

## I. Where We Are.

- A.** During the first half of the semester, we used mathematical models of rational decision making (in the "utility maximizing" sense) to model human decision making in a choice settings in which one person's welfare (utility) is essentially independent of the decisions of other persons.
- B.** Last week we shifted to a new choice setting in which one person's utility (or net benefits or profit) depends in part on the decision depends in part on the decisions of other players. That choice setting is the main focus of Game Theory.
- C.** We began last week with the simplest possible game setting, in which there were two person and two strategies, and explored how different payoff structures affect the equilibrium outcomes of a variety of such games.
- i. For example, we demonstrated that the PD-type of payoff structure implies the existence of pure dominant strategies, and that the typical Nash equilibrium is not Pareto optimal.
  - ii. We also noted that suboptimal equilibria may also arise in assurance games.
  - iii. In general, whether the Nash equilibrium outcome of a particular game is Pareto Optimal or not depends on the relative sizes of the payoffs associated with each players strategies.
  - iv. These are partly determined by nature of the game (which is characterized by strategy sets and payoff functions) and partly by the strategy choices of the other players.
- D.** We then showed how the 2x2 game matrix can be generalized to analyze settings in which there are a more than two strategies, and more than two players.
- i. For example, we extended the 2x2 game to three strategies, and then to a continuum of strategies (in the lottery game).
  - ii. We used the lottery game to illustrate how decisions can be made in a setting in which there are an infinite (continuum) of strategies.
    - To find the equilibrium in such games, we first found the **best reply functions** of the individual players.
    - Then we found the combination of strategies that occurred if all players were simultaneously on their best reply functions.
    - This combination of strategies was a Nash equilibrium of the game, because the fact that every player was simultaneously on their best reply function implied that each had maximized his utility (payoff)--given the choices of the other players.
  - iii. We also used the lottery game to illustrate how games in which a various numbers of players (N players) may participate in the game, which allowed us to characterize an infinite strategy and infinite player game.
- E.** For the next two lectures, we will apply and generalize these game theoretic models to settings in which there are sequences of choices and information constraints.
- i. Our focus will be mainly on games in which "continuous strategy" sets exist.
  - ii. Our illustrations will be from economics and political science.

## II. An Application from Economics: Cournot Duopoly

- A.** There are three widely used models of duopoly: (1) *Cournot* (based on symmetric quantity competition), (2) *Bertrand* (based on symmetric price competition), and (3) *Stackelberg* (based on asymmetric quantity competition with a first and second mover).
- i. The Cournot model is the most widely used in economics textbooks and is where we'll begin.

- ii. The Stackelberg model is less widely used, but the same modeling structure is often used to model games in which players make decisions in a sequence of some kind, and chose their strategies in part because of anticipated reactions on the part of the other players.
- iii. The Bertrand model is less widely used, but should not be totally neglected, because it yields simple and direct predictions about pricing and output.

**B.** In the **Cournot** model (some times called Cournot/Nash duopoly), two firms produce identical goods and make their *output* decisions independently of one another. Each takes the other's output given, and selects its own best output given that assumption given the *downward sloping market demand curve* for the product in question. Total output and market price are represented as equilibria to the "noncooperative" production game between the two firms.

