

## Chapter 5: Optimization by Firms Facing Downward Sloping Demand Curves

### I. Price-Taking Versus Price-Making Firms

- A. Essentially all mathematical and geometric models of the behavior of firms and consumers in competitive markets assume that both are “price takers.” That is to say that they assume that market prices are exogenously determined and that consumers and firms simply adjust to those prices. Consumers do so by purchasing their utility or net-benefit maximizing quantity of goods. Firms do so by producing their profit maximizing quantities of goods (given technology and input prices) given the prevailing price of the product(s) that they sell.
- B. Market clearing prices in such models “emerge,” although it is not clear how they emerge since no person or firm in those models actually can influence market price through their behavior. This is, of course, a weakness in those models. However, as long as prices are at least a bit controlled by firms and consumers—as through decisions about discounts or premiums that can alter the price at which goods are actually sold are possible—it is reasonable to assume that this “haggling” process allows prices to adjust incrementally in the direction that will clear the market—and so this process can be (and usually is) ignored for purposes of modeling the end result of that process: namely, market clearing prices.
- When Walras developed his idea of a general equilibrium (a vector of prices that simultaneously clears all markets in the economy of interest) he imagined a local “searching” or “tâtonnement” process that gropes around and generates the equilibrium prices. But that process is not included in many mathematical or geometric models of price determination.
  - In the models that do include such processes, it can be shown that some such search procedures do generate market clearing prices, but not all.
  - In lab and classroom experiments, it turns out that market clearing prices are very likely to emerge from the process of offers and acceptance along with other bargaining. See, for example, VL Smith (1962). The latter provides one justification for leaving such processes out of models of price determination to simplify the models without significant loss.
  - Nonetheless, it seems obvious that a model of price determination should actually characterize the process through which prices adjust and eventually clear the market. Although, this is rarely done in micro-economic textbook or in published research on perfectly competitive markets.
- C. One way to bring price determination into economics models is to drop the Marshallian assumption that firms are all the same and sell the same (identical) products. If firms in an industry produce slightly different products or are in different locations, each firm faces a downward sloping demand curve. And given that demand curve, they would naturally choose a price and output combination that maximizes their profits—other things being equal.

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- Firms that compete with other firms selling similar but not identical products would have relatively horizontal, “flat,” or price-elastic demand curves. Such markets are sometimes termed “monopolistically competitive” markets.
  - Prices in such markets would tend to be fairly similar because their products are similar. Prices would be fairly close to their marginal costs (as in perfectly competitive markets) and profits would be modest and mostly be within a fairly narrow range (as in perfectly competitive markets).
  - These are the firms that neoclassical theory of monopolistic competition attempts to model. The more similar the products and better informed the consumers in such markets are, the less firms are able to charge prices above their marginal costs. Such markets can be regarded as “very competitive” rather than perfectly competitive.
- D. Firms that dominate a market without good substitutes (or serious rivals producing identical products) are called monopolists. They usually face a “steeper” downward sloping (less-elastic) demand curve. They are said to have significant “monopoly power,” because they can set their prices well above the marginal cost of production.
- If the market is relatively large, their profits would tend to be well above average and their prices would be significantly greater than marginal cost. These are the firms that the neoclassical theories of monopoly attempts to model.
- E. **Very similar mathematical models can be used to characterize the pricing and output decision of both kinds of firms**, because what distinguishes them from firms in perfectly competitive markets is simply that they face (and know that they face) downward sloping demand curves.
- How “downward” their demand curves are differs, but it turns out that the same or very similar mathematics can characterize choices made when confronting very steeply downward demand curves and slightly downward sloping curves.
- F. From the last chapter we know that production costs vary with technology and input prices as does the mix of inputs used to produce various outputs.
- We also know that costs tend to rise with output and with the prices of inputs, but tends to fall with improvements in technology.
  - These properties can be incorporated into a cost function—as with  $C=c(Q, w, r, t)$ —although for the purposes of this chapter it is enough to simply assume that production costs rise with quantity.
- i. It is often assumed that the shape and properties of a typical firm’s cost function determine what economists call market structure, the number of firms that exist in a particular industry.
- When the bottom of each firm’s  $ATC^{LR}$  is reached at a relatively low level of output relative to market demand, a large number of firms is supported.

