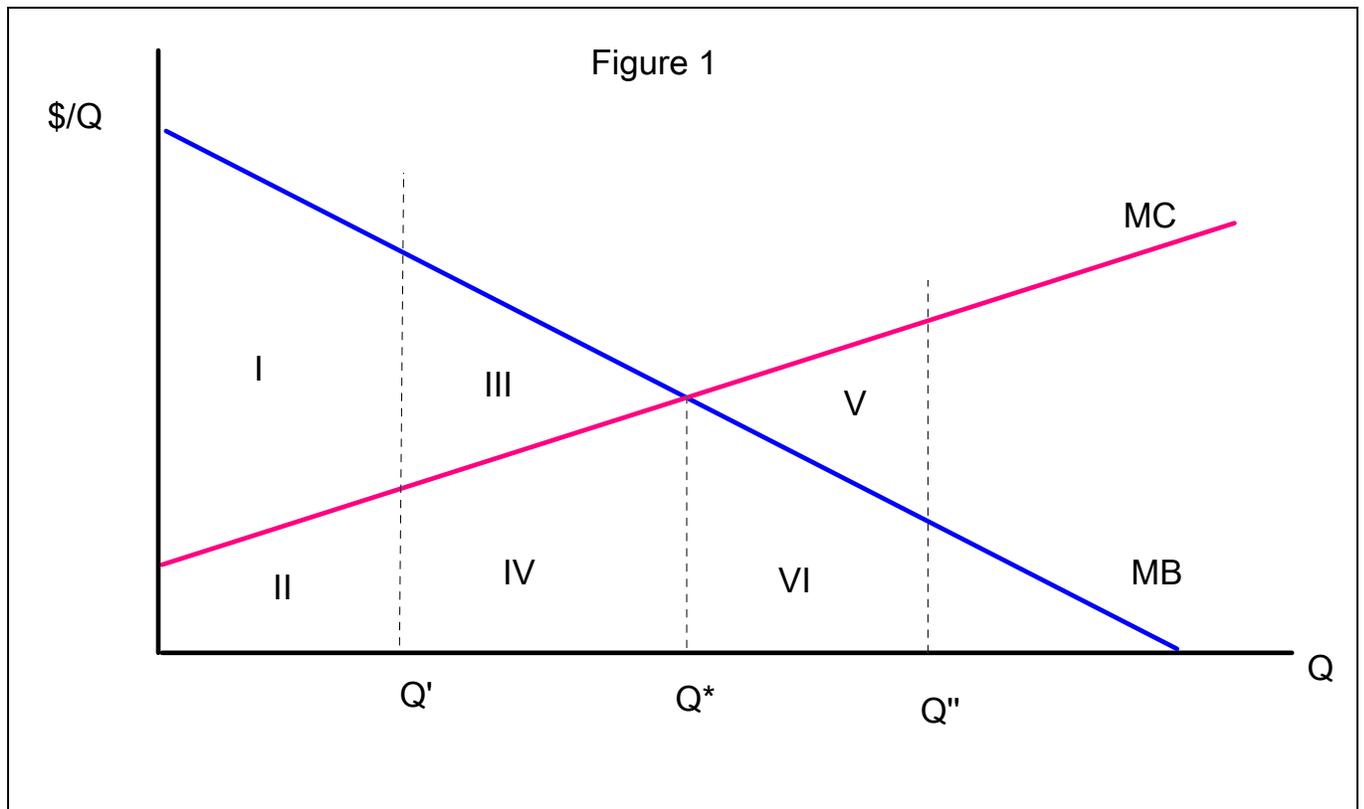


I. The Essential Logic and Geometry of Net Benefit Maximizing Choice

- A.** Nearly all economic models can be developed from a fairly simple model of rational decision making that assume that individuals maximize their private net benefits.
- i. Consumers maximize consumer surplus: the difference between what a thing is worth to them and what they have to pay for it. $CS(Q) = TB(Q) - TC(Q)$
 - ii. Firms maximize their profit: the difference in what they receive when they sell their products and what it costs to produce them. $\Pi = TR(Q) - TC(Q)$
- B.** The change in benefits, costs, revenue, etc. with respect to quantity consumed or produced is called Marginal benefit, Marginal cost, Marginal revenue, etc..
- i. DEF: Marginal "X" is the change in Total "X" caused by a one unit change in quantity. It is the slope of the Total "X" curve. $X \in \{\text{cost, benefit, profit, product, utility, revenue, etc.}\}$
 - ii. *Important Geometric Property:* Total "X" can be calculated from a Marginal "X" curve by finding the area under the Marginal "X" curve over the range of interest (usually from 0 to some particular quantity, Q).
 - iii. This property allows us to determine consumer surplus and/or profit from a diagram of marginal cost and marginal benefit curves and marginal cost and marginal revenue curves.
- C.** Marginal Benefits and Marginal Costs vary for systematic reasons.
- i. The marginal cost of an object that can be purchased in a market varies with its price.
 - The higher the price, the higher is the marginal cost to a consumer-purchaser of the good.
 - ii. The marginal benefit of a good varies with a persons "subjective" assessment of the value of the object, the quantity already possessed, and the amount of money that the person has in his possession that can be used to pay for it.
 - iii. Note that marginal cost is often affected by legal setting.
 - For example, if the object is illegal in addition to the price of the good there is the risk of being punished (probability of being arrested and convicted) and the punishments that may be imposed.
 - These create other costs--the marginal expected fine as we will see later in the course.
 - In cases in which there are no legal systems, there may also be marginal costs associated with protecting the goods acquired from theft and destruction.
 - iv. Marginal benefits may also be affected by legal setting, because the law often determines how a good can be used.
 - A bicycle has lower marginal benefits if one is not permitted to ride it on roads or sidewalks.
 - Note that if enforcement of this restriction is less than perfect, then it may be said instead that such laws affect the cost rather than benefits of owning and using a bicycle.