- i. As an illustration, suppose that the market in question has a demand curve:  $Q = 1000 - 10P$ .
  - Assume also that firms are profit maximizers and that profit is simply revenues from sales less the cost of producing the goods sold; thus, firm "A's" profit can be written as  $\Pi^A = PQ^A - C^A$ .
  - In order to know (or estimate) their profits they will have to know price, but market price depends on the total amount brought to the market by all firms, not simply that brought by firm "A."
  - Demand curves "slope downward," which means that the more output is "brought to the market" the lower prices tend to be.
- ii. Given our assumed demand curve,  $Q = 1000 - 10P$ , and the affect of total market output on market price can be written as:  $P = 100 - 0.10*Q$ .
  - (This way of writing a demand curve is often called an inverse demand curve, because it goes from quantities into prices, rather than from prices into quantites.)
- iii. If there are just two firms, A and B, then  $Q = Q^A + Q^B$  and firm A profits can be written as:
  - $\Pi^A = [100 - 0.10*(Q^A + Q^B)] Q^A - C^A$
- iv. To simplify a bit more, let us also assume that the cost function is the same for each firm exhibits constant returns to scale and can be written as:
  - $C = 5Q$
  - so the profit of firm A is simply  $\Pi^A = [100 - 0.10*(Q^A + Q^B)] Q^A - 5Q^A$
  - or (multiplying),  $\Pi^A = 100 - 0.10(Q^A)^2 - 0.10Q^A Q^B - 5Q^A$
  - Notice that firm A's profit (his payoff in this game) is affected by the other player's output decision, as is typical in a game setting.
- v. To find firm A's profit maximizing output, we need to find the "top" of the profit function, which can be found where the slope of the profit function is zero.
  - Differentiating with respect to  $Q^A$  and setting the result equal to zero yields:  
 $100 - 0.20 Q^A - 0.10Q^B - 5 = 0$
- vi. Solving this "first order condition" for  $Q^A$  allows us to characterize the output level that maximizes firm A's profit for each output level that B might choose:
  - $100 - 10Q^B - 5 = 0.20 Q^A$
  - $95 - 0.10Q^B = 0.20 Q^A$
  - $475 - 0.50Q^B = Q^A$
  - or  $Q^A = 475 - 0.50Q^B$

- (This equation is the best reply function of firm A. It tells firm A how much to produce to maximize his own profits for any output level of firm B.)
- vii. A similar best reply function can be found for firm B (Do this as an exercise!)
- $Q^B = 475 - 0.50Q^A$
- viii. At the Nash equilibrium, both firms will be on their best reply function--that is to say both firms will be maximizing their profits given the other firm's output.
- This implies that
  - $Q^B = 475 - 0.50Q^A$  and
  - $Q^A = 475 - 0.50Q^B$
  - will simultaneously satisfied.
- ix. Substituting for  $Q^B$  into  $Q^A$  allows us to find the Nash equilibrium levels of output for firm A. allows us
- $Q^{A*} = 475 - 0.50Q^{B*}$  which implies that  $Q^A = 475 - 0.50(475 - 0.50Q^A)$
  - multiplying through yields  $Q^A = 475 - 475/2 + 0.25Q^A$
  - gathering terms  $0.75 Q^A = 475/2$
  - or that  $Q^{A*} = (4/3)(475/2) = (950/3) = 316.6$
  - 
  - By symmetry (show this as an exercise) we also know that  $Q^{B*} = 316.6$ ,
  - which implies that total output,  $Q^* = 633.3$ , namely, the sum of the outputs of the two firms.
- x. Substituting back into the profit function allows us now to determine each firm's payoff:
- the profit of firm A is simply  $\Pi^A = [100 - 0.10*(Q^{A*} + Q^{B*})] Q^{A*} - 5Q^{A*}$
  - or, substituting for the "Qs" we have  $\Pi^A = [100 - 0.10*(633.3)] 316.6 - 5(316.6)$
  - $\Pi^A = (95)(316.6) - 63.3(316.6)$  or  $(31.3) 316.6$
- C.** In equilibrium both firms are simultaneously maximizing profits
- Geometrically this occurs at the intersection of the two firm's best reply curves.
- i. As an exercise, show that total output increases relative to the monopoly market and prices fall.
- Recall that a monopolist will simply maximize profit, which in this case is:
  - $\Pi = [100 - 0.10*(Q)] Q - 5Q$
  - Find the profit maximizing output for the monopolist by differentiating its profit function with respect to Q, setting the result equal to zero, and solving for Q.
  - 
  - Next find the profit levels of the two firm Cournot Duopoly).
  - Which industry earns higher profits?
  - (Hint: the result should be 475 which, of course is less than 633.3.)
  - In which case are consumers better off?
- D.** The Cournot model can be generalized in a number of ways. For example, the inverse demand curve and cost function can be made more abstract and/or general. One can also extend the Cournot approach to characterize markets in which more than two firms interact.

