<div align="center">**Time, Uncertainty, and Decision**</div>

Topics: Present Discounted Value, Expected Value
Applications: theory of the firm, consumer choice, economic regulation, cost benefit analysis.

I. Uncertainty and Time in Decision Making

A. In economics there are many settings in which decision makers confront problems that involve both uncertainties and long time horizons.

   i.   For example, there is the firm's choice of output levels and production methods when input prices are uncertain or regulations are imperfectly enforced.

   ii.   There are consumer choices to invest, take a vacation, or engage in other risky activities such as gambling, rock climbing, smoking, crime, etc..

   iii.   In the field of public economics, there is the determination of the proper level of regulation, fine schedules, and enforcement efforts.

   iv.   Indeed, one can argue that most choices involve uncertainties of various kinds.

      a. Not all "facts" and "relationships" are know with certainty.

      b. Many causal relationships are partly random. For example, many production costs depend on the weather (wind and rain).

      c. Other uncertainties arise because information is incomplete or imperfect.

      d. Together, these imply that the costs and benefits of alternatives are uncertain or probabilistic in both the short and long term.

   v.   Moreover, many choice settings involve of benefits and costs that take place over relatively long periods of time, rather than simple one-shot consumer experiences.

      a. The benefits and costs of many capital purchases occur over many years.

      b. The benefits and costs of career choices span decades.

      c. Regulatory and constitutional choices often involve both uncertainties and long time horizons. The issues associated with global warming span centuries or millennia.

B. Neither uncertainty nor long time horizons make rational calculations or choices impossible, but they do make such choices more difficult and more prone to errors of various kinds.

    • This handout provides an overview of the most widely used tools that economists (and most other policy analysts) employ to analyze decisionmaking under uncertainty and long term decisionmaking.

    • The same tools can be extended to assess the costs and benefits of alternative economic policies in settings where those policy decisions have long term consequences that are, at least partly, uncertain.

II. Decision Making under Uncertainty

A. In many areas of choice, the benefits and costs of particular choices (or policies) are at least partly the consequence of chance.

   i.   The most common models of decisionmaking in settings of uncertainty are the expected utility and expected net benefit maximizing models.

    ii. **Expected value**, itself, is an idea taken from statistics and means the average result that would be expected from a series of "draws" from a stable random process of some kind.

B. DEF: The **mathematical expected value** of a set of possible outcomes, 1, 2, ... N with values V_1, V_2, ... V_N and probabilities of occurrence P_1, P_2, ... P_N is

$$E(V) = \sum_{i=1}^{N} P_i V_i$$

    i. The **mathematical expected value** represents the long term *average* value of the distribution of values.

    ii. Recall that a probability distribution has the properties: $\sum Pi = 1$ (something has to happen) and $Pi \geq 0$ for all i (all possibilities, i, have positive probabilities of occurrence $1 \geq Pi > 0$, and impossibilities, j, have $Pj = 0$).

    iii. The *expected* utility associated with a probabilistic setting is calculated in a similar manner:

$$E(U(V)) = \sum_{i=1}^{N} P_i \, U(X_i)$$

    where the *N* "value possibilities" are now measured in benefit terms associated with the affected individuals.

C. To use expected value calculations, one has to assume that all the outcomes of the "uncertain" events of interest can be listed and probabilities assigned to them.

    i. This is not an unreasonable assumption in many circumstances and is a reasonable first approximation of many others.

    ii. Here, it bears noting that the probabilities assigned may be the result of careful empirical work (frequentist) or (Bayesian) intuitions about the likelihood of particular events that are updated as more evidence is gathered.

    iii. (Frank Knight's *Risk, Uncertainty and Profit* argued that "risk" occurs when one can assign realistic probabilities to outcomes and "uncertainty" occurs when one cannot. He argued that true economic profits can only arise from "uncertainty" in perfectly competitive markets.)

    iv. Most economists and most economic models are quite willing to assume that all the possible outcomes are known and that probabilities can be assigned to them.

        ♦ In most policy areas, the probabilities are themselves estimates that are updated as research or persuasive campaigns take place.

D. Illustration of an expected value calculation: the roll of a die

    i. Suppose that a single die is to be rolled. The face that turns up on top is a random event.

    ii. Suppose that you will be paid a dollar amount equal to the number on the face that winds up on top.

    iii. Since the probability of a particular face winding up on top is 1/6 and the value of the outcomes are 1, 2, 3, 4, 5, 6, arithmetic implies that the expected value of this game is $3.50 = (1)(1/6) + (2)(3.5) + (3)(1/6) + ...(6)(1/6)$.

      a. If you played the game dozens of time, your average payoff per roll would be approximately $3.50.

      b. Note that the expected value of a single roll of a die is 3.5, a number that actually is impossible, rather than "expected" in the usual sense in ordinary English.

     c. This is not always the case, but this example illustrates that the meaning of "expected value" is a technical one: long term average result.

     d. There are many probability distributions in which the average value is also the mode, so it is also the most likely value to be observed, as with a normal distribution.

    iv. In most theoretical work benefits are calculated in "utility" terms.

     a. Utility functions that can be used to calculate expected utility values that properly rank alternative outcomes (according to expected utility) are called **Von-Neumann Morgenstern utility functions**.]

     b. Von-Neuman Morgenstern utility functions are bounded and continuous.

     c. Von-Neuman Morgenstern utility functions are also "unique" up to a linear transformation (and considered by some to be a form of *cardinal* utility).

E. Most, but not all, theoretical work assumes that people are risk averse.

F. DEF: An individual is said to be *risk averse* if the expected benefit of some gamble or risk is less than the utility generated at the expected value (mean) of the variable being evaluated.

    i. A risk averse person is one for whom the expected benefit of a gamble (risky situation) is less than utility of the expected (mean) outcome, if the latter could be obtained with certainty.

     a. In mathematical terms, a person is risk averse if and only if $U(x)^e < U(X^e)$ where X is a random event and X^e is its expected value.

      ♦ That is to say, when: $U(x)^e > U(X^e)$

     b. Note that this implies that **any benefit or utility function that is strictly concave with respect to income, exhibits risk aversion**.

      ♦ (Why? Note that expected utilities are convex combinations of utilities. Draw a one dimensional illustration.)

    ii. A *risk neutral* individual is one for whom the expected benefit of a gamble (risky situation) and utility of the expected (mean) outcome are the same.

      ♦ $U(x)^e = U(X^e)$

    iii. A risk preferring individual is one for whom the expected utility of a gamble is greater than the utility of the expected (mean) outcome.

      ♦ $U(x)^e > U(X^e)$

    iv. The degree of risk aversion is often measured using the *Arrow-Pratt* measure of (absolute) risk aversion: $r(Y) = -U''(Y)/U'(Y)$]
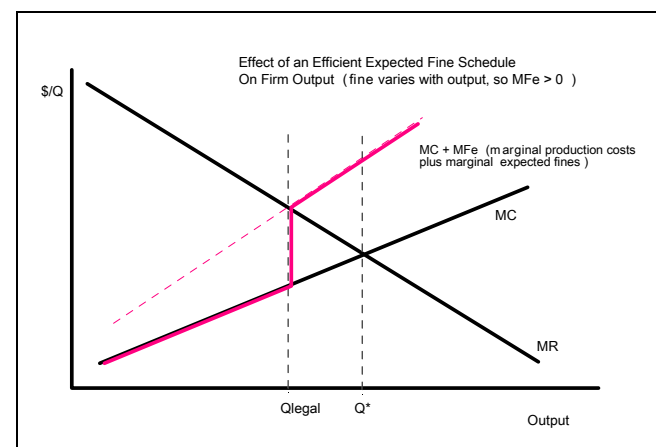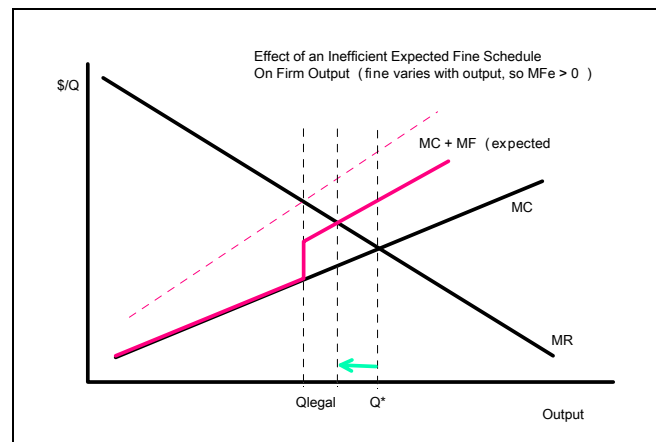
III. Application: Expected Values and the Logic of Crime and Punishment

    i. The economic analysis of crime derives from a classic paper written by Gary Becker, who subsequently won a Nobel prize in economics. In that paper, and in many others published since then, a criminal is modeled as a rational agent interested in maximizing his EXPECTED income or utility, given some probability of punishment.

    • This type of model can be used to model theft and violations of other laws.

    • For example, in the area of environmental laws, firms will take account of their overall net benefits from pollution including both cost savings and anticipated criminal sanctions when choosing their production methods.

ii. In the absence of fines or fees for pollution and in the absence of enforcement of fines greater than 0, firms will *choose production methods to minimize their production costs.*

- (This does not necessarily mean that firms will pay no attention to air or water pollution, but will do so only insofar as it affects the firm's expected profit through productivity and cost effects.)
- That is to say, air or water quality that *effects the productivity of the firms workforce* will be taken account of, but not spillovers on others outside the firm.)

iii. In the real world, laws are only imperfectly enforced, and firms know this.

iv. Consequently, it is not simply the magnitude of the fine or penalty schedule that affects a firm's decision to "pollute illegally or not," but also the probability that a person that violates the law will be caught, convicted and punished.

v. Analyzing enforcement on a firm's choice of production method or output level requires taking account of the "expected cost" and "expected marginal cost" of any fines or penalties that might be associated with those decisions.

vi. In a regulatory environment with fines, the firm's expected profits equal its Revenues less its Production Costs less its Expected Fines. $\Pi = R - C - F^e$ where $F^e = PF$

vii. To the extent that extra output increases Revenues, MR > 0. To the extent that extra output increases production costs MC>0, and to the extent that extra output over the legally allowed amount increases fines or the probability of being punished, $MF^e > 0$.

A. Illustrations: "damage-based fines schedules." (A fine schedule such as $F = f(Q-Q_{legal})$, where f could be an increasing function of $Q - Q_{legal}$, or is simply a constant fine on each unit of output or emission over the allowed level, affects the expected marginal costs of outputs greater than Q_{legal})

i. Note the similarities between Pigovian taxes and optimal enforcement with fines.

ii. If the regulation attempts to achieve Pareto efficiency, Q**, then the smallest fine sufficient to induce the target Q** has the **same expected value** as a Pigovian tax at Q**.

iii. Puzzle: Write down an expected profit function for a firm facing a fine schedule that is imperfectly enforced, but where the fine increases as Q exceeds Q legal. Find the first order conditions and compare them to the above diagrams.

iv. Puzzle: draw examples of a perfectly enforced and imperfectly enforced "fixed fine schedule." (Such fines do **not** affect expected marginal costs.) Compare your graph with the mathematics of expected profit maximization in this case. Are such fines always irrelevant?