D. The Geometry of Rational Choice



- i. Given the marginal cost and marginal benefit curves in Figure 1, it is possible to calculate the total cost of Q' and the total benefit of Q' .
 - The total cost of Q' is the area under the MC curve from 0 to Q'
 - The total benefit of Q' is the area under the MB curve from 0 to Q'
 - $TC(Q') = II$; $TB(Q') = I + II$.
- ii. Similarly, one can calculate the net benefits of any output or activity level Q by finding total benefit and total cost for the quantities of interest, and subtracting them.
 - Thus the net benefit of output Q' is $TB(Q') - TC(Q') = [I + II] - [II] = I$.
- iii. Use Figure 1 to determine the areas that correspond to the total benefit, cost and net benefit at outputs Q^* and Q'' .
- iv. Answers:
 - $TB(Q^*) = I + II + III + IV$, $TC(Q^*) = II + IV$, $NB(Q^*) = I + III$
 - $TB(Q'') = I + II + III + IV + VI$, $TC(Q'') = II + IV + V + VI$,
 $NB(Q'') = I + III - V$
 - (It should be clear that a net benefit maximizing individual will prefer Q^* over Q' and Q'' ! [why?])

- E.** *If one attempts to maximize net benefits, it turns out that generally he or she will want to consume (or produce) products and services at the point (quantity where marginal cost equals marginal benefit (at least in cases where Q is very "divisible").*
- i. All net-benefit maximizing decision makers, thus, tend to choose activity levels where **their own marginal costs equal their own marginal benefits**--not because they care about "margins" but because this is how one maximizes net benefits.
 - ii. There is a nice geometric proof of this for cases in which MB crosses MC from above.
 - ("C.iv" above implies this; see also your class notes.)
 - iii. This characterization of net benefit maximizing decisions is **quite general**, and can be used to model the behavior of both firms and consumers.
 - We will use this property to derive individual level demand and firm level supply curves.
 - These individual demand and supply curves can then be used to derive market demand and supply curves, by adding up the individual curves for the markets of interest.
 - A very similar logic and geometry can be used to characterize the policies that maximize net benefits for a given society, insofar as "all" relevant costs and benefits can be computed.
 - iv. It is important to realize, however, that the logic of maximizing net benefits is **not always** simply a matter of looking for where $MC=MB$.
- F.** The **"consumer surplus" version of the net-benefit maximizing model** can be used to derive a consumer's demand curve for any good or service (and marginal benefit curves)
- i. One does this by assuming that one knows the consumer's marginal benefit curve and then: (i) choosing a price, (ii) finding the implied consumer MC curve, (iii) finding the net-benefit (CS) maximizing quantity of the good or service, and (iv) plotting price and the CS maximizing Q^* . (v) This process is repeated several times with different prices to trace out an individual's demand curve.
 - ii. The demand curve tells you how many units of the good or service that an individual will buy at a given price. It is a function from P into Q .
 - iii. It turns out that when the marginal benefit curve is downward sloping (and $MB>0$) that the demand curve will go through exactly the same points as the MB curve.
 - This implies that one can use estimates of individual demand curves to estimate a person's MB curves. (A person's MB curve is a function from Q into benefits in $\$/unit$ terms.)
 - iv. Although these curves go through the same points, they are not the same functions.
 - A demand function goes from price ($\$/unit$) into quantity (units); whereas MB goes from quantity (units) into benefits per unit ($\$/unit$).
 - That is to say, the functions are inverses of one another.

- G.** Similarly, one can use a net benefit (**profit**) maximizing model to derive a competitive firm's short run, medium, and long run supply curves.
- i. To derive a firm-level supply curve, one assumes that the firm's short, medium, or long run marginal cost curve is known. Given that curve, one: (i) chooses a price. (ii) uses that price to characterize the firm's MR curve, (iii) finds the profit maximizing output, (iv) plots P and Q^* . (v) This process is repeated with different prices to trace out the firm's supply curve.
 - ii. The supply curve tells you how many units of the good a firm will produce at a given price. It is a function from P into Q.
 - iii. Note that when the MC is upward sloping (and $MC > 0$) that a firm's supply curve goes through the same points as his MC curve.
 - iv. This implies that one can use estimates of a firm's supply curve to estimates a firm's MC curve. (MC is a function from Q into cost in terms of \$/unit.)
 - (As in the case of demand curves, a firm's supply curve is the inverse of its MC curve. Supply goes from prices (\$/unit) into quantities; whereas MC goes from quantity into \$/unit measures of how costs increase as output increases.)
- H.** That each person maximizes their own net benefits does not imply that every person will agree about what the ideal level or output of a particular good or service might be.
- i. Most individuals will have different marginal benefit and/or marginal cost curves, and so will differ about "ideal" service levels.
 - ii. Moreover, sometimes people miss-estimate their benefits and costs and make mistakes.
 - iii. Because each individual's and firm's choices can be represented with marginal benefit and cost curves, allow these curves can be used to model all sorts of economic and political behavior, all sorts of market outcomes, and the effects of public policies on individuals and markets.