- E.** Consider the case in which the inverse demand curve is linear:  $P = a - bQ$ ,  $C_i = cQ_i$ , and there are  $n$  identical firms participating in the market. Each firm makes its decision independently of one another. (That is to say, there are no cartel-like meetings or coordination of output strategies).
- i. Firm 1's profit in this case is:  $\Pi_1 = a - b(\sum Q_i) Q_1 - c Q_1$
  - ii. Differentiating with respect  $Q_1$  yields:
    - $a - 2bQ_1 - b\sum_2 Q_i - c = 0$
  - iii. This can be solved for  $Q_1$  as a function of parameters of the demand function, the outputs of the other firms and firm 1's cost function. A little algebra yields:
    - $Q_1^* = (a-c)/2b - \sum_2 Q / 2$
  - iv. This is firm 1's best reply function and similar functions can be derived for all the other firms as well.
  - v. The easiest way to find the Nash equilibrium is to assume that there is a symmetric equilibrium in which case all the firm outputs are the same.
  - vi. Simplifying  $Q_1^* = Q_2^* = \dots = Q_N^*$  into the above yields:
    - $Q_1^* = (a-c)/2b - (n-1)Q_1^*/2$
    - Multiplying by 2 and gathering terms yields:
    - $2Q_1^* + (n-1)Q_1^* = (a-c)/b$
  - vii. which implies that at the symmetric Nash Equilibrium:
    - $Q_1^{**} = (a-c)/b(n+1) = [(a-c)/b] [1/(n+1)]$
    - Total market output is  $n Q_1^{**} = [(a-c)/b] [n/(n+1)]$
  - viii. Substituting this into the inverse demand curve, gives us the market price:
    - $P^* = a - bQ = a - b\{[(a-c)/b] [n/(n+1)]\} = a - [(a - c)][n/(n+1)]$
    - (Note that this formula works for the 2 firm case done above.)
  - ix. Note also that as  $N$  approaches infinity  $n/(n+1)$  approach 1 and  $P^*$  approaches  $c$ . That is to say, marginal cost pricing emerges as the number of firms grows large.
    - [Do the algebra for this short proof as an exercise.]
    - [It can also be shown that firm profits converge to zero as  $N$  gets large. Show this as an exercise.]
- F.** This  $n$ -firm model of market competition is often referred to as Monopolistic Competition
- i. There are many ways that the Cournot and monopolistic competition models can be generalized. For example the demand function may be generalized a bit to take account of consumer income.
  - ii. Suppose each firm has cost function  $C = cQ$  and let the inverse market demand be:
    - $P = XY - eQ$  where market production is  $Q = [Q^a + (N-1)Q^o]$
  - iii. The first order condition that characterizes maximal profits for firm "a" is now
    - $(XY - e(N-1)Q^o - 2eQ^a) = c$
  - iv. Firm a's reaction function is thus:  $Q^a = [XY - e(N-1)Q^o - c] / 2e$
  - v. and the Cournot Nash equilibrium output for a typical firm at the symmetric equilibrium is :
    - $Q^* = \{[XY - c] / (N+1)e\}$  and total output is  $N$  times as large:
    - $Q^* = \{[XY - c] / e\} \{N/(N+1)\}$

- vi. As  $N$  approaches infinity, the total output again approaches the competitive equilibrium.
- vii. *Perfect Competition, thus, is the limiting case that can be generated by entry in a Cournot-Nash type model.*

**G.** In general there are a wide variety of models of imperfect competition, which vary mainly with respect to the manner in which players anticipate or fail to anticipate reactions of other players in the game.

- [The expectational differences are often referred to as “the” **conjectural variation.**]

### III. The Bertrand duopoly model

**A.** The Bertrand duopoly model assumes that each firm **competes on price** rather than output. In effect, each firm bids for the business of consumers in a market with homogeneous products by choosing a price for its products.