Effect of an Inefficient Expected Fine Schedule
On Firm Output (fine varies with output, so MFe > 0)

$/Q

MC + MF (expected

MC

MR

Qlegal   Q*

Output

Effect of an Efficient Expected Fine Schedule
On Firm Output (fine varies with output, so MFe > 0)

$/Q

MC + MFe (marginal production costs
plus marginal expected fines)

MC

MR

Qlegal   Q*

Output

B. Note that there is always policy-tradeoff between the probability of conviction and the optimal level of punishment. [Recall that the expected fine is $F^e = PF$]

   i.   The larger the fine, the smaller the probability of capture can be to generate the same effect on individuals.

      a. The larger is the probability the smaller the fine can be and still have the same effect.

      b. The effect is determined by the expected fine, PF, in this case.

      c. The probability that an illegal activity is detected and punished varies with the resources used to enforce the law and the flagrency of the violation, so the probability of being caught and punished tends to vary with law enforcement budgets and the size of the violation.

   ii.   The politics of enforcement and penalties are partly determined by error rates in detecting criminal activities--sometimes the wrong person is singled out for punishment.

      a. Given this, how would you pick the appropriate punishment for speeding? for theft? for murder? etc.

      b. How would the relative importance of the probability of detection and the expected fine be affected by the process of a jury trial and a long delay between being detected and being fined? (Some hints are provided in the next section of the notes.)

IV. Intertemporal Choice: Time Discounting and Present Values

A. Many decisions involve long term flows of costs and benefits that need to be evaluated by a decisionmaker or group of decisionmakers. These flows are easiest to compare if one can construct a common "metric" for the purposes of comparison.

     ◆ The expected value calculations above can be thought of as such a metric for settings of uncertainty.

B. To calculate and compare streams of benefits or costs that flow through time, most economists use a concept called "**present discounted value**," a concept from finance.

   i.   The present value of a series of benefits and/or costs through time is the amount, **P**, that you could deposit in a bank at interest rate **r** and used to replicate the entire stream of benefits or costs, $B_1, B_2, B_3, ... B_T$.

   ii.   That is to say, you could go to the bank in year 1, and withdraw the amount (B_1) for that year, return in year 2, pull out the relevant amount for that year (B_2) and so on...

   iii.   DEF: Let **Vt** be the value of some asset or income flow **"t"** time periods from the present date. Let **r** be the interest rate per time period over this interval.

   iv.   The present value of Vt is $P(V_t) = V_t/(1+r)^t$

     ◆ **P** is the amount that if invested today at interest rate **r** which would yield exactly amount **Vt** after **t** years thanks to compound interest.

     ◆ (Note that r is entered into the formula as a fraction, e. g. 4%=.04)

   v.   More generally, the present value of a series of income flows (which may be positive or negative) over **T** years when the interest rate is **r** (as a fraction) per period is:

$$P = \sum_{t=0}^{T} (V_t/(1+r)^t)$$

> **vi. That is to say the present discounted value of any series of values is the sum of the individual present values of each element of the series.**
>   - **This is a very useful property which allows the present values of even very complicated series of values to be calculated.**

C. In cases where a constant value is received through time, e.g. $V_t = V_{t+1} = v$, a bit of algebra allows the above formula to be reduced to:

$$P = v \left[((1+r)^T - 1)/r\,(1+r)^T \right]$$

    i. Derivation:

- ◆ Multiply $P = \sum\limits_{t=1}^{T} (v/(1+r)^t)$ by $1+r$ which yields $(1+r)P = \sum\limits_{t=0}^{T-1} (v/(1+r)^t)$

- ◆ Subtract P from $(1+r)$ P which yields: $rP = v\,[\,1/(1+r)^0 - 1/(1+r)^T)\,]$ (Note that all the terms in the two sums are the same except for the first and last one, so they cancel out.)

- ◆ Recall that $1/(1+r)^0 = 1$ so $rP = v\,[\,1 - 1/(1+r)^T\,]$

- ◆ This can be written as $rP = v\,[\,(1+r)^T - 1\,] / [(1+r)^T]$

- ◆ Dividing both sides by r yields $P = v\,[(1+r)^T - 1] / [r\,(1+r)^T]$ QED.

    ii. Note that this constant flow of benefits (or costs) formula has a limit as T approaches infinity, namely: **$P = v/r$** (show this).

