I. Answers for the Problem Set from Lecture 1

A. Suppose that Al always prefers larger apartments to smaller ones, but is unable to discern difference of ten sq. ft. or less. Are Al's preferences transitive? Explain.

- *Al's preferences are not transitive. Take two apartments: A having 1000 sq. ft, B having 1009 square feet and C having 1018 feet.*
- *Note that Al is A (I) B and B(I) C but A ~(I) C where (I) means "indifferent between."*
- *Similarly Al is A (R) B and B(R) C but A ~(R) C where (R) means "is at least as good as" (the weak preference operator)*
- *(Perhaps surprisingly, the strong preference operator is transitive. If A "is at least ten sq. feet larger than B" and "B is at least ten square feet larger than C" then "A is always at least ten square feet larger than" C !)*

B. Determine whether the following sets of the following are convex sets or not.

i. Al has a budget set W = PaA + PbB, where A and B are both non negative numbers. Pa is the price of good A and Pb is the price of good B. Is Al's budget set convex?

- *An illustration is almost enough for this purpose*
- *Budget sets are defined over goods, here A and B. The proposition can be demonstrated as follows suppose that {A' , B'} and {A", B"} are elements of this set.*
- *Is it necessarily the case that the convex combination of these points (αA' + (1-α)A", αB'+(1-α)B") is also a member of this set?*
- *To demonstrate that it every convex combination of a point within the original set is also a member of the set, pick two arbitrary points (A',B') and (A", B") that are members of the set and demonstrate that*
- *W > Pa [αA' + (1-α)A"] + Pb [αB'+(1-α)B"]*
- *Note that the above can be rewritten as: α [Pa A' + Pb B'] + (1-α) [PaA" + Pb B"]*
- *We know from our initial assumption that (A',B') and (A", B") are elements of the budget set and have to be less than α W + (1-α) W which is just equal to W*
- *Hence Pa [αA' + (1-α)A"] + Pb [αB'+(1-α)B"] < W*
- *QED*

ii. Barbara has a bliss point "B" characterized in terms of all goods relevant over which her preferences are defined. Consider bundle "C" which has less of every good than B. Is the better set for "C" convex? (Depict two dimensional examples.)

- *It depends on the shape of the indifference curves and better set. A good answer will illustrate both cases.*
- *Circles and Ellipses generally do have this property*
- *But, "bent jelly bean" shaped better sets do not.*
- *(What every answer you construct has to show a bliss point.)*

C. Consider the function $f(X) = 1/X^2$ for $X \neq 0$ and $f(X) = 1$ for $X = 0$.

i. Is the domain of f compact?

- *The domain is not compact since it consists of the real number line which is not unbounded. (See LF 8.15 and 8.16)*

ii. Is f a continuous function?

- *F is not continuous since the limit of a series that converges on zero will be "infinity" rather than 1.*

iii. Is f monotone increasing for x>0 ?

- *Monotone increasing means that the function always increases as x increases. However, this function always DECREASES as x increases.*

- *That is to say, the function $1/X^2$ is monotone decreasing in X rather than monotone increasing.*

iv. Prove that the f does not have a limit at X=0.

- *For there to be limit point at $X = 0$, some number Y^* must exist such that the function of a sequence $x_1, x_2, \ldots$ in X that has a limit point at 0 can by used to generate a sequence $f(x_1), f(x_2) \ldots$ thathas a limit point at $f(0) = Y^*$. That is to say there must be an infinite number of points from the f(xi) sequence within an arbitrarily small distance of Y^**
- *One sequence that converges to 0 is 1/N, where N is an integer, $N = 1, 2, 3 \ldots$.*
- *The corresponding sequence in the range of the function is F(N) is $1/(1/N)^2 = N^2$*
- *Let D be a neighborhood about Y^*.*
- *Let N' be the largest N such that $N^2 < Y^* - D$ and let N" be the smallest N such that $N^2 > Y^* + D$.*
- *The number of elements the neighborhood about Y^* is clearly finite since it is $N'' - N' - 1$.*
- *Consequently, only a **finite** number of elements of the sequence 1/N can be within distance D of Y^*. Hence, f is not continuous at 0.*

D. For most purposes, economists assume that utility functions are continuous and twice differentiable.

i. What does this differentiability imply about the commodity space over which the utility function is defined?

- *The domain has to be such that limits of sequences in x exist, that is to say it has to be closed.*
- *Essentially this implies that commodities are infinitely divisible--as might be said of time or space, but perhaps not pairs of shoes or automobiles.*

ii. What does twice differentiability imply about the shape of the utility function?

- *Twice differentiability implies that whenever the utility function's slope changes it does so continuously, that is to say smoothly.*
- *This assumption rules out sharp kinks, cusps, and discontinuities in utility.*

iii. Are their any important limitations of models which rely upon the assumption of differentiable utility functions? Discuss.

- *In some cases, discreteness may be a critical aspect of the choice at hand, in which case the continuity and smoothness assumptions could generate misleading conclusions or hide what is fundamentally interesting.*
- *The methodological/scientific question boils down to how good an approximation these mathematical assumptions are.*
- *(The strict concavity assumptions that we deal with in Lecture 2 add another series of restrictions on the nature of utility itself.)*
- *(Of course, that reverse assumption, that only discrete possibilities exist, can also yield very misleading results when the underlying phenomena is actually more or less continuous.)*

iv. Is differentiability an important modeling assumption or a mathematical convenience? Explain.

- *It is both. In most of the cases studied by economists the convenience of differentiability (reduced cost of manipulation) appear to exceed its costs.*
- *However, in some cases, all or nothing decisions have to be made and can not really be handled very well with continuous models because the behavior called for by the solutions (maximum or minima) are not even approximately feasible.*