- If consumers have complete information, each firm can secure essentially the entire market by charging a price just below that of its competitor.
- A sequence of profit maximizing prices offers, thus, declines, until at the Nash equilibrium, neither firm earns a positive economic profit.
- In the case where each produces via constant returns to scale, this implies that the prevailing market *price under this form of duopoly is exactly the same as that of a perfectly competitive market.*

### IV. The Stackelberg Duopoly Model

**A.** In the **Stackelberg** model, a market is shared by two firms. However, in contrast to the Cournot model, there is a "natural" order to the game.

- i. You can think of the Cournot example as representing the decisions of two farmers who independently bringing baskets of potatoes to a local market, without knowing exactly what the other does.
  - At the Cournot-Nash equilibrium, each is satisfied with the quantity of potatoes that they brought to market given what the other farmer has done.
- ii. In the Stackelberg case you can imagine farmer B bringing his potatoes to market after observing the quantity brought by farmer A.
  - The Stackelberg case demonstrates that the sequence of decision making can have important affects on strategy choices and game equilibria.
  - This simple two-step model, thus, represents a significant generalization of the usual "one shot" games.
- iii. In the usual Stackelberg model, one firm (game player) announces its output first (the leader) and the other announces his second (the follower).
  - **The first mover will try to anticipate how his or her decision will affect the decision of the second mover.**
- iv. The Stackelberg two-step game can be used to represent a wide variety of choice settings in which there is a "natural" first and second mover, as we will see later in the course.

**B. A typical Stackelberg model.** Suppose that there are two firms but firm a's decision will be known by firm b when it makes its output decision.

- i. Firm a, knows that firm b will choose its output to maximize its profits given what firm a has done. So, it chooses its profit maximizing output with the "profit maximizing output schedule" of its competitor in mind.

- ii. Firm a is said to be the “leader,” although this vocabulary is used less today than it was two decades ago.
  - iii. The definition of profit has not changed, so:  $\Pi^A = PQ^A - C^A$ 
    - If we use the same specific functional forms that we used in the Cournot Duopoly problem we will have:
    - $\Pi^A = [100 - 0.10 \cdot (Q^A + Q^B)] Q^A - C^A$
    - which is exactly the form that we used above in the Cournot problem.
  - iv. However, in a Stackelberg model case, A does not take  $Q^B$  as given, but knows that B's behavior will be affected by his own decision about output.
    - In fact, A knows that B will choose the quantity that maximizes his own profits given the quantity brought to market by A.
  - v. From our previous effort on the Cournot duopoly model, we know (and so does A) that B will follow the rule:  $Q^B = 475 - 0.50Q^A$ , as per B's best reply function.
  - vi. Thus, A anticipated B's response when bringing his own goods to market, and maximizes:
    - $\Pi^A = [100 - 0.10 \cdot (Q^A + Q^B)] Q^A - C^A$  where  $Q^B = 475 - 0.50Q^A$  and  $C^A = 5Q^A$ , thus,
    - $\Pi^A = [100 - 0.10 \cdot (Q^A + (475 - 0.50Q^A))] Q^A - 5Q^A = 100Q^A - 0.10(Q^A)^2 - 47.5Q^A + 0.05(Q^A)^2 - 5Q^A$
  - vii. In order to find the profit maximizing output, we again differentiate  $\Pi^A$  with respect to  $Q^A$  and set the result equal to zero:
    - $52.5 - 0.10 Q^A - 5 = 0$  (in economic terms this is  $MR - MC = 0$ )
    - or  $Q^A = 475$
    - Note that the marginal revenue function now includes the effect of the leader's anticipated effect on firm b's change in output and on the prevailing price, as well as the leader's own effect on price.
  - viii. To find out total market supply we now go back to firm B's best reply function and substitute for  $Q^A$ :
    - $Q^B = 475 - 0.50Q^A = 475 - 0.50(475) = 237.5$
    - So, the total output is now  $237.5 + 475 = 612.5$  which is between the monopoly output and the Cournot output.
    - So profits will be higher under Stackelberg than under Cournot, and they will be higher for the first mover than for the second mover.
    - (Calculate the profit levels for A and B as an exercise.)
- C. The Stackelberg type of model can be used to model a wide range of settings in which there is a "natural order" to the decisions of interest or in the information sets of the players of interest.**
- i. For example in crime control, normally the legal system "moves first" by imposing a schedule of fines and allocating resources to the police and courts. The criminal moves second by choosing his crime rate.
  - ii. Clearly the best strategy for the "legal system" (as desired by the median voter) is to take account of how the typical criminal will respond to its decision regarding penalties and policing effort.