II. Competitive Markets and Social Net Benefits

- A.** Market **Demand** can be calculated by **adding up** all the amounts that consumers are interested in buying at given prices. Similarly, Market **Supply** (in the short, medium, and long run) can be calculated by adding up all the amounts that firms are willing to sell at given prices (in the short, medium, and long run).
- i. **Market Demand** curves for ordinary private goods are "horizontal" sums of individual demand curves
 - ii. **Market Supply** curves for ordinary private goods are "horizontal" sums of individual firm supply curves (in the short and medium run).

- B.** Market prices tend to move to levels where the total quantity supplied by all firms equals the total amount demanded by consumers. (This defines P^* , and Q^*)
- This model of price clearing market predicts that output and prices tend to move toward P^* and Q^* as prices adjust to "ration" the quantities produced to consumers. (Walras)
 - In equilibrium, prices adjustments cause "markets to clear" at which point supply = demand.
 - Note that this adjustment process is, in principal, an entirely decentralized process requiring governments to do nothing more than enforce property rights and contracts.
- C.** Having derived the individual market demand and supply curves using the net benefit maximizing model, it should now be clear that:
- the market demand curve is (approximately) the sum of the marginal benefit curves of the individual consumers
 - and that the market supply curve (short run) is the sum of the marginal cost curves of individual firms in the market.
 - Consequently, market demand and (SR) supply curves can be used to estimate the net benefits realized by all firms and consumers in an industry.**
 - When no others are affected, the sum of the consumer and firm net benefits are the social net benefits associated with producing and consuming particular products and services.**
- D. In the absence of externalities or market concentration, markets tend to produce social net benefit maximizing outcomes.**
- Note that the geometry of "market clearing" price implies that markets to produce the **output levels that set marginal social benefits (demand) equal to marginal social costs (supply).**
 - Note that in cases in which the demand curve includes all benefits (to consumers, etc) and the industry supply curve accounts for all of the costs of production (to firms, etc.), markets maximize social net benefits.
 - That is to say, competitive markets tend to produce the social net-benefit maximizing level of output** (if there are no externalities, e.g. relevant spillover costs or benefits).
 - This conclusion is one very widely used **normative** argument favoring competitive markets as an "efficient" welfare maximizing form of social organization.
 - The claim that markets are "efficient" is usually based on this sort of logic and geometry.

III. Markets, Externalities and Social Net Benefits

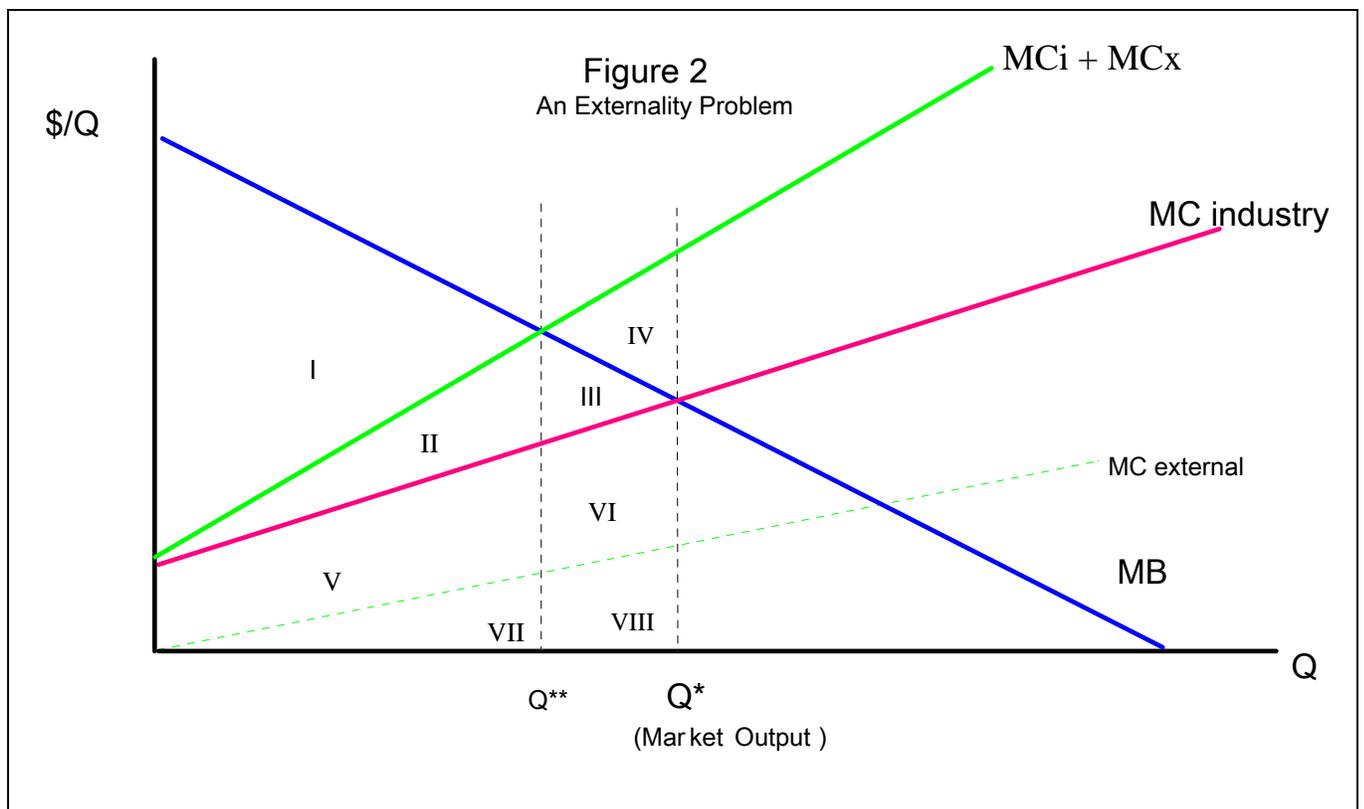
- A.** In cases where **external costs** exist, there are costs imposed on others outside the market of interest.