- ◆ This is another very convenient formula.

- ◆ There are many long term investments and regulatory policies that have very long lives that can be thought of as infinitely lived investments.

D. Intertemporal utility maximization problems generally express the relevant budget constraints in present discounted value terms.

- ◆ A good deal about the nature of intertemporal choices can be generated from simple two period models of choice.

E. Mathematically, if $U = u(C_1, C_2)$ and $Y_1 + Y_2/(1+r) = C_1 + C_2/(1+r)$, where Y_1 and Y_2 are incomes in period 1 and 2, and C_1 and C_2 are consumption levels in the two periods. Note that this peron's wealth, W, is the present value of current and future income, and r is the relevant interest rate.

    i. Both the Lagrangian or substitution methods can be used to characterize the optimal consumption expenditures in each period.

        a. Note that first order condition(s) imply that the marginal rate of substitution between future and current consumption is equal to one plus the interest rate, $(1+r)$.

        b. The implicit function theorem allows consumption in both periods to be characterized as a function of interest rates.

    ii. Geometrically, this problem looks like an ordinary consumer choice problem except the axes represent consumption now and consumption in the future.

- • The marginal rate of substitution between future and current consumption is sometimes called the *subjective rate of time* discount.

- Note that in simple choices, the subjective rate of time discount will be set equal to the interest rate. (why?)
- Savings rates are the differences between current income and current consumption, which may be negative.

iii.   Given C_1^* and C_2^*, The envelop theorem can be used to characterize the effect of changes in interest rates on a person's maximal utility levels.

F.   The present value formula can also be used for many practical purposes:

- It can be used to size up investment opportunities.
- It can be used to determine the implicit rate of return on bonds and stocks.
- It can be used to determine mortgage payments on a loan, which is just the reverse of a present value.
- With a loan, you know P (the amount borrowed) and need to solve for "v" given your monthly interest rate r and the number of months over which the payments will be made, T.
- What you are doing with a bank loan is providing the bank with a cash flow approximately equal to the present value of the loan.
- (The bank profits by charging you somewhat more than the bank's own market rate of interest. It can normally borrow at a lower rate than you can.)

G. The present value and expected value formula can be combined to deal with uncertain flows of future benefits and costs.

i.   For example consider the purchase of a lottery ticket in a "million dollar" game. Suppose that the winner receives $50,000/year for twenty years, the interest rate is 5%, the probability of winning is 1/1,000,000 and the lottery ticket costs 1 dollar.

ii.   Suppose also that there are just two outcomes: winning and losing.

iii.   The present value of winning the lottery is the present value of $50,000/year for twenty years.

- $(50,000) [(1.05)^{20} - 1) / (.05 (1.05)^{20})] = (50,000)(12.4622) = \$623,110.52$
  when the current interest rate is 5%/year.
- This is, of course, **much less than** the $1,000,000 value that lottery sponsors usually claim for the prize of such contests.
- Subtracting one dollar for the cost of the lottery ticket yields:$623,109.52

iv.   The present value of losing is simply zero minus the cost or -$1.00.

v.   The expected present value of the ticket is thus:
       [1/1000000][623,109.52] + [999999/1000000][-1.00] = -$0.37

vi.   This ticket is a bad bet. It has a negative expected discounted value.

- ◆ (By the way, this hypothetical lottery is a better deal than most state lotteries, which have expected present values of less than -$0.50)

V. An Overview of Benefit-Cost Analysis

A. One of the most widely used tools of policy analysis is benefit-cost analysis. In principle, benefit-cost analysis attempts to determine whether a given policy or project will yield benefits sufficient to more than offset its costs.

i.   Cost-benefit analysis, ideally, attempts to find policies that maximize social net benefits measured in dollars.

ii.   Every diagram that includes a dead weight loss triangle is implicitly using cost benefit analysis.

a. Economists used this approach to find characterize externality and monopoly problems.

      b. It is also used to characterize ideal and less than ideal public policies and taxes.

   iii. Unfortunately the data do not always exist for these calculation to be made.