## V. Externalities and Pigovian Taxation

- A.** In cases where an externality is generated by an activity, it will often be the case that the privately optimal activity levels will ones that are **not Pareto efficient**.
- B.** The easiest way to demonstrate this mathematically is with a two person non cooperative game (group) illustration.
- Suppose that Al and Bob are neighbors. Both own barbecues, and that neither enjoys the smell of smoke and such associated with the other use of their barbecue. Let us refer to Al as Mr. 1 and Bob as Mr. 2.
  - Let  $U_i = u_i(C_i, B_i, B_j)$  for each person  $i$  (here:  $i = 1, 2$ ) with  $C_i$  being food cooked indoors and  $B_i$  being food cooked outdoors by  $i$ , and  $B_j$  being food cooked outdoors by the neighbor ( $i \neq j$ ). To make the model tractable, assume that Mr.  $i$  allocates his "kitchen time"  $T_i$  between cooking and barbecuing so that  $T_i = C_i + B_i$  for all  $i$ .
  - Mr.  $i$ 's barbecuing time can be determined by maximizing  $U$  subject to the time constraint. Substituting, the constraint into the objective function to eliminate  $C_i$  yields:  $U_i = u_i(T_i - B_i, B_i, B_j)$ .
  - Differentiating with  $B_i$  yields:  $U_{iC_i}(-1) + U_{iB_i} = 0$ . Each person will use the barbecue up to the point where the marginal cost in terms of reduced satisfaction from indoor cooking equals the marginal utility of further outdoor cooking.
  - The implicit function theorem implies that  $B_i^* = b_i(B_j, T_i)$ . This can be interpreted as Mr.  $i$ 's best reply function.
  - In a Nash game between the two neighbors, equilibrium will occur when:
    - $B_1^{**} = b_1(B_2^{**}, T_1)$  and  $B_2^{**} = b_2(B_1^{**}, T_2)$
- C.** The matter of whether this Nash equilibrium is Pareto Efficient or not may seem obvious. Since each imposes costs on the other that are neglected, odds are they wind up in a setting where both would be better off if they produced less smoke.
- D.** That Al or Bob could be made better off by coordinating their behavior or not can be demonstrated in a number of ways.
- One way to determine this is to show that a general **social welfare function**,  $W = w(U^A, U^B)$  is maximized at by the relevant choices, given the same constraints. This can be ascertained by determining whether the first order conditions for maximizing "social welfare" are the same as those which maximize individual welfare.
  - Another method of determining this without using a social welfare function (taken from Baumol) is to consider whether one person could be made better off at the Nash equilibrium without making the other worse off.
    - For example: maximize  $L = u_1(T_1 - B_1^*, B_1^*, B_2^*) - \lambda (U_2 - u_2(T_2 - B_2^*, B_2^*, B_2^*))$  by varying  $B_1$  and  $B_2$
    - Differentiating with respect to  $B_1$  and  $B_2$ , and appealing to the envelop theorem (to eliminate effects of  $B_1^*$  on  $B_2^*$  and vice versa) yields:
 
$$U_{1C_1}(-1) + U_{1B_1} - \lambda U_{2B_1} = 0$$

$$U_{1B_2} - \lambda (U_{2C_2}(1) + U_{2B_2}) = 0$$
- E.** Note that these first order conditions are different than those met for either person insofar as they imply that the externality will be internalized at the margin for both parties.