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- When there is not bottom of the  $ATC^{LR}$  function or it emerges at an output level that is larger relative to market demand, markets can support only a small number of firms—or in some cases just a single firm (natural monopoly).
  - Although average costs may initially diminish as the quantity produced for sale increases, diminishing returns imply that (in most cases) it will rise again after the initial economies of scale have been realized.
    - The typical case is one in which a firm’s marginal and average cost curves are “J” or “U” shaped. In such cases, the efficiently sized firm produces at the output that minimizes its average cost (because of competitive pressures).
    - In the long run, the number of firms in an industry (under Marshallian assumptions that all firms are the same) is the long-run equilibrium quantity sold in that market divided by the output that characterizes the minimum of the typical firm’s average total cost function.
  - Exceptions to that rule of thumb do exist. For example, under “Ricardian” assumptions where firms all have somewhat different cost functions because of differences in location and in the quality of management and other inputs. In that case, the number of firms supported in the LR is determined by the efficiency ( $ATC^{LR}$ ) of the least efficient firm (firm with the highest minimum  $ATC^{LR}$ ) in the market.
- ii. However, for the purposes of this chapter, we’ll ignore the effects of cost function on market structure and simply assume that costs rise with output. For devoted Marshallians, the results can be thought of as short run results where profits can be larger than zero. In the long run, all firms in a Marshallian market realize the same rate of return on their entrepreneurship, which is said to be zero profits. For Ricardian markets, the results of the model developed in this chapter can also be interpreted as the long run results for “inframarginal firms” after a stable number of firms has emerged. (Only the marginal firm in the Ricardian environment has a zero or near zero profit in the long run.)

### II. On the Mathematics of Selecting Prices and Outputs when a Firm Faces a Downward Sloping Demand Curve.

- A. To illustrate the mathematics, we’ll go through a very concrete example. Then we’ll draw a diagram of that solution, and finally redo the solution in somewhat more abstract form.
- Also, for the reasons discussed above, we’ll assume a cost function rather than derive it from the firm’s production function, because we are less focused on the logical foundations of a firm’s cost function in this chapter.
- B. Suppose the demand curve faced is linear and downward sloping  $Q^D = a - bP$  and the firm’s cost curve is upward sloping  $C = cQ^2$ .
- i. The firm’s profit is  $\Pi = PQ - C$

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- We want to express  $P$  in terms of  $Q$  so that the effects of the firm's output on market prices can be taken into account when it makes its output decision.
- This can be found by solving the demand curve for  $P$  as a function of  $Q$ .
  - $Q = a - bP$  (the demand function) implies that
  - $P = (Q - a)/(-b) = a/b - Q/b$
- This result and the cost function can be substituted into the profit function to make:
  - $\Pi = PQ - C = (a/b - Q/b)Q - cQ^2$
- Differentiating with respect to  $Q$  and setting the result equal to zero yields:
  - $d\Pi/dQ = [a/b - 2Q/b] - 2cQ = 0$  at  $Q^*$
  - The first term (the one in brackets) is marginal revenue, and the last term is marginal cost.
  - Solving this expression for  $Q$  characterizes this firm's ideal output level (which we denote as  $Q^*$ ). One method of solving for  $Q$  is the following.
  - First, shift all of the  $Q$  terms to the lefthand side.
  - $a/b = 2Q/b + 2cQ$
  - Factor the righthand side, then then solve for  $Q$ .
  - $a/b = Q(2c + 2/b)$
  - $Q^* = (a/b)/(2c + 2/b)$
  - This expression can be simplified a bit by multiplying the top and bottom of the fraction (the one characterized by the terms in the two parentheses) by  $(b/b)$  to get
  - $Q^* = a/(2bc + 2)$
- The equilibrium price is found by substituting this quantity (or expression for quantity) into the pricing equation derived above from the firm's demand function (curve)
  - $P = a/b - Q/b$  Substituting our result for  $Q^*$  into this function yields.
  - $P^* = a/b - Q^*/b = a/b - [a/(2+2bc)]/b$
  - (On an exam, in most cases, you could stop here.)
  - This expression (the one to the right) can be simplified by bringing "b" inside the brackets.
  - $P^* = a/b - a/b(2+2bc)$
  - Then placing both terms over the same denominator  $b(2+2bc)$
  - $P^* = a(2+2bc)/b(2+2bc) - a/b(2+2bc)$
  - Then combining both terms

