

## Answers for Problems Assigned Week 5

### I. Problems (collected next week)

- A. Suppose that Acme is a profit maximizing monopolist facing the inverse demand curve  $P = p(Q, Y)$  and production costs  $C = c(Q, w, r)$  where  $Y$  is average consumer income,  $w$ , is the average wage rate of those employed by Acme, and  $r$  is the prevailing market rate of interest. (Assume that  $p$  and  $c$  have the conventional first and second derivatives.)
- Characterize Acme's profit maximizