EC421: Chapter 7: Intertemporal Choices

I. Introduction

A. Most neoclassical models are timeless in the sense that "time" is left out of the model. That is not because time is never important, but because for some purposes leaving time out of a model or analysis does not undermine our understanding of the puzzle or phenomena being addressed. If a consumer decides that he or she will spend next month's wages in a particular way, the fact that the actions associated with that decision do not take place for a month (or year or decade) does not necessarily influence the optimization process that led to that decision. The period of analysis is simply assumed to be the one that is relevant for the decision and usually its associated action to take place.

B. However, there are cases in which time matters. For example, a consumer's decision to spend a certain amount of money in the future may affect the extent to which he or she works (and how they work) today. Or, it may affect consumption decisions today insofar as he or she may decide to save money today so that it can be spent in the future---or engage in the opposite type of behavior, and borrow against future wages to pay for today's consumption today.

C. Time cannot be ignored when actions are taken today that affects one's possibilities in the future—if one is rational and forward looking. Indeed, the phrase "forward looking" implies that one takes some account of the effects of one's present actions on one's future possibilities.

D. The same logic applies to decisions made by economic organizations (firms) who may borrow against future profits to pay for capital goods that will be used in production today or in the near future—or attempt to build a cash reserve today that can be used to address problems that may arise in the future because of Knightian uncertainty or simply to smooth out predictable fluctuations in one's cash flow over the course of a year. (Many businesses have sales patterns that are connected with the seasons, but hold onto their employees for the entire year—often more persons than they need in the short term—and so need a "wages fund" to pay their wages during periods of low demand that will be more than made up with during periods of high demand (as with the demand for toys or holiday foods and beverages).

- Similarly intertemporal decisions are also made by consumers when they borrow to purchase a house, "save for a rainy day," invest in human capital, or save for retirement.

E. This chapter develops some mathematical methods and models that can be used to characterize such intertemporal decision making. Again the focus is on optimization, but in the case explored in this chapter the timing of decisions and actions are the main focus of attention rather than put aside to simplify the analysis.

F. For the most part, optimal decisionmaking through time rests on the notion of **present discounted value**—the mathematics of which emerges naturally when it is recognized that both borrowing and saving have opportunity costs.

II. Intertemporal Choice: Time Discounting and Present Values

A. The simplest way to think about "present discounted value" is to think about the amount in the present (PV) that you would be indifferent to having now rather than some other value (F) in in T years.

- One way to estimate this, if one thinks in money terms, is to calculate the amount of money that one would have to invest today to have F dollars T years in the future.
- If the interest rate or rate of return is r, one can just apply the compound interest formula. $PV (1+r)^T = F$
- Solving for PV yields $PV = F_T/(1+r)^T$ which is the basic formula for calculating the present value of some value in the future.
- To make the formula concrete, suppose that F is $20,000 that T=2 and R=3% or 0.03. In that case, $PV = (20{,}000)/(1.03)^2 = \$18851.92$
- Notice that PV of future among F goes down when the interest increases and when the time period increases.
- The PV of $20,000 in two years at an interest rate of 5% is
  $$PV = (20{,}000)/(1.05)^2 = \$18{,}140.59$$
- The PV of $20,000 in ten years at an interest rate of 5% is
  $$PV = (20{,}000)/(1.05)^{10} = \$12{,}278.27$$
- If one thinks purely in financial or money terms, one would be indifferent between $12,278.27 today and $20,000 in 20 years.
- This assumes that no inflation occurs (or that F_T is in inflation adjusted terms) and that there is no risk involved about whether the future amount will be paid or not.
- When one takes account of inflation either everything should be in inflation adjusted (real) terms (including the interest rate, where the real interest rate is the nominal rate of interest less the average annual inflation rate over the period of interest)—or everything should be in nominal (ordinary dollar) terms.
- When there is the risk that amount F will not be paid, then one needs to also take account of the risk using the methods developed in chapter 6.