## VI. Electoral Competition in a Representative Democracy

A. If two candidates can choose policy positions, and voters will vote for the candidate closest to their preferred policy, it turns out that the candidate who is closest to the median voter will win the election.

- i. (This "distance based" model of voter behavior is sometimes called the spatial voting model.)
- ii. If the candidates may freely choose policy positions, there is a tendency for electoral competition to cause them to select essentially identical policy positions which maximize the median voter's welfare.
- iii. At this Nash equilibrium, the strong form of the median voter theorem obtains.

B. To see this consider the case where  $v(G)$  is the distribution of voter preferences and  $V^0$  is the median voter's ideal point, that is to say the voter whose preferences lies exactly in the middle of the distribution, with half the voters preferring larger and half smaller values of  $G$ .

- i. The votes received by a candidate 1 can be written as  $V_1(G_1) = \int_{(G_1+G_2)/2}^{G_1} v(G) dG$  and that of candidate 2 as  $V_2(G_2) = \int_{(G_1+G_2)/2}^{G_2} v(G) dG$ .
- ii. Each candidate wants to adopt the platform that maximize their own votes, given that chosen by the other candidate.
- iii. The optimal decision of candidate 1 can be found by differentiating his vote function with respect to  $G_1$ , which yields:  $V_{G_1} = v((G_1+G_2)/2) / 2$  which is not zero except where  $G_1 = G_2$ .
  - (The above is positive for  $G_1 < G_2$ , and is negative,  $-v((G_1+G_2)/2) / 2$ , for  $G_1 > G_2$ .)
  - This implies that a candidate should adopt the same platform as his competitor.
- iv. However, this is not really an equilibrium except at  $G_1 = G_2 = V_0$ .
  - At every other place where the candidates adopt the same position, one of the candidates can always do better than the other (rather than accept a tie) by moving closer to the median voter.
  - Thus, at the Nash equilibrium both candidates take the same position, the platforms maximize both candidate's votes (given what the other candidate has done), and also **maximize the welfare of the median voter.**

## VII. Using the Lottery Game to model Status Seeking and Rent Seeking

A. The rent-seeking literature has used a game theoretic frame of analysis, which like that of the Chicago models, has been more focused on interest group politics than on elections.

- i. The core rent seeking model regards the process of lobbying for special privileges to be analogous to a lottery.
  - That is to say, the response of government is thought to be random, but favors those who have invested the most in lobbying.
  - The process by which these prizes are awarded is considered to be complex.
  - A wide variety of unpredictable personalities and events may ultimately determine who gets which prize.
- ii. The special favors obtained from government--tax breaks, subsidies, protection from foreign competition, contracts at above market rates etc.-- are the prize sought by rent seekers.
  - The prize is usually assumed to be known before hand (exogenous), although a few models assume that it is produced in part by the lobbying process (endogenous).



- The lottery structure implies that the more resources are devoted to securing preferential treatment (e. g. the better prepared and more widely heard are the "rationalizations" for special preference) the more likely it is that a particular rent-seeker will be successful.
- It also implies that the greater the efforts of alternative rent-seekers, the less likely a particular rent-seeker is to succeed.
- As a **first approximation** of this political influence game, investments in political influence are often modeled as if they were purchases of lottery tickets.