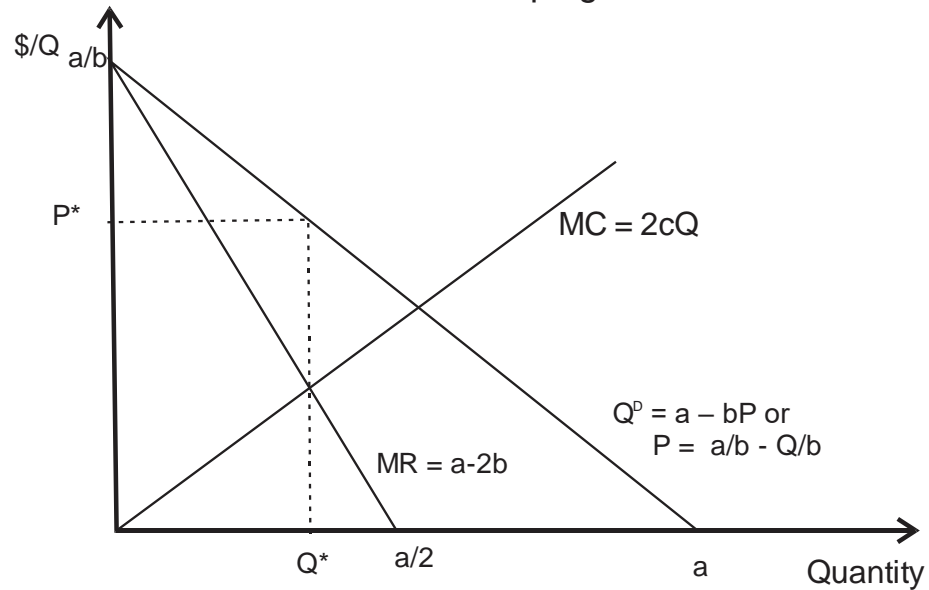
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- $P^* = (2a+2bca - a)/b(2+2bc)$
- Subtracting the “a” from the “2a” and simplifying
- $P^* = (a+2bca)/b(2+2bc) = a/2b(1+bc) + 2bca/2b(1+bc)$  which simplifies to
- **$P^* = a/2b(1+bc) + ac/(1+bc)$**
- The marginal cost at  $Q^*$  is simply the first derivative of the cost function evaluated at  $Q^*$ 
  - The marginal cost function is  $dC/dQ = 2cQ$
  - The value of marginal cost at  $Q^*$  is  $MC(Q^*) = 2c(Q^*) = 2c[a/(2+2bc)]$   
 $= 2ca/2(1+bc)$
  - Which simplifies to  $MC(Q^*) = ac/(1+bc)$
  - Recall that this was the second term in the expression for  $P^*$  derived above (written in blue).

Note that the algebra implies that  $P^*$  is greater than marginal cost at  $Q^*$  by amount  $a/2b(1+bc)$

  - Note that this “mark up” declines as the downward slope ( $-1/b$ ) of the demand curve (here, in its inverse form, with  $P$  on the left) increases. Written in this way, which is the usual way of thinking about the slope of a function on a diagram, as “b” increases, the demand curve gets flatter. It becomes more and more horizontal.
- ii. **An Illustration of this instance** of the simultaneous determination of prices and outputs is provided below for firms facing downward sloping demand curve.
  - Note that the original demand curve (with  $Q$  on the lefthand side) is a linear expression with an intercept on the horizontal axis at “a,” the value of  $Q$  when  $P$  is zero.
    - When expressed with  $P$  on the lefthand side ( $a/b - Q/b$ ) the vertical intercept is  $a/b$ , the value of  $P$  when  $Q$  is zero.)
  - Marginal revenue is:  $MR = a/b - 2Q/b$  (in this case, the first two terms in the first order condition for profit).
    - Note that  $MR$  also has the value  $a/b$  when  $Q = 0$ . And  $MR$  is itself zero when  $Q=a/2$ . It emerges from the vertical axis at the same point as the linear demand curve and falls twice as fast.
  - The first order condition derived above using calculus is satisfied when  $Q$  is such that  $MR = MC$ .
  - The price at which the goods are sold is that on the demand curve at  $Q^*$

Pricing when a Firm Faces a Downward Sloping Demand Curve



**III. Practice Problems**

- i. Find the price-making firm's market clearing price and output when  $Q^D = 1000 - .05P$  and  $C = Q^5$ .
- ii. Find the price-making firm's market clearing price and output when  $Q^D = 1000 - .05P$  and  $C = cQ$ .
- iii. Find the price-making firm's market clearing price and output when  $Q^D = 1000 - .1P$  and  $C = cQ$ .
- iv. Repeat the above derivation for the case in which  $Q^D = a - 2bP$