B. Many decisions involve long term flows of costs and benefits that need to be evaluated by a decision maker or group of decisionmakers. These flows are easiest to compare if one can construct a common "metric" for the purposes of comparison.

i. The present value of a series of benefits and/or costs through time is the amount, **P**, that you could deposit in a bank at interest rate **r** and used to replicate the entire stream of benefits or costs, F_1, F_2, F_3, ... F_T.

ii. That is to say, you could go to the bank in year 1, and withdraw the amount (B_1) for that year, return in year 2, pull out the relevant amount for that year (B_2) and so on...

iii. The present discounted value of a series of future amounts is simply be the sum of the present values of each element of the series—which is calculated as above.

C. **DEF:** Let **Ft** be the value of some asset or income flow "**t**" time periods from the present date. Let **r** be the interest rate per time period over this interval.

- **The present discounted value** of Vt is $P(F_t) = F_t/(1+r)^t$
- **The present value (here P) of a series of future income flows** (which may be positive or negative) over **T** years when the interest rate is **r** (as a fraction) per period is:

$$\boldsymbol{P = \sum_{t=1}^{T} F_t/(1+r)^t}$$

i. That is to say **the present discounted value of any series of values is the sum of the individual present values** of each element of the series.

ii. This formula always "works" but it is somewhat cumbersome as T becomes relatively large.

iii. Another useful formula is one that characterizes the present discounted value of a steady flow of values on off into the next T years.

D. In cases where a **constant value** is received through time, e.g. V1 = V2 … = Vt … = V_T = v, a bit of algebra allows the above present value formula to be reduced to:

$$P = v\,[\,((1+r)^T - 1)/r\,(1+r)^T]$$

i. Derivation:

- Multiply $\boldsymbol{P = \sum_{t=1}^{T} V_t/(1+r)^t}$ by (1+r) which yields
  $(1+r)\boldsymbol{P = \sum_{t=0}^{T-1} V_t/(1+r)^t}$
- Subtract P from (1+r) P which yields: $rP = v\,[\,1/(1+r)^0 - 1/(1+r)^T)]$
  - (Note that all the terms in the two sums are the same except for the first and last one, so they cancel out.)
- Recall that $1/(1+r)^0 = 1$ so $rP = v\,[\,1 - 1/(1+r)^T)]$
  - Putting the lefthand term over a common denominator yields
    $rP = v\,[\,(1+r)^T - 1]\,/\,[(1+r)^T]$
- Dividing both sides by r yields $\mathbf{P = v\,[(1+r)^T - 1]\,/\,[r\,(1+r)^T]}$ QED.

ii. Note that this constant flow of benefits (or costs) formula **has a limit** as T approaches infinity, namely: **$P = v/r$** .

- This is another very convenient formula.
- There are many long-term investments and regulatory policies that have very long lives that can be thought of as infinitely lived investments as a "first approximation".

E. Illustrative Applications

i. These formulae can, for example be used for cost benefit analysis. Suppose that a dam can be built that cost $1,000,000 and will produce $50,000/year in electricity for 40 years. Is the dam worth building if the interest rate is 5%/year?

- Use the PV formula: $P = v\ [\ ((1+r)^T - 1)/r\ (1+r)^T]$
- The PV of the future benefits are
  $P = 50{,}000[((1.05)^{40} – 1)/(.05)(1.05)^{40}\ ] = \$857{,}954.31$ answer NO
- What if the interest rate is 2%/year? In this case PV = $1,367,773.96 answer YES
- Discount rates matter. Note that the benefits off in the distant future are worth far less when $r = 0.05$ than when $r = 0.02$
- Note that if the dam would provide electricity forever, then
  $P = v/r = \$50{,}000/0.05 = \$1{,}000{,}000$ in that case the dam project exactly breaks even (ignoring any maintenance expenses) But, also note that the all the years after year 40 add relatively little to the present discounted value of the future benefits.

ii. Suppose that Al can afford to pay $5000/year in car payments for 5 years toward a new automobile. If the bank's opportunity cost rate of return is 7%, what is the largest amount that the bank will loan Al given his budget?

- Use the PV formula: $P = v\ [\ ((1+r)^T - 1)/r\ (1+r)^T]$
- $P = 5000\ [\ ((1.07)^5 – 1)/r(1.07)^5] = \$20{,}500.99$
- That is the bank's opportunity cost of tying up P dollars during the 5 years the loan will be repaid.

III. Intertemporal Utility Maximization

A. Intertemporal utility maximization problems generally express the relevant budget constraints in present discounted value terms.