### B. The Basic N-Player Rent-Seeking Game

- Suppose that N risk neutral competitors participate in a rent seeking game with a fixed prize,  $\Pi$ .
- Each player may invest as much as he wishes in the political contest.
- The prize is awarded to the player whose name is "drawn from a barrel" containing all of the political lottery "tickets." So, the expected prize for player i is  $\Pi [ R_i / (R_i + R_o) ]$ , where R is the value of the prize,  $R_i$  is the investment in rent seeking by player i, and  $R_o$  is the investment by all other players.
- If the rent seeking resource, R, cost C dollars each, the number of tickets that maximizes player 1's *expected reward* for a given purchase by all other players can be determined by differentiating the expected rent  $\Pi^e = \Pi [ R_i / (R_i + R_o) ] - CR_i$  with respect to  $R_i$  and setting the result equal to zero.
  - $\Pi [ 1 / (R_i + R_o) - R_i / (R_i + R_o)^2 ] - C = 0$
- Which implies that:  $\Pi [ R_o / (R_i + R_o)^2 ] - C = 0$  or  $\Pi R_o / C = (R_i + R_o)^2$
- So player 1's best reply function is  $R_i^* = -R_o \pm \sqrt{(\Pi R_o / C)}$  Of course, only the positive root will be relevant in cases where  $R_i$  has to be greater than zero.
- In a symmetric game, each player's best reply function will be similar, and at least one equilibrium will exist where each player engages in the same strategy.
- Thus, if there are N-1 other players, at the Nash equilibrium,  $R_o^{**} = (N-1)R_i^{**}$ . which implies that  $R_i^{**} = -(N-1)R_i^{**} \pm \sqrt{(\Pi (N-1)R_i^{**} / C)}$ .
- which implies that  $NR_i^{**} = \sqrt{(\Pi (N-1)R_i^{**} / C)}$  or squaring both sides, dividing by  $R_i^{**}$  and  $N^2$  and gathering terms, that:  $R_i^{**} = [(N-1)/N^2] [\Pi / C] = [(1/N) - (1/N^2)] [\Pi / C]$

### C. So for example with N = 2 and C = 1, $R_i^{**} = (\Pi/4)$

- Total rent seeking effort is N times the amount that each player invests
- Thus in the two person unit cost case,  $\mathbf{R} = \Pi/2$ .
  - Half of the value of the prize is consumed by the process of rent seeking. [Draw an illustrating figure]
  - In the more general case,  $\mathbf{R} = [(N-1)/N] [\Pi / C] = [1 - 1/N] [\Pi / C]$  [See our earlier results on lottery games.]
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- D.** The effect of entry on individual and total rent seeking expenditures can be determined by inspection or by differentiation Ciii and Bx above with respect to  $N$ .
- i. It is clear that individual contributions fall as the number of rent seekers increase, but also clear that the total amount of rent seeking "dissipation" increases.
  - ii. In the limit, as  $N \Rightarrow \infty$  the total rent seeking investment approaches the level where the value of those resources,  $RC$ , equals to the entire value of the prize, note that  $\mathbf{R}^{**} C = [\Pi/C] C = \Pi$ .
  - iii. The effect of increases in the cost of participating in the political influence game and/or changes in the value of the regulation to the rent-seeker can also be readily determined in this game.
- E.** The basic lottery model can be generalized to cover cases in which the prize is **endogenous** and where the probability of securing the prize varies, and to cases where the prize is shared rather than awarded to a single "winner take all" winner.
- i. For example,  $R_i^e = P(R_1, R_2, \dots, R_N) \Pi_i(\mathbf{R})$  encompasses many of these features.
  - ii. The affects of economies of scale may also be examined in this general framework and in the earlier explicit one.

### VIII. Homework Problems

- A.** Use a linear version of the Stackelberg model to determine whether the leader or follower has a larger market share and profit.
- B.** Critique the Bertrand model. Are two firms always sufficient to generate marginal cost pricing? Why or why not?
- C.** Use a Cournot-type model to represent a two person rent-seeking game. Assume that two parties compete for complete control of a market by making campaign contributions to elected officials who control the monopoly license. Suppose that the probability of gaining the monopoly privilege,  $P_1 = p(R_1, R_2)$ , increase with ones own contribution,  $R_1$ , but falls with that of other firm's,  $R_2$ .
- i. Characterize the equilibrium level of rent seeking engaged in by two firms who could realize a profit of  $\Pi$  dollars if they win the monopoly privilege.
  - ii. How much of the prize is "consumed" (dissipated) by the competitors.
- D.** Discuss why behavior in such games tends to be suboptimal from the point of view of the participants and society at large. Are there cases where rent-seeking efforts do not generate a dead weight loss?