

Answers for Problems Assigned Lecture 10

I. Review Problems

A. Stackelberg Game. Suppose that Acme and Apex are duopolists who have identical total cost functions, $C = 100 + 10Q$, and "share" the same market (inverse) demand curve: $P = 1000 - 20Q$.

i. Acme makes its output decision first. What output should it choose if it knows that Apex will simply maximize its own profits given Acme's output?

a. Acme knows that Apex will maximize its profits given what ever Acme produces,

b. so Apex will set its output Q_x at the level which maximizes

$$\Pi_x = (1000 - 20Q_x - 20Q_m)Q_x - 100 - 10Q_x$$

c. the f. o c. is $1000 - 40Q_x - 20Q_m - 10 = 0$

d. which implies that $Q_x^* = (99 - 2Q_m)/4$

e. Under the Stackelberg assumption, Acme will maximize profit by taking account of Apex's reactions to its output Q_m to maximize:

$$\Pi_m = (1000 - 20Q_x^* - 20Q_m)Q_m - 100 - 10Q_m$$

f. or substituting for Q_x^*

$$\Pi_m = (1000 - 20[(99 - 2Q_m)/4] - 20Q_m)Q_m - 100 - 10Q_m$$

g. Differentiating with respect to Q_m yields:

$$1000 - 495 + 20Q_m - 40Q_m - 10 = 0$$

h. or $495 = 20Q_m$ which implies that $Q_m^* = 24.75$

i. substituting into $Q_x^* = (99 - 2Q_m)/4$ implies that $Q_x^* = 12.375$

ii. Is the resulting market equilibrium sub-game perfect? Explain.

a. It is sub-game perfect.

b. When Acme choose $Q_m = 24.75$, it is based on the assumption (calculation or promise) that Apex will choose to set its output at $Q_x^* = (99 - 2(24.75))/4 = 12.375$.

c. It could be said that Apex's promise to produce 165 units of output is completely "credible" when Acme produces 165 units.

iii. Does the Stackelberg equilibrium differ from the Cournot equilibrium for this pair of firms? Demonstrate and explain.

a. At the Cournot-Nash equilibrium both firms are simultaneously on their reaction/best reply functions so:

b. $Q_x^* = (99 - 2Q_m)/4$ and $Q_m^* = (99 - 2Q_x)/4$ (derived as above)

c. substituting yields: $Q_x^* = (990 - 2[(99 - 2Q_x)/4])/4$

d. or: $Q_x^* = (49.5 + Q_x)/4 = 49.5/4 + Q_x/4$

e. so $(3/4)Q_x^* = 49.5/4$ which implies that $Q_x^{**} = 16.5$

f. and by symmetry $Q_m^{**} = 16.5$

g. [Note that total output is higher under Stackelberg than under the Nash arrangement and aggregate profits lower. Although Acme is more profitable, Apex is less profitable.]

B. Continuous Dealings. Suppose that Al uses George's garage for all car repairs. Al tells George that if he ever believes that he has been cheated by George that he will never return. Suppose further that, ex post, cheating can always be determined by Al. George gains \$25.00 each time he honestly services Al's car and \$50.00 if he cheats. If Al leaves before service is obtained his payoff is 0. Al receives \$15.00 of consumer surplus if he uses George and \$5.00 if he uses another garage (known to be honest, but a bit further away and more expensive). However, if George cheats, Al loses \$15.00 (of surplus).

i. Analyze this as a one shot game. Should George cheat Al and/or should Al use George? Explain.

a. If Al shows up George should cheat Al (e. g. $50 > 25$)

b. But, knowing this, Al would not use George's services (e. g. $0 > -15$)

ii. Now consider the setting in which the garage game is to be repeated *ad infinitum*. Is the game now sub-game perfect in non-cheating by George and use of George's by Al? Demonstrate and explain.

a. The solution to this problem is a variation of the folk theorem.

b. That is, if Al can convince George that he will never return if George cheats him, then George now will provide good service ((e. g. $50 < 25/r$ as long as interest rate $r < 50\%$)

c. However, there are many other possibilities.

d. Moreover, Al's behavior is fully credible, since his alternative is not a perfect substitute. That is to say as long as George cheated less than half the time he is still better off with George than the alternative.

e. (Note that Al's threat to leave forever would be fully credible if the alternative were honest and inexpensive, e. g. in that case, he would have nothing to lose by leaving George and using the other shop. Competition can make continuous dealings more effective.)