B. The most widely used methods for dealing with uncertainty and time in Benefit-Cost analysis are "Expected Value" and "Present Value" calculations as discussed above.

   i. Cost-benefit analysts carefully estimate the benefits, costs, and risks (probabilities) associated with of alternative policies through time.

   ii. If several policies are possible, cost-benefit analysis allows one to pick the policy that adds most to social net benefits (in expected value and present value terms) or that has the highest social rate of return.

      a. If only a limited number of projects can be built or policies adopted, then one should invest government resources in the projects or regulations that generate the most net benefits.

      b. For example, cost-benefit analysis might be used to determine whether a particular dam yields sufficient benefits (electricity generation, recreation use of the lake, etc.) to more than offset its cost (materials used to construct dam, lost farmland and output, habitat destruction, homes relocated, etc.).

   iii. For example, one can also use cost-benefit analysis to evaluate alternative environmental policies.

      a. The question is: Does the policy of interest generate sufficient benefits (improved air quality, health benefits, habitat improvements etc.) to more than offset the cost of the policy (the additional production costs borne by those regulated plus any dead weight losses and the administrative cost of implementing the policy)?

      b. The *net-benefit maximizing* norm implies that both good projects, and good regulations, should have **benefit-cost ratios** that exceed one, $B/C > 1$. That is to say, the benefits of a project should exceed its costs if it is worth undertaking.

   iv. However, many of the goods and services generated by environmental regulations *are not sold in markets* and so *do not have prices* that can be used to approximate benefits or costs at the margin.

      a. These "implicit prices" can be estimated, but the estimates may not be very accurate.

      b. A good deal of the policy controversy that exists among environmental economists is over the proper method of estimating non-market benefits and costs.

        ♦ For example, the recreational benefits of a national forest may be estimated using data on travel time. However, this estimate is biased downward. We know that the benefit must be somewhat greater than the opportunity cost of driving to the forest!

        ♦ Survey data can also be used, but people have no particular reason to answer truthfully (or carefully) to such questions as how much would you be willing to pay to access "this national forest," "to protect this wetland," or to "preserve this species."

      c. In cases where the benefits and costs are not entirely predictable, the probability of benefits and costs also have to be estimated.

      d. In cases in which the benefits or costs are largely subjective and concern things that are not sold in markets, these benefits and costs also have to be estimated (but without very reliable data).

   v. So instead of attempting to find the best (social net benefit maximizing) policies, cost benefit analysis often simply attempts to determine whether the benefits of a policy exceed its costs.

      a. A policy is said to improves a situation if it generates Benefits greater then its Costs. [Explain why.]

        ♦ To what extent is this consistent with the maximize social net benefit norm?

      ♦   Discuss why "opportunity" costs matter in such calculations.

      b.   To what extent is this norm inconsistent with the Pareto norms?

      c.   [For more on this and other issues, see the various debates and critiques of Hicks-Kaldor social welfare functions.]

C. In spite of these difficulties, benefit-cost analysis has several advantages as method of policy analysis:

    i.   It forces the consequences of policies to be systematically examined.

    ii.   It provides "ball park" estimates of the relevant costs and benefits of regulations for everyone who is affected by a new regulation or program.