- Fortunately, a good deal about the nature of intertemporal choices can be generated from simple two or three period models of choice. This greatly reduces the mathematical complexity of such models.

B. Suppose that Al's utility function is $U = u(C_1, C_2)$ and her intertemporal budget constraint is $Y_1 + Y_2/(1+r) = C_1 + C_2/(1+r)$, where Y_1 and Y_2 are incomes in period 1 and 2, r is the interest rate or opportunity cost rate of return, and C_1 and C_2 are consumption levels in the two periods. Note that Al's person's wealth, W, is the present value of current and future income, and r is the relevant interest rate. (Let's also assume that either there is no inflation or that the income flows and interest rates are in "real" or inflation adjusted dollars.

i. Both the Lagrangian and substitution methods can be used to characterize Al's optimal consumption expenditure in each period.

ii. Concrete functional forms such as the Cobb-Douglas and its variations with exponents that do not sum to one allow consumption in both periods to be characterized as a function of wealth, interest rates and prices in the two periods.

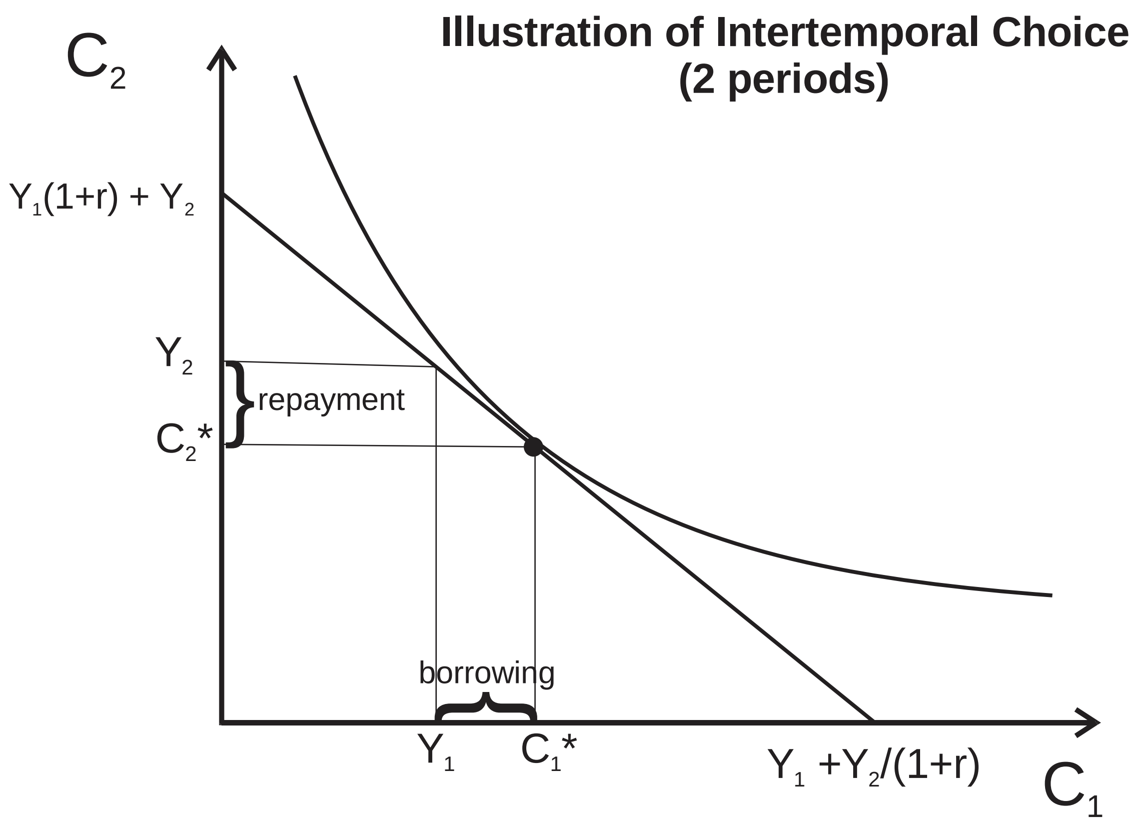
iii. Illustrating example. Let $U = C_1^a C_2^b$ and let $Y_1 + Y_2/(1+r) - C_1 - C_2/(1+r) = 0$