B. In such cases, competitive market outcomes will (often) fail to maximize social net benefits.

- i. In cases where significant external costs exist at the margin (Q^*), markets will tend to **over produce** the output of interest.
- ii. This is the **normative** basis for government policy in environmental areas.
- iii. (Intuitively, the problem generated by air and water pollution is that firms impose costs on individuals living downwind or down stream, that are not captured by market prices.)

C. Figure 2 illustrates the "problem of external costs."

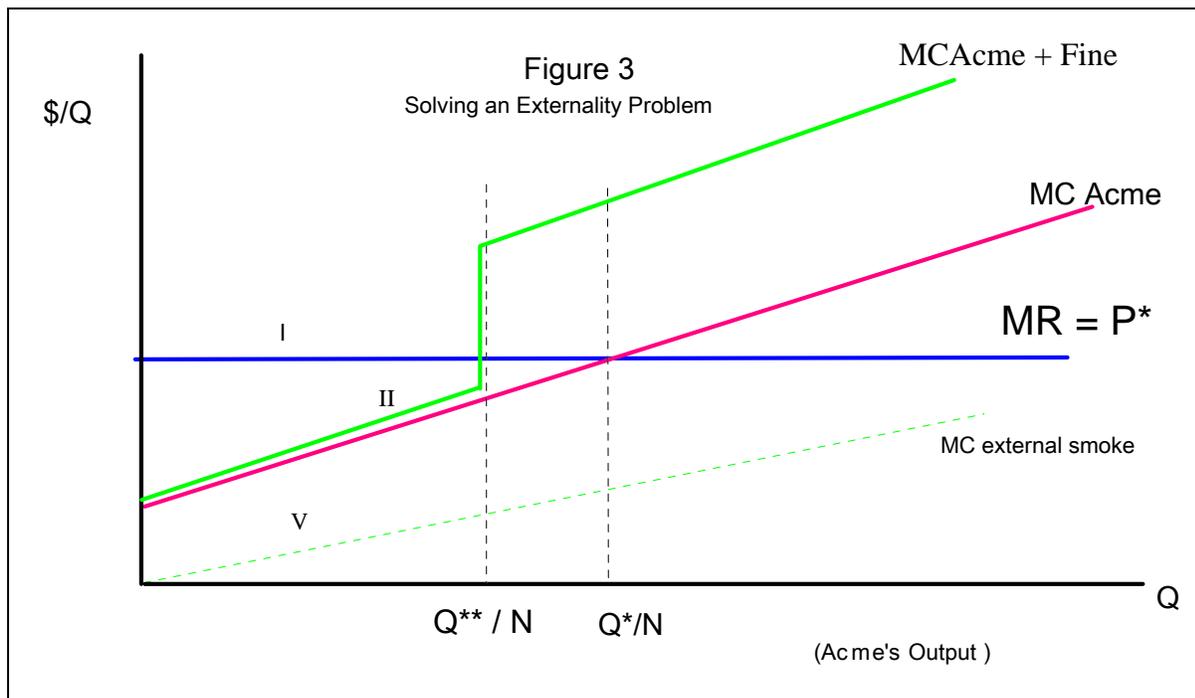
- i. In this case, the industry's marginal cost curve does not include all the true costs of production.
- ii. In order to find the social marginal cost, the "external" marginal costs should be added to the industry's marginal costs ($MC = MC_i + MC_x$)
- iii. After properly accounting for all of the costs of production, we find that the social net benefit maximizing output, Q^{**} , lies below the market output, Q^* .



- iv. Social net benefits are calculated by finding total social benefits (the area under the social marginal benefit curve) and total social costs (the area under the social marginal cost curve) and subtracting TSC from TSB. At Q^{**} social net benefits equal area I, whereas at Q^* social net benefits equal I - IV.

IV. Using Laws to Control Externalities

- A. Laws are rarely targeted at markets, but they can be targeted at individuals, firms, or other organizations. Of course, externality problems like that characterized above are the combined result of lots of consumer and firm decisions, one can reduce the externality by reducing the net benefits for firms and or consumers in the market.
- i. In a standard public economics or environmental economics class, an economist might, for example, suggest placing an appropriate Pigovian tax on the production of this product or service.
 - ii. However, in this class, we might simply make producing more than a certain level of externalities illegal. Recall that the diagram implies that there is an “optimal” (social net benefit maximizing) level of the externality.
 - iii. To give you the geometry behind this idea, suppose the firms are all about the same size and have similar cost curves (this is not necessary for the analysis, but makes it easier).
 - iv. If there are N firms, in principle, we could have a law that punished outputs greater than Q^{**}/N with a fine or jail time for senior managers. However, since our interest is really controlling the externality (smoke, noise, accidents, etc.) it would make more sense to punish firms for producing more than $1/N$ the total Smoke produced at Q^{**} .
 - v. When such a law is well enforced, it changes the choice facing the typical firm by changing its MC curve (for illegal levels of production), as illustrated below:



- B. Here a fine is imposed on Acme for producing more smoke than normally produced at output Q^{**} .
- C. Note that Acme's new profit maximizing choice is Q^{**}/N and if all N firms do more or less the same thing the new output of smoke will be that associated with Q^{**} (N time $Q^{**}/N = Q^{**}$).
- D. By punishing smoke rather than output, it allows the possibility that firms may innovate and produce less smoke (or what ever else might be causing the external costs).
- E. In this way, a well design law and punishment for violating the law can quite efficiently solve externality problems of a wide variety, as will be seen in this course.