D. A Relatively Simple Illustration of an Environmental Cost-Benefit Analysis: suppose that Acme produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Acme saves $5,000,000/year by using this disposal method, rather than one which does not endanger the ground water.

    i.   What is the present discounted value of Acme's savings (much of which is passed on to consumers) if the interest rate is 10% and Acme expects to use this method for 30 years?

      a.   The easiest method is to use the formula $P = v [((1+r)^T - 1)/r (1+r)^T]$ altlhough the additive formula, $P = \Sigma (Vt/(1+r)^t)$, can also be used,

      b.   Here: $P = (5,000,000) [((1+.10)^{30} - 1)/(.10) (1+.10)^{30}] = \$49,574,072.44$

      c.   One could also approximate the present value of Acme's cost savings using the present value of an infinite series formula $(P=F/r)$ which yields $(5,000,000/0.1 = \$50,000,000.00$
        • Note that this simpler calculation produces nearly the same answer, and so is often a good way to check one's math.

    ii.   Suppose that an environmental law is passed which requires firms like Amex to adopt the more costly but safer technology. If the fine assessed is $10,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate (interest rate) is 10% ?

      a.   The expected fine in a given year has to be greater than the expected cost savings,

      b.   so $P*10,000,000 > 5,000,000$ for the fine to affect Acme's choice.

      c.   (In this case the interest rate is essential for finding the solution, although we could also use present values for both the penalties and cost savings.)

      d.   The smallest probability of punishment that "works" is 0.5, because this makes the expected fine equal to the expected cost savings.

   iii.   Suppose that administering the enforcement regime costs $1.000,000/year that produces a 0.75 probability of punishment. What is the smallest annual external damage that can justify the program?

      a.   Given ii, we know that this program will induce Acme to clean up, so the only important question is when the present value of the damages (net of administration costs) are greater than the present value of the cost savings realized by Acme.

      b.   Intuitively, we can see that if the damage per year (D) less the administrative costs ($1,000,000/year) are greater than the cost savings then the program is worthwhile in cost-benefit terms. (D - $1,000,000 > $5,000,000)

      c.   This implies that the damages must be greater than $6,000,000 per year.

      d.   If the damages vary a bit through time, then we would need to use present values to figure this out.

      e. In that case the present value of the damages avoided minus the present value of the administrative costs would have to be greater than the present value of the cost increase imposed on Acme (and its consumers).

  iv. If the damages were random, perhaps because rainfall is random, then we would have to compare the expected damage reductions (net of administrative costs) with the cost of "cleaning up."

      a. For example, suppose that on rainy days the "dirty" waste disposal system causes $20,000,000 of damages and that on dry days, the "dirty" waste disposal causes no damages to the local ground water supply. Suppose that it rains one third of the time.

      b. In this case the expected damages from the "dirty" waste disposal system has expected damages, D^e = (.33) ($20,000,000) + (.67) (0) = $6,666,666 per year.

      c. In this case the cost of eliminating the damage is the cost of the clean up (more expensive waste disposal system) plus the administrative costs ($5,000,000 +$1,000,000) while the benefits are the expected reduction in damages: ($6,666,666 per year).

  v. The **expected present value** of the social net benefits from the program over a thirty years can be calculated with formula $P^e = v \, [\, ((1+r)^T - 1)/r \, (1+r)^T]$ given a planning horizon (T) and discount rate (r). Let T= 30 and r = 10% again.

      a. P^e = ($666,666) $[((1+0.1)^{30} - 1)/(0.10) (1+0.1)^{30}]$ = ($666,666) (9.4269)

      b. so P^e = $6,284,603.40

      c. This program will produce a bit more than 6.28 million dollars of expected net benefits over a thirty year period (in present value terms).

E. A Few Practice Exercises

  i. Suppose that Al wins the lottery and will receive $100,000/year for the next twenty five years.

      a. What is the present value of his winnings if the interest rate is 6%/year?, 5%/year, 3%/year?

      b. How much more would a prize that promised $100,000/year forever be worth?

      c. Suppose that Al can purchase lottery tickets for $5.00 each and that the probability of winning the lottery is P. If Al wins, he will receive $100,000 dollars per year for 20 years. The twenty year interest rate is 3%/year.

      d. What is the highest price that Al will pay for a ticket if he is risk neutral?

      e. Determine how Al's willingness to pay for the ticket increases as P, the probability of winning, increases and as the interest rate diminishes.