- Form a Lagrange equation and then differentiate with respect to C_1, C_2, and λ.
- $L = C_1^a C_2^b + \lambda(Y_1 + Y_2/(1+r) - C_1 - C_2/(1+r))$
- $L_{C1} = aC_1^{a-1}C_2^b - \lambda = 0$
- $L_{C2} = bC_1^a C_2^{b-1} - \lambda(1/(1+r) = 0$
- $L_\lambda = Y_1 + Y_2/(1+r) - C_1 - C_2/(1+r) = 0$
- Shift the lambda terms in the first to equation to the right and divide the first equation by the second and simplify.
- $aC_1^{a-1}C_2^b / bC_1^a C_2^{b-1} = \lambda/(\lambda(1/(1+r))$
- $aC_2/bC_1= 1+r$
- (This can be interpreted as the marginal rate of substitution between current and future consumption is equal to 1 plus the interest rate, which in a diagram would occur at the tangency between the highest indifference curve that can be reached and the budget line.)
- Solve for C_2 as a function of C_1 and then substitute that into the constraint (L_λ).
- $C_2 = [b(1+r)C_1/a]$
- Substituting yields: $Y_1 + Y_2/(1+r) - C_1 - [b(1+r)C_1/a]/(1+r) = 0$
- Shift the C1 terms to the right (e.g. add the negative of their values to each side) and factor.
- $Y_1 + Y_2/(1+r) = C_1 + [b(1+r)C_1/a]/(1+r) = C_1 (1+b/a) = C_1 [(a+b)/a]$
- Divide and reverse to find Al's demand curve for present consumption:

- $C1 = [a/(a+b)][Y_1 + Y_2/(1+r)]$
- Note that this is analogous to the usual $C_1^* = [a/(a+b)]W/P$ of the usual non-intertemporal demand functions derived from this family of functions, but here $W = [Y_1 + Y_2/(1+r)]$ and $P = 1$, since we measured current consumption in dollar rather than in goods and services. (In effect we are holding the prices of current and future consumption constant.)

iv. The marginal rate of substitution between future and current consumption is sometimes called the *subjective rate of time* discount.

v. The extent of Al's savings is the differences between current income and current consumption, $Y_1 - C_1^*$, which will be negative if he or she borrows against future income to increase his/her current consumption.



vi. Geometrically, the above model of intertemporal choice looks like an ordinary consumer choice problem except the axes represent consumption now and consumption in the future.

- Note that in simple choices, the subjective rate of time discount will be set equal to the interest rate. (why?)

- Answer, the indifference curve tangency implies that the slope the highest indifference curve that can be reached is equal to the slope of the intertemporal budget line.

- (This is one interpretation of the first steps of solutions to a Lagrangian representation of the choice, as noted above, and it can be derived from the substitution method as well.)

vii. (Optional) Given an abstract function form for U, calculus can be used to characterize the effect of changes in interest rates on a person's maximal utility levels and to characterize C_1* and C_2*.

- The first order condition(s) will again imply that the marginal rate of substitution between future and current consumption is equal to one plus the interest rate, (1+r).

- If we have time at the end of the course, we'll show how the envelope theorem can be used to do this for abstract functional forms.

IV. Intertemporal Choices and Uncertainty

A. The **present value and expected value formula can be combined** to deal with uncertain flows of future benefits and costs or uncertain future income levels.

i. For example, consider the purchase of a lottery ticket in a "million dollar" game. Suppose that the winner receives $50,000/year for twenty years, the interest rate is 5%, the probability of winning is 1/1,000,000 and the lottery ticket costs 1 dollar.

ii. Suppose also that there are just two outcomes: winning and losing.

iii. The present value of winning the lottery is the present value of $50,000/year for twenty years.

- $(50{,}000)\ [\ (1.05)^{20} - 1)\ \ /\ (\ .05\ (1.05)^{20}\ )] = (50{,}000)(12.4622) = \$623{,}110.52$ when the current interest rate is 5%/year.

- This is, of course, **much less than** the $1,000,000 value that lottery sponsors usually claim for the prize of such contests.