V. Decision Making under Uncertainty

- A. In many areas of law and in all areas of law enforcement, the cost and benefits of a law and/or of law enforcement are partially consequences of chance. A law may be violated or not. A criminal may be caught or not.
 - i. To understand decisionmaking by individuals and to engage in policy analysis in such settings requires taking account of the **range of possible outcomes** that any single choice or policy might generate and the **probability** that each outcome might arise.
 - ii. The method used for most such analysis is based on statistical theory of averages, "mathematical expected values."
- B. **DEF:** The **mathematical expected value** of a set of possible outcomes, 1, 2, ... N with values V_1, V_2, \dots, V_N and probabilities of occurrence P_1, P_2, \dots, P_N is

$$E(V) = \sum_{i=1}^N P_i V_i$$
 - i. The expected value represents the long term *average* value of the distribution of values.
 - ii. A probability distribution has the properties: $\sum P_i = 1$ (something has to happen) and $P_i \geq 0$ for all i (where all "possibilities" have positive probabilities of occurrence and "impossibilities" have $P = 0$).
- C. The *expected benefit* associated with such a setting is calculated in a similar manner:

$$E(B(X)) = \sum_{i=1}^N P_i B(X_i)$$

where the "value possibilities" are now measured in benefit terms associated with the affected individuals.

- i. Note that the "expected benefit" requires multiplying the benefit associated with each outcome, $B(X_i)$, times the probability of the outcome, P_i . *The expected benefit is NOT the benefit associated with the average outcome!*
- ii. Similarly, the "expected cost" is the sum of the costs associated with each outcome multiplied by the probability of each outcome, $E(C(X)) = \sum P_i C(X_i)$.

D. Illustration: expected benefits associated with the roll of a die

- i. Suppose that a single die is to be rolled. Clearly which face turns up on top is a random event.
- ii. Suppose also and that you will be paid 3 dollar if number 1 or two turn up on top, 0 dollars if numbers three or four turn up on top, and 12 dollars if number five or six turn up on top.
- iii. Since the probability of a particular face winding up on top is $1/6$ and the value of the outcomes are 3, 3, 0, 0, 6, 6, arithmetic implies that the expected benefit, B^e , of this game is

$$E(B(x)) = B^e = (3)(1/6) + (3)(1/6) + 0 + 0 + (1/6)(12) + (1/6)(12) = \mathbf{\$5.00}.$$

- iv. The expected outcome (x^e) of the game is

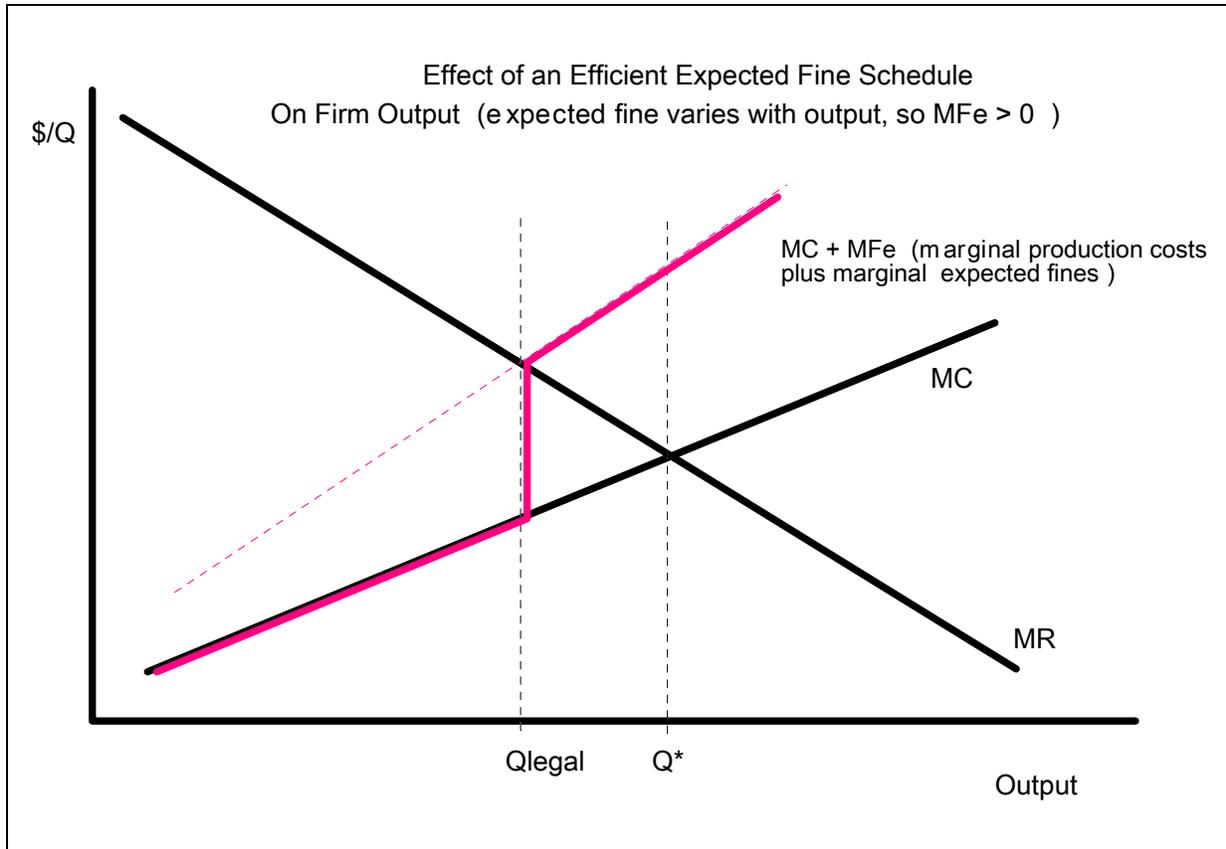
$$E(x) = (1)(1/6) + (2)(1/6) + (3)(1/6) + (4)(1/6) + (5)(1/6) + (6)((1/6) = \mathbf{3.5}.$$

