I. Overview of the Course

The main aim of this course has been to provide a "tool bag" of mathematical methods, models, and results to help bridge the gap between undergraduate economics programs that are mainly geometric and graduate programs that are mainly mathematical. More could, in principle, be done, but not in a semester long course. The course is also potentially useful for economics and other majors who are not heading for graduate school but hope to land a position in industry or government where modeling and estimation is part of the job. The "tool bag" reviewed in the course provides a sufficient foundation for a wide variety of rational-choice based modeling. Although model-buiilding takes some practice and imagination to master, the tools covered in the course are still widely used in applied theory papers in economic journals.

To recapitulate the course: we began with a series of explicit function models of demand and supply. The first were applications of the net-benefit maximizing model (as with a net-benefit maximizing consumer or profit maximizing firm). We were able to show a very tight connection between those models and models of market demand and supply and price determination. The latter provides the foundations of neoclassical price theory. We extended those models in various ways—as did the economists that originally developed neoclassical economics in the late nineteenth and early twentieth centuries. We developed utility function-based analyses of consumer choices that allowed tradeoffs among purchases to be better characterized. And, we showed how production functions could be used to generate a tight theory of the cost of production and thereby a logically tight theory of the supply decisions of "price-taking" firms. No "hand waving" was necessary or allowed in that theory—something that is not always clear in geometric modeling.

We also considered how market structure affects output and prices. We first extended the basic model of firms in competitive markets to monopolistic markets and then (after the game theory part of the course) to levels of competition between monopoly and perfect competition using the Cournot duopoly model and modest extensions of it. We found that such a model predicts the existence of a full range of prices that span the gap between monopolistic and perfectly competitive models.

In the second half of the course, we moved beyond the relatively narrow economic models of intermediate micro-economic courses to consider the effects that time, uncertainty, and interdependencies have on decisions made by rational individuals, whether in their roles as consumes or producers or in other settings. We developed "present discounted value" models of the choices with consequences that take place through time and "expected utility maximizing" models of choices in settings where one cannot know exactly what will happen after a choice is made or the initial setting in which the choice is made. These were uncertain circumstances (of the relatively tame variety in which probabilities can be assigned to all possible events). We used the former case to think a bit about cost benefit analysis with respect to public policies and the latter to think about markets for insurance and decisions where choosers have limited information.

We considered cases in which outcomes are not completely under the control of single decision makers using the tools of non-cooperative game theory. We demonstrated that game theory can be used to model all manner of small number (and even relatively large number) choice settings in which outcome emerge as consequences of the more or less simultaneous decisions of many individuals—each attempting to advance their own interests. In some cases, as in many trade settings, the results can be judged "good" from the perspective of the individuals involved. However, in others, the results may be "less good" from that perspective than other possibilities that could have emerged. The latter were referred to as social dilemmas, although this was not emphasized in the course, but simply treated as a special case (as in Prisoner's dilemma games) of the types of outcomes that can emerge when no single person fully controls the outcomes they experience, even each person exercises some control over those outcomes.

In the end, we showed how such models can be used to think about a wide variety of market phenomenon where such models can be applied—namely imperfect competition: the domain between perfect competition and monopoly.

We also showed how "exogenous shocks" can change the equilibria of all these sorts of equilibria, in markets, in games, and so on. That analysis was termed "comparative statics," and while we did some of this during the semester, we spent a little time towards the end of the course thinking about how we can use mathematical tools to undertake such an analysis. It terms our that the effects of shocks on equilibrium conditions can be characterized as derivatives with respect to exogenous parameters of the choice settings faced by consumers and firms.

The course finished with a few lectures on even more general models that rely upon broader families of functions (mostly abstract strictly concave functions). We demonstrated that in many cases such models are mathematically easier to construct and analyze than models with explicit functional forms. To do so we relied upon three additional results from mathematics: the sufficiency theory, the implicit function theorem, and the implicit function differentiation rule. With these additional tools, we were able to determine the general shape of demand and supply curves and construct general models with fewer "ad hoc" assumptions.

Students that head to graduate school in economics or similar fields will see these sorts of mathematical models developed for all sorts of economic and non-economic behavior. This course will stand all those who head to graduate school in good stead. It will provide very useful mathematical foundations for what you'll learn during your years in masters and Ph D programs.

II. The Next Step, Model Building – Going Beyond the Class Notes

To move beyond the class notes requires a combination of mastery of the tools, confidence, and imagination.

To build a model, requires (a) a puzzle that you or others find of interest, (b) an intuition that there is a way to model it using the mathematical methods and results that you have learned along the way, (c) sufficient confidence and interest to try to "get your head around the puzzle" sufficiently to develop a model, and (d) enough tenacity and gumption to work through the math—and all the various dead ends that you encounter—to find results that seem plausible and interesting—and, perhaps, deeper than you imagined at the start of the project.

In class, I have illustrated the process by developing a few abstract models in class that may never have been worked out before. The models that I developed all start from the perspective of methodological individualism. That methodology argues that social phenomena emerge from the joint effects of individual choices. Thus, the models all start with the decision of an individual—a firm owner, a consumer, a worker, a voter, etc.. Given this, a models begins by answering the following questions: (i) What are the main or essential considerations that an individual will or should take account of in the choice setting of interest? (ii) How does one write down the goal and constraints as either a net benefit or utility maximizing problem?

(iii) As a rule of thumb, initially, it usually wise to keep everything as simple as possible to determine how the choice problem "fits together," e.g. what goes into a utility function, what goes into constraints (if anything), and what goes into "technology" the manner in which a goal can be advanced or that generates the payoffs. (iv) "Turn the crank" on the simple model to see how the pieces fit together and what implications that model has for decisionmaking. (If you can't "turn the crank," the model is probably not simple enough yet.) What do the first order conditions look like, do they capture the most relevant costs and benefits of the choice at hand? (In economic models, these are normally practical interests, but a more general model may include others.) What exogenous factors influence the choice? Do the comparative statics include both intuitive and surprising results? Do the possible "exogenous variables" appear to be the most important or not? If not, what is missing?

Sometimes a simple model is sufficient to shed a good deal of light on the factors that influence individual decisions and thereby real-world outcomes and their comparative statics. Often a "quick and dirty" model is sufficient for the purposes at hand. Other times, a richer model is required.

In both cases, it is worth thinking about ways to generalize the model, to bring more factors in, to provide a more realistic objective function or technology or set of constraints. Again, these do not have to be perfectly realistic to be useful. The point, after all, of a model is to abstract from the many entangling factors present in the real world in order to focus on a few factors that seem likely to be most important, so that you can better understand the key relationships that determine choices and thereby the social phenomena of interest.

So, the fifth step (v) is to generalize and enrich the model when it seems worthwhile to do so. And then repeat from (iv) onward until you are satisfied with the model and results or reach a dead end beyond which you lack the tools to proceed further. (In those latter cases, you might spend some time mastering some new tools, but this is not always worth the time and energy.) When writing your papers, this is one way to "stretch" the prose. Start with a simple model that lays the foundations and then develop an extension in the second part.

People have been working on such puzzles and writing their solutions down for at least 2500 years and there is still more to learn. So, do take on a bit of model building yourselves. It's one of the ways that humanity progresses.