- **The expected present value of such a lottery ticket** is:

  [1/1000000][623,109.52] + [999999/1000000][-1.00] = -$0.37

iv. This ticket is a bad bet. It has a negative expected discounted value.

- (By the way, this hypothetical lottery is a better deal than most state lotteries, which have expected present values of less than -$0.50)

B. Applications to Policy Analysis: Benefit-Cost Analysis

i. One of the most widely used tools of policy analysis is benefit-cost analysis. In principle, benefit-cost analysis attempts to determine whether a given policy or project will yield benefits sufficient to more than offset its costs.

ii. Cost-benefit analysis, ideally, attempts to find policies that maximize social net benefits measured in dollars.

- Every diagram that includes a dead weight loss triangle is implicitly using cost benefit analysis.
- Economists use this approach to characterize externality and monopoly problems.
- It is also used to criticize ideal and less than ideal public policies and taxes.
- Unfortunately, the data do not always exist for these calculation to be made.

iii. The most widely used methods for dealing with uncertainty and time in Benefit-Cost analysis is to use various combinations of "Expected Value" and "Present Value" calculations as developed in this chatper.

- Cost-benefit analysts carefully estimate the benefits, costs, and risks (probabilities) associated with of alternative policies through time.
- If several policies are possible, cost-benefit analysis allows one to pick the policy that adds most to social net benefits (in expected value and present value terms) or that has the highest social rate of return.

iv. If only a limited number of projects can be built or policies adopted, then one should invest government resources in the projects or regulations that generate the most net benefits (the highest rates of return in terms of social net benefits).

- One can also use cost-benefit analysis to evaluate alternative environmental policies.
- When many projects can be adopted, the policy question is essentially a yes or no question is: Does the policy of interest generate sufficient benefits (improved air quality, health benefits, habitat improvements etc.) to more than offset the cost of the policy (the additional production costs borne by those regulated plus any dead weight losses and the administrative cost of implementing the policy)?

v. The *net-benefit maximizing* norm implies that both good projects, and good regulations, should have **benefit-cost ratios** that exceed one, $B/C > 1$. That is to say, the benefits of a project should exceed its costs if it is worth undertaking.

- However, many of the goods and services generated by environmental regulations *are not sold in markets* and so *do not have prices* that can be used to approximate benefits or costs at the margin.
- These "implicit prices" can be estimated, but the estimates may not be very accurate.
- Thus, a good deal of the policy controversy that exists among environmental economists is over the proper method of estimating non-market benefits and costs.

- For example, the recreational benefits of a national forest may be estimated using data on travel time.
- However, this estimate is biased downward.
- We know that the benefit must be somewhat greater than the opportunity cost of driving to the forest!

- Survey data can also be used, but people have no particular reason to answer truthfully (or carefully) to such questions as how much would you be willing to pay to access "this national forest," "to protect this wetland," or to "preserve this species."

vi. In cases where the benefits and costs are not entirely predictable, the probability of benefits and costs also have to be estimated.

- In cases in which the benefits or costs are largely subjective and concern things that are not sold in markets, these benefits and costs also have to be estimated (but without very reliable data).
- The probabilities assigned to the various outcomes also are often difficult to estimate.
- Thus, although arguably better than nothing, benefit-cost analysis tends to be quite inaccurate.

vii. So instead of attempting to find the best (social net benefit maximizing) policies, cost benefit analysis often simply attempts to determine whether the benefits of a policy exceed its costs.

- A policy is said to improves a situation if it generates Benefits greater then its Costs. [Explain why.]
- For example, cost-benefit analysis might be used to determine whether a particular dam yields sufficient benefits (electricity generation, recreation use of the lake, etc.) to more than offset its cost (materials used to construct dam, lost farmland and output, habitat destruction, homes relocated, etc.).
- To what extent is this yes/no evaluation consistent with the maximize social net benefit norm?
- Discuss why "opportunity" costs matter in such calculations.
- To what extent is this normative approach consistent with or inconsistent with the Pareto norms?
- [For more on this and other issues, see the various debates and critiques of Hicks-Kaldor social welfare functions.]