- v. (Notice that the "expected" outcome--3.5--is **actually impossible** on a single roll, but is what will turn up **on average** if you roll the dice many times.)
- vi. (Note also that the expected benefit of the "average" outcome is somewhere around 0 dollars which is quite different than 5 dollars.)
- vii. If a "player" maximized his **expected net benefits**, what is the highest price that you would pay to play this game?

E. A law, whether perfectly or imperfectly enforced, changes behavior **only** if it changes the expected marginal or total costs associated with different activities or activity levels.

F. The economic analysis of the effect of imperfect law enforcement on crime (or any other externality generating behavior) is based on a fine paper written by Gary Becker, 1974, who subsequently won a Nobel prize in economics, partly for that paper.

- i. In that paper, and in many others published by other economists since then, a potential "criminal" is modeled as a rational agent interested in maximizing his EXPECTED income or utility, given some probability of punishment.



- ii. When applied to environmental regulations, this model implies that "potential polluters" will take account of their overall net benefits from pollution including both cost savings and anticipated criminal sanctions when choosing production methods and output levels.
- iii. In such models there are tradeoffs between the probability that a law violator (criminal) will be caught and punished and the size of the punishment. $F^e = PF$
- iv. The higher the probability of being caught and punished the smaller the fine (or other punishment) can be.
- v. Choose some numerical examples of probability and fine combinations that can generate an expected fine of \$200.

G. In the absence of fines or fees ($F > 0$) for pollution, theft, murder, etc and in the absence of enforcement of ($P > 0$), self-interested agents may *choose whatever methods they which to minimize their production costs* and/or maximize their net benefits.

- i. (This does not necessarily mean that firms will pay no attention to other norms that the entrepreneur may have internalized or those of his customers or employees, but will do so only insofar as it increases his or her profits.)
- ii. (That is to say, a firm will adopt its own rules to increase the productivity of the firm, which will take account of inside the firm advantages of rules, but not spill overs outside the firm.)

H. In the real world, fines are only imperfectly enforced, and firms know this.

- i. Thus, it is not simply the magnitude of the fine or penalty schedule that affects a person or group's decision to engage in an illegal activity or not.
- ii. The probability that the person or group will be caught, convicted and punished whenever it violates the law also matters.
- iii. (Initially, we'll ignore internal norms that a person may have. An honest person may not care about the fine or probability of being caught, but simply that an activity is illegal or immoral. Most economic analysis of the law assumes that people do not have such internalized norms, which is not always a good characterization of behavior.)

I. Analyzing the effect of enforcement on a firm's choice of production method or output level requires taking account of the "expected cost" and "expected marginal cost" of any fines or penalties that might be associated with those decisions.

- i. In a setting with random enforcement of the law, the firm's expected profits equal its (certain) Revenues less its (certain) Production Costs less its Expected Fines.

$$IF = R - C - F^e \quad \text{where } F^e = PF$$

- ii. Extra output normally increases revenues, so $MR > 0$.
- iii. Extra output also normally increases costs, so $MC > 0$.
 - For example in the case of environmental law, only outputs that generate effluents greater than allowed by law imply that the firm may have to pay fines, and does so only to the extent that the firm is likely to be caught and punished.
 - Effluents over the legally allowed amount increases both fines and the probability of being punished, so beyond Q_{legal} , $MF^e > 0$.

J. The illustration below shows how such fines affect the firm's **expected MC curve**.

K. Note that there is a **tradeoff between the probability of conviction and the optimal level of punishment**.

- i. Recall that the expected marginal fine is $F^e = PF$, where F is the "marginal fine."
- ii. The higher the marginal fine, the lower the probability of punishment that is sufficient to affect the firm's behavior.
- iii. (Evidently the persons who drafted the code of Hammurabi believed that high "fine" (the death penalty) would be sufficient. Would it?)

L. For those who have had public finance or public economics, there is clearly a similarity between Pigovian taxes and efficient enforcement of an effluent mandate or standard.

