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## Study Guide II

1. Identify and/or Define the following:
a. compact set
j symmetric game
b. convex set
c. initial endowment
d. first order condition
e. second order condition
f. excess demand function
g. Cournot Duopoly
h. Nash Equilibrium

I .dominant strategy
k. PD game

1. PDE game
m. Pareto efficiency
n. coordination game
o. Stackelberg Duopoly
p. repeated game
q. sub-perfect equilibrium
r. mixed strategy
2. Is the utility function, $\mathrm{U}=\mathrm{X}^{\mathrm{a}} \mathrm{Y}^{\mathrm{b}}$ with $\mathrm{b}>\mathrm{a}>0$ strictly concave?
a. In what range is it not strictly concave.
b. Use second derivatives to demonstrate.
3. Use an abstract strictly concave utility function (defined over at least two goods) and an abstract linear budget constraint to derive the demand function for some good.
a. Is your demand curve downward sloping?
b. Is the good represented a normal good?
4. Suppose that $A C M E$ is a monopoly firm that sells its output at a single price. ACME faces the following inverse demand curve for its product. $\mathrm{P}=\mathrm{p}(\mathrm{Q}, \mathrm{Y})$.
a. Find ACME's profit maximizing output if its total cost function is $C=c(Q, w, r)$ where $w$ is the wage rate and $r$ is the rental cost of capital.
b. What partial derivatives are necessary to assure that your first order conditions characterize the profit maximizing output?
c. What happens to Acme's output level if consumer income increases?
d. What happens to Acme's output level if the cost of capital increases?
e. What happens to Acme's profit if the cost of capital increases?
f. How can you modify this model to analyze the effect of an exogenous change in production technology on Acme's output?
g. How would you alter your model to characterize the effect of entry by a new firm on Acme's output level and profits?
5. Suppose that Apex enters Acme's market. To simplify, assume that both Acme and Apex have cost function, $C=Q w$. and face inverse demand function $P=Z-a Q+b Y$
a. Find the best reply functions of Apex and Acme.
b. Find the Cournot-Nash equilibrium outputs of the two firms.
c. Find the individual and total profit of the two firms.
d. Is output and profit higher or lower than in the monopolistic case?
6. Suppose that Acme anticipates Apex's entry, and adjusts its output beforehand to account for Apex's likely production.
a. How much does each firm produce?
b. Compare the Stackelberg equilibrium with the Cournot equilibrium. Are firms better or worse off with the Stackelberg solution?
c. What about consumers?
7. Use a two-person two-strategy matrix of payoffs to characterize the following games and/or social settings:
a. The Prisoner's Dilemma
b. The gains from a mutually beneficial exchange
c. Free Riding: where a pure public good is to be provided. Assume that the subjective opportunity cost of producing a single unit of the public good (worth $\$ 5$ to each party) is $\$ 6$.
d. The Samaritan's dilemma
e. The Pork Barrel Dilemma: where coalitions favoring two negative net benefit projects in Nash equilibrium manage to get both projects adopted, although each would have been better off if neither had been built.
f. The Tragedy of the Commons: where output from a common property resource falls once use exceeds $M$. Once $M$ is reached, further use reduces total output.
8. Repeat number 7 with,
a. a three-strategy characterizations of the games (with similar equilibria).
b. (concrete) continuous strategies and payoff functions.
c. abstract continuous strategies and payoffs.
d. Note equilibria that occur, whether the result is Pareto optimal or not, and whether equilibria in pure strategies exist.
9. Use a two-by-two game matrix to illustrate the logic of the free rider problem. Suppose that the public good of interest provides benefits of $\$ 5.00$ to each "player" and costs a total of $\$ 6.00$ to produce. The cost is shared if both contribute, but must be paid entirely by one person if that person provides the public good while the other free rides.
a. Label all payoffs, and explain the logic of the game.
b. What is the "problem" illustrated by your game?
c. Now suppose that voluntary provision of the good is subsidized. How much would the subsidy have to be to eliminate the free rider problem in your illustration?
d. Now suppose that shirking is punished or taxed in some way. How high would a "shirking tax" have to be to solve the free-rider problem?
10. Construct an N-firm Nash-Cournot model of monopolistic competition for a market in which the demand curve is linear and there are constant returns to scale in production.
a. Write down the profit function for a typical firm.
b. Find a typical firm's best reply (reaction) function.
c. Find the Cournot-Nash equilibrium.
d. Show that as the number, N , of firms increases, price falls until it equals marginal cost and profits fall to zero.
11. Suppose that Al allocates his or her time between legal $(\mathrm{H})$ and illegal activities ( S ) to maximize his expected income. Assume that "his" opportunity cost wage is W and that he has T hours to allocate between H and S . Assume that criminal income increases with time devoted to
crime, but at a diminishing rate. Assume also that both the probability of conviction and the penalty received increase with the time devoted to criminal activity.
a. Write down a function that characterizes the income generated by legal activity S .
b. Write down a function that characterizes the expected net income generated by criminal activities.
c. Characterize the amount of time Al that "invests" in the criminal activity if he/she attempts to maximize net income.
d. How is that investment affected by an increase in wage rates from the legal activity?
e. Describe and discuss the conditions under which corner solutions are chosen.
f. (Optional) Suppose that the probability that a criminal is caught rises with public expenditures on law enforcement and that social losses increase with the level of criminal activity. What level of expenditure would minimize the expected (social) cost of crime?
12. Suppose that Al and Bob have Cobb-Douglas utility functions defined over personal income Y and leisure L. Assume that team production takes place via a Cobb-Douglas production function defined over Al and Bob's labor. Al and Bob are each be paid their full marginal product (which is partly determined by the other's effort) and can allocate 16 hours between work and leisure. (To simplify assume that all of the exponents take the value 0.5.)
a. Find the Nash equilibrium of this team production game.
b. Compare this equilibrium with the Pareto optimal level of effort and production the maximizes the sum of their utilities.
c. Draw diagrams of the solutions in the $\mathrm{L}^{\mathrm{A}} \times \mathrm{L}^{B}$ plane.
d. Can these results be represented as a two by two game matrix?
e. What is the problem here?
f. What side payment from Al to Bob and/or from Bob to Al can solve the problem?
13. Suppose that a dictator's expected return to holding power is $\mathrm{R}^{\mathrm{e}}=\mathrm{P}(\mathrm{tY}-\mathrm{G})+(1-\mathrm{P})(\mathrm{L})$ where P is the probability of continuing in office and L is the income realized if overthrown. Suppose that
national income Y increases with G but falls with t , and that P increases with national income.
a. Characterize the dictator's optimal tax rate for a given level of government service G. Explain why this tax level does not maximize national income.
b. Characterize the dictator's optimal service level for a given tax rate, $\mathrm{T}=\mathrm{T}^{\circ}$. Determine whether this tax rate maximizes national income.
c. Does the dictator's ideal tax and service combination differ from the combination characterized when $\mathrm{G}^{\circ}$ is the solution to " b " and $\mathrm{T}^{\circ}$ is the solution to a?. Explain.
d. Now consider the case where national income, Y , is produced in two regions "a" and " b " so that $\mathrm{Y}=\mathrm{Ya}+\mathrm{Yb}$. The dictator's probability of staying in office varies with regional incomes such that $\mathrm{P}=\mathrm{p}(\mathrm{Ya}, \mathrm{Yb})$ where $\mathrm{P}_{\mathrm{Ya}}>0$ and $\mathrm{P}_{\mathrm{Yb}}<0$. Suppose that income in both regions increases with government services, $G$, but diminishes with the tax rate, t .
14. Suppose that Al and Bob are in the running for a new position at their firm. The position brings with it an increase in salary of R dollars. The individuals can spend their time producing unobservable output, Q , for the firm, or in transmitting evidence of their productivity, E , to their boss. The latter does not, itself, increase the firm's output. Initially assume that the employees are indifferent between "signalling" and "working." Consider two cases.
a. In case one, the signals, $\mathrm{E}^{\mathrm{A}}$ and $\mathrm{E}^{\mathrm{B}}$, are all that the firm's owner bases his decision on with the probability that Al gets the new salary being $\mathrm{P}^{\mathrm{A}}=\mathrm{p}\left(\mathrm{E}^{\mathrm{A}}, \mathrm{E}^{\mathrm{B}}\right)$ and the probability that Bob gets the new salary being $\mathrm{P}^{\mathrm{B}}=1-\mathrm{P}^{\mathrm{A}}$.