C. In spite of these difficulties, benefit-cost analysis has several advantages as method of policy analysis:

i. It forces the consequences of policies to be systematically examined.

ii. It provides "ballpark" estimates of the relevant costs and benefits of regulations for everyone who is affected by a new regulation or program.

D. **A Relatively Simple Illustration of an Environmental Cost-Benefit Analysis**: suppose that Acme produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Acme saves $5,000,000/year by using this disposal method, rather than one which does not endanger the ground water.

i. What is the present discounted value of Acme's savings (much of which is passed on to consumers) if the interest rate is 10% and Acme expects to use this method for 30 years?

ii. The easiest method is to use the formula $\mathbf{P = v\ [(1+r)^T - 1]\ /\ [r\ (1+r)^T]}$ although the additive formula, $P = \Sigma\ (Vt/(1+r)^t)$, can also be used,

- Here: $P = (5{,}000{,}000)\ [\ ((1+.10)^{30} - 1)]/[(.10)\ (1+.10)^{30}] = \$49{,}574{,}072.44$

iii. One could also approximate the present value of Acme's cost savings using the present value of an infinite series formula (P=F/r) which yields (5,000,000/0.1 = $50,000,000.00

- Note that this simpler calculation produces nearly the same answer, and so is often a good way to check one's math.

iv. Suppose that an environmental law is passed which requires firms like Amex to adopt the more costly but safer technology. If the fine assessed is $10,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate (interest rate) is 10% ?

- The expected fine in a given year has to be greater than the expected cost savings,
- Thus, P*10,000,000 > 5,000,000 in order for the fine to affect Acme's choice.

a. (In this case the interest rate is not necessary for finding the solution because it is assumed that violations would be detected and fines paid annually.

- Although we could also use present values for both the penalties and cost savings.)

b. The smallest probability of punishment that "works" is 0.5, because this makes the expected fine equal to the expected cost savings.

v. Suppose that administering the enforcement regime costs $1.000,000/year that produces a 0.75 probability of punishment. What is the smallest annual external damage that can justify the program?

a. Given the fine and probability of being caught and punished, we know that this program will induce Acme to clean up, so the only important question is when the present value of the damages (net of administration costs) avoided are greater than the present value of the extra costs borne by Acme (and its consumers).

b. Intuitively, we can see that if the damage per year (D) less the administrative costs($1,000,000/year) are greater than the cost imposed then the program is worthwhile in cost-benefit terms. (D - $1,000,000 > $5,000,000)

c. This implies that the damages must be greater than $6,000,000 per year.

- If the damages vary a bit through time, then we would need to use present and expected values to figure this out.
- In that case the present value of the damages avoided minus the present value of the administrative costs would have to be greater than the present value of the cost increase imposed on Acme (and its consumers).

iv. If the damages were random, perhaps because rainfall is random, then we would have to compare the expected damage reductions (net of administrative costs) with the cost of "cleaning up."

a. For example, suppose that on rainy days the "dirty" waste disposal system causes $20,000,000of damages and that on dry days, the "dirty" waste disposal causes no damages to the local ground water supply. Suppose that it rains one third of the time.

b. In this case the expected damages from the "dirty" waste disposal system has expected damages, D^e = (.33) ($20,000,000) + (.67) (0) = $6,666,666 per year.

c. In this case the cost of eliminating the damage is the cost of the clean up (more expensive waste disposal system) plus the administrative costs ($5,000,000 +$1,000,000) while the benefits are the expected reduction in damages: ($6,666,666 per year).

v. The **expected present value** of the social net benefits from the program over thirty years can be calculated with formula $P^e = v\,[\,((1+r)^T - 1)/r\,(1+r)^T]$ given a planning horizon (T) and discount rate (r). Let T= 30 and r = 10% again.

a. P^e = ($666,666) $[((1+0.1)^{30} - 1)/(0.10)\,(1+0.1)^{30}]$ = ($666,666) (9.4269)

b. Thus, P^e = $6,284,603.40

c. Given all these details, this program will produce a bit more than 6.28 million dollars of expected net benefits over a thirty year period (in present value terms).

E. Some Practice Exercises (See also the Ecampus homework problem set.)

i. Suppose that Al wins the lottery and will receive $100,000/year for the next twenty five years.