  ii. Suppose that Amex produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Amex saves $1,000,000/year by using this disposal method rather than one which does not endanger the ground water.

      a. What is the present discounted value of this waste disposal technology to Amex if the interest rate is 6%? if it is 4%?

      b. Suppose that an environmental law is passed which requires firms like Amex to adopt the more costly but safer technology. If the fine assessed is $2,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate is 5%? if it is 10% ?

  iii. Suppose that global warming is caused (at the margin) by CO_2 emissions and that to reduce CO_2 emissions enough to affect future temperatures requires policies that will reduce economic output by 5% per year. U. S. GNP is currently about 15 trillion dollars and is expected to grow by about

2.5% per year in the future. How large do expected damages have to be to justify such an aggressive environmental policy?

a. Hint 1: in this case, the future value of GNP is $Y_t = 15*(1+.025)^t$, because of economic growth, which works like compound interest. The reduction in non-environmental income in year t is thus $V_t = (.05)15*(1+.025)^t$

b. Hint 2: This implies that present values can be calculated using the summation formula $P = \Sigma \ (V_t/(1+r)^t$ by substituting for $V_t = (.05) \ 15*(1+.025)^t$

   ♦ That is to say, $P = \Sigma \ ((.05) \ (15 \text{ trillion}) \ (1+0.025)^t/(1+0.05)^t$

c. Hint 3: more generally one can write this expression as $P = \Sigma \ (V_o \ (1+g)^t/(1+r)^t$ where g is the economic growth rate, r is the discount rate (interest rate), and V_o is the initial value of the "thing" that is growing at rate g.

d. Hint 4: It turns out that in a present value problem with an infinite planning horizon, one can use a relatively simple formula to calculate the present values of a series of values that grow by a constant percentage each year:

   ♦ **P = Vo / (r-g)** where Vo is the initial value, r is the discount rate (or interest rate) and g is the long term growth rate.)

iv. [**Now you can easily calculate** the present discounted value of the cost of reducing CO_2 emissions in this way, which is approximately 30 trillion dollars.]

VI. Problems (collected next week)

A. Suppose that Acme is a profit maximizing monopolist facing the inverse demand curve $P = p(Q, Y)$ and production costs $C = c(Q,w,r)$ where Y is average consumer income, w, is the average wage rate of those employed by Acme, and r is the prevailing market rate of interest. (Assume that p and c have the conventional first and second derivatives.)

   i.   Characterize Acme's profit maximizing output level.
   ii. Characterize how Acme's output will increase as household income increases.
   iii. Characterize how Acme's profits change as interest rates increase.
   iv. Characterize how Acme's profits change as household income increases.

B. Suppose that Al wins the lottery and will receive $100,000/year for the next twenty five years.

   i. What is the present value of his winnings if the interest rate is 4%/year?
   ii. How much more would a prize that promised $100,000/year forever be worth? (Hint: find the limit of formula IIB as T approaches infinity.)

C. Suppose that Al can purchase "replacement" insurance to eliminate the down side risk of fire in his home. Suppose that in its current state, the house is worth 200k and that after the fire it would be worth 100k. The probability of a fire is P. Replacement insurance increases the after-fire value of the house back to 200k.

   i. If Al is risk neutral, what is the highest price that he will pay for replacement insurance?
   ii. If Al is risk averse with $U = V^{.75}$, where V is the value of the house, what is the highest price that he would be willing to pay?

D. Suppose that Al can purchase lottery tickets for $5.00 each and that the probability of winning the lottery is P. If Al wins, he will receive $50,000 dollars per year for 20 years. The twenty year interest rate is 3%/year.

    i.   What is the highest price that Al will pay for a ticket if he is risk neutral?

    ii.   Determine how Al's willingness to pay for the ticket increases as P, the probability of winning, increases and as the interest rate diminishes.

Next Week: More on the intertemporal choice and decision making under uncertainty.