If a regulation is set to achieve Pareto efficiency, Q^{**} , then the **smallest fine** sufficient to induce the target Q^{**} has the **same** expected value as a **Pigovian tax** at Q^{**} .

VI. Decision Making and Time: Present Values

- A.** In addition to uncertainty about the punishments associate with an instance of violating the law, the timing of punishments is often important. A punishment may not occur until well into the future or it may take the form of a series of future payments or time spent in jail.
- B.** To think about how such future events affect the net benefits of violating the law, one needs to convert all benefits and costs into "present value" terms.
- C.** Some useful formulas:
- i. The present value of future amount F_T that will be received in T years is: $P = F_T / (1+r)^T$ where r is the interest rate over the period of interest or the individual's subjective rate of time discount. (In the latter case, one can think of r as an index of "impatience," the more impatient one is, the higher one's " r " is and the lower the present discounted value of a future event.)
 - ii. The present value of a series of future events $F_0, F_1, F_2 \dots F_T$ is the sum of the present values of each element of the series: $P = \sum F_t / (1+r)^t$
 - iii. The present value of a series of equal future values, F , that continue for ever is $P = F/r$.
- D.** Intuition behind the formulae:
- i. Imagine that you are thinking about making a bank deposit today so that you will have a particular amount a year from now. Suppose that that amount is \$110 and the interest rate is 10%. Given the interest you earn, you'll have to deposit the amount, P , such that $P(1+.1)$ equals the amount that you want to have next year. P such that $P(1.1) = 110$ which implies that $P = 110/1.1 = 100$. When the interest rate is 10% the present value of 110 in one year is 100 dollars.
 - ii. Now suppose that you want to have \$121 in two years. The same logic applies, but this time there will be two years of interest, so $P(1.1)(1.1) = 121$. A bit of division yields: $P = 121 / [(1.1)(1.1)] = 100$.
 - iii. If we leave the interest rate abstract as " r " where r is a fraction corresponding to the interest rate as a percent (as with 10% = 0.1) then $P(1+r)(1+r) = 121$ which implies that the amount that I need to deposit today to have \$121 dollars in two years is $P = 121 / (1+r)^2$.
 - iv. We can also make the future amount abstract, say amount F in 2 years, in that case $P = F / (1+r)^2$.
 - v. Note that as we moved the future amount further into the future we added extra $(1+r)$ terms to calculate the compound interest that will accumulate. To calculate how much we

Net Benefit Maximizing Choice, Expected Values and Fines, PD Games, and Present Values

need to place in the bank at interest rate r to have amount F in ten years requires: $P(1+r)^{10} = F$ which implies that we need to deposit amount $P = F/(1+r)^{10}$ today to have amount F in 10 years.

- vi. In general, the present value of amount F_T received T years from now is $P = F_T/(1+r)^T$.
- vii. Note also that if one wanted to deposit an amount today so that you could return to the bank at the end of the year and withdrawal \$110 and then withdraw \$121 dollars after another year had passed, it would be sufficient to deposit 200 dollars in one account (or 100 in two separate accounts) as we calculated under i and ii.
- viii. In general, the present discounted value of a series of future amounts is simply the sum of the present values of each element of the series.
- ix. In other words, the **present value** of a series of benefits and/or costs through time is the amount that you could deposit today in a "bank" at interest r and use to replicate the entire stream of benefits or costs.
 - That is to say, if the present value of a flow of future values: $V_1, V_2, V_3, \dots, V_T$, is P , you could deposit amount P in the bank, and return to the bank in year 1, and withdraw the amount V_1 , return in year 2 and withdraw the amount V_2 , return year 3 and withdraw amount V_3 for that year and so on ... finally withdrawing amount V_t after T years.
 - After your withdrawal of V_T , the account would have a balance of exactly zero dollars!
 - (In spite of all the compound interest that needs to be accounted for, it is surprisingly easy to calculate P .)

E. In cases where a constant value is received for T years, e.g. $v_t = v_{t+1} = v$, the above formula can be simplified to:

$$P = v [((1+r)^T - 1) / r (1+r)^T]$$

- i. This present value formulas have many uses in ordinary personal finance.
- ii. For example the present value of a lottery prize of \$50,000/year for twenty years is

$$(50,000) [(1.05)^{20} - 1] / (.05 (1.05)^{20}) = (50,000)(12.4622) = \$623,110.52$$

when the current interest rate is 5%/year.

- iii. This is, of course, much less than the \$1,000,000 that lottery sponsors often claim for such contests.

F. Application of "discounting" to Law and Economics

- i. The discounting formulae imply that the way future events such as punishments affect present day calculations about violating the law depend on how long it will be before one is punished or fined (T) and also on one's discount rate (r). The further in the future the punishment will come and the higher the discount rate, the smaller the present value (present discounted cost) that the fine imposes on the criminal.

- ii. This implies that impatient persons (those with high discount rates) are more likely to commit crimes than more patient persons.
- iii. Combined with the effect of imperfect law enforcement, the lower the probability of being punished, the higher is the discount rate, and the more distant the punishment, the smaller is the effect of a given fine (or punishment schedule) on a person's decision to engage in crime or not.