- Write down the expected net benefits of "signalling" activity for Al.
- Write down the expected net benefit of signaling activity for Bob.
- Derive their best replay functions.
- Is there a Nash equilibrium in pure strategies?
- If so what is it and is it good for the firm?
b. In case two, firm owners can independently estimate actual output, but cannot do so precisely. Now both personal output and signaling affect the probability of getting the raise.
- Characterize the individual employee time allocation decision and the Nash equilibrium of the game.
- (Go through the same steps as in part a.)
c. (Optional) Now analyze how the firm owner can manipulate the size of R in these two cases to maximize the profits generated by these two employees in a setting where the firm's output can be sold at price M.
- Write down an expression for the firm owner's profits.
- (You might assume that both employees are initially paid the same salary, Y. )
- What R maximizes the firm owner's expected profits?
- Discuss your results.

15. Develop an extended form (two period) game in which a contract between two individuals is self-enforcing. Explain the key properties of your game that make your contract self enforcing.
16. Use backward induction to demonstrate that the "rational" method of playing a finite-repeated prisoners dilemma game ( say 2,3 , or T times) is defection. How would an uncertain ending point change your answer?
17. Show that in a two strategy zero sum game, a mixed strategy can be the only possible Nash equilibrium.
a. Find a Nash equilibrium mixed strategy pair for your game.
b. Demonstrate that it is an equilibrium.
c. Develop a zero sum game in which equilibria in pure strategies exist.
18. Determine whether the following functions have "fixed" points within the unit square and their values if a fixed point exists. (Hint: a fixed point occurs when $\mathrm{x}=\mathrm{f}(\mathrm{x})$.)
a. $\mathrm{Y}=\mathrm{X}^{2}$
b. $\mathrm{C}=.6 \mathrm{Q}$
c. $\mathrm{N}=10 \mathrm{Q}-0.75 \mathrm{Q}^{.5}$
d. $\mathrm{U}=100-(\mathrm{Q}-10)^{2}$