- What is the present value of his winnings if the interest rate is 6%/year?, 5%/year, 3%/year?
- How much more would a prize that promised $100,000/year forever be worth?

ii. Suppose that Al can purchase lottery tickets for $5.00 each and that the probability of winning the lottery is P. If Al wins, he will receive $100,000 dollars per year for 20 years. The twenty year interest rate is 3%/year.

- What is the highest price that Al will pay for a ticket if he is risk neutral?
- Determine how Al's willingness to pay for the ticket increases as P, the probability of winning, increases and as the interest rate diminishes.

iii. Suppose that Amex produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Amex saves $1,000,000/year by using this disposal method rather than one which does not endanger the ground water.

- What is the present discounted value of this waste disposal technology to Amex if the interest rate is 6%? if it is 4%?

iv. Suppose that an environmental law is passed which requires firms like Amex to adopt themore costly but safer technology. If the fine assessed is $2,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate is 5%? if it is 10% ?

v. Suppose that global warming is caused (at the margin) by CO_2 emissions and that to reduce CO_2 emissions enough to affect future temperatures requires policies that will reduce economic output by 5% per year. U. S. GNP is currently about 15 trillion dollars and is expected to grow by about 2.5% per year in the future. How large do expected damages have to be to justify such an aggressive environmental policy?

- Hint 1: in this case, the future value of GNP is $Yt = 15*(1+.025)^t$, because of economic growth, which works like compound interest. The reduction in non-environmental income in year t is thus $Vt = (.05)15*(1+.025)^t$

- Hint 2: This implies that present values can be calculated using the summation formula $P = \square\ (Vt/(1+r)^t$ by substituting for $Vt = (.05)\ 15*(1+.025)^t$
- { That is to say, $P = \Sigma\ ((.05)\ (15 \text{ trillion})\ (1+0.025)^t/(1+0.05)^t$
- Hint 3: more generally one can write this expression as $P = \square\ (Vo\ (1+g)^t/(1+r)^t$ where g is the economic growth rate, r is the discount rate (interest rate), and Vo is the initial value of the "thing" that is growing at rate g.
- Hint 4: It turns out that in a present value problem with an infinite planning horizon, one canuse a relatively simple formula to calculate the present values of a series of values that grow by a constant percentage each year:
    - **P = Vo / (r-g)** where Vo is the initial value, r is the discount rate (or interest rate) and g is the long term growth rate.)
- [**Now you can easily calculate** the present discounted value of the cost of reducing CO_2 emissions in this way, which is approximately 30 trillion dollars, given all the assumptions made.]

V. Additional Problems

A. Suppose that Acme is a profit maximizing monopolist facing the inverse demand curve P = p(Q, Y)and production costs C = c(Q,w,r) where Y is average consumer income, w, is the average wage rate of those employed by Acme, and r is the prevailing market rate of interest. (Assume that p and c have the conventional first and second derivatives.)

i. Characterize Acme's profit maximizing output level.

ii. Characterize how Acme's output will increase as household income increases.

iii. Characterize how Acme's profits change as interest rates increase.

iv. Characterize how Acme's profits change as household income increases.

B. Suppose that Al wins the lottery and will receive $100,000/year for the next twenty five years.

i. What is the present value of his winnings if the interest rate is 4%/year?

ii. How much more would a prize that promised $100,000/year forever be worth? (Hint: find the limit of formula IIB as T approaches infinity.)

C. Suppose that Al can purchase "replacement" insurance to eliminate the down side risk of fire in his home. Suppose that in its current state, the house is worth 200k and that after the fire it would be worth 100k. The probability of a fire is P. Replacement insurance increases the after-fire value of the house back to 200k.

i. If Al is risk neutral, what is the highest price that he will pay for replacement insurance?

ii. If Al is risk averse with $U = V$, where V is the value of the house, what is the highest price that he would be willing to pay?

D. Suppose that Al can purchase lottery tickets for $5.00 each and that the probability of winning the lottery is P. If Al wins, he will receive $50,000 dollars per year for 20 years. The twenty-year interest rate is 3%/year.

i. What is the highest price that Al will pay for a ticket if he is risk neutral?

ii. Determine how Al's willingness to pay for the ticket increases as P, the probability of winning, increases and as the interest rate diminishes.