VII. Some Elementary Game Theory: The Prisoner's Dilemma

- A.** There is a classic game theoretic tool that emerged from early work on game theory at the Rand Corporation (a think tank) in the early 1950s. If this game were just about the “prisoner’s dilemma” we would develop the tool much later in the course when we talk about law enforcement. However the basic logic of the game extends to a wide range of settings that we will be analyzing in this course.
- i. The “classic” prisoner’s dilemma problem is the following.
 - ii. Two people, Al and Bob, have been caught for a minor crime, but are believed to be guilty of a major crime.
 - The two suspects are separated into different rooms and told that if they give evidence about the other, they will get preferential treatment, a reduced sentence.
 - Each person imagines what will happen if he alone give testimony (“rats” or “confesses”) and what will happen if the other person gives testimony.
 - It turns out that no matter what the other person does, each person is best off testifying (“ratting” on the other).
 - As a result both people give testimony and each persons is convicted of the more serious crime, rather than the minor crime they were initially charged with.
 - iii. This is an intuitively plausible outcome, but what is important is not the story or the setting but that this choice setting can be illustrated with a “game matrix,” which can be used to analyze a very wide range of choices.
- B.** The game matrix represents the “payoffs” that each person gets under all four possible outcomes (neither gives testimony), (Al gives testimony, but not Bob), (Bob gives testimony but not Al), (both give testimony).

Classic Prisoner's Dilemma game (payoffs in net benefits or utility levels)		
	Bob does not give testimony	Bob testifies against AL
Al does not give testimony	3, 3	1, 4
Al testifies against Bob	4, 1	2, 2

•

- i. The two numbers in each “cell” are the payoffs received by Al (first number) and Bob (second number).
 - Note that if Al imagines that Bob will not testify, then he’ll get a payoff of 3 if he does not testify (upper left payoff cell, first number) and a payoff of 4 if he testifies against Bob (lower left cell, first number). Clearly $4 > 3$, so if Al is a net benefit maximizer, she will choose to testify in that case.
 - Note that if Al imagines that Bob will testify, then he’ll get a payoff of 1 if he does not testify (upper right payoff cell, first number) and a payoff of 2 if he testifies against Bob (lower right cell, first number). Clearly $2 > 1$, so if Al is a net benefit maximizer, she will choose to testify in that case, as well.
- ii. This game has a **dominant strategy**, that is a strategy that maximizing net benefits (payoffs) regardless of what the other person does.
- iii. Note that the **same logic applies to Bob** as well. (This is a “symmetric” game.)
- iv. If both players play their dominant strategies, the outcome will produce the payoffs of the lower right-hand cell.
 - This outcome is stable, because neither person can do better, given what the other person has done.
 - This sort of equilibrium in a game is called a Nash equilibrium after John F. Nash, and American game theorist in the 1940s and 1950s.
- v. Note that there is a problem for these two prisoners. They would both have been better off if neither had provided testimony $(3,3) > (2,2)$.
 - That is, there is a Pareto Superior move from the lower right-hand cell to the upper left-hand cell.
 - (Remember that a Pareto Superior move always makes at least one person better off and no one worse off.)
 - Relative to this norm, both players are worse off than they could have been!
 - **This is why the game is called the Prisoner’s Dilemma.**

C. This game has been generalized in a variety of ways and produced a very important strand of modern game theory.

- i. For example:
 - The number of players may be increased from 2: to 3, to N, to infinity.
 - The number of strategies may be increased from 2: to 3, to N, to infinity.
 - The game may be repeated rather than a “one-shot” game.
 - Players may have to play the game or be free to pick and choose which game to play.
 - The payoffs may have a variety of relationships (zero sum, positive sum, etc.)
 - etc etc.
- ii. Many of these more complicated games (choice settings) have the properties that are similar to the simple prisoner’s dilemma game. Such games are all referred to as **PD games** (that is Prisoner Dilemma games), even though they have nothing to do with prisoners.

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Net Benefit Maximizing Choice, Expected Values and Fines, PD Games, and Present Values

- Some of these games may be thought of as “externality problems.”
 - Others as “commons problems”
 - Still others as contests in which unproductive forms of competition occur.
- iii. They are all social dilemmas in the sense that at there Nash equilibrium (or equilibria, some games have more than one equilibrium), that are not Pareto efficient.
- That is, at the equilibrium, there are Pareto superior moves.
 - Such equilibria are not “Pareto efficient” and normally do not maximize the total net benefit received by the players.
- iv. Of course, such problems do not always exist.
- For example, this is not true of trading games or competitive markets in which all participants (firms and consumers) are price takers.
 - Not every game is a PD game or a social dilemma.

D. Generalizations like the following are important for this course:

Generalized Prisoner’s Dilemma game (payoffs in net benefits or utility levels)			
Al \ Bob	Strategy 1	Strategy 2	
Strategy 1	3, 3	1, 4	
Strategy 2	4, 1	2, 2	

- i. Note that the payoffs are all the same (at least relative to each other) and so the equilibrium that emerges is still (2, 2) and there is still a Pareto superior move at the equilibrium.
- ii. There are lots of PD-games that have payoffs like this, many of which are relevant for Law and Economics.
- iii. Indeed, the law and law enforcement are often ways of solving PD dilemmas.

E. As an exercise, write down a 2-person 2-strategy game in which the outcome is a social dilemma.

- Double all payoffs and note that the equilibrium is not changed.
- Subtract 10 from each payoff and find the new equilibrium
-
- Write down a series of payoffs in which the Nash Equilibrium is Pareto efficient.
-
- Write down a two persons three strategy game with a PD like outcome.

- F.** We will return to the geometry of net benefit calculations, PD games, expected values, and (to a lesser extent) present value calculations as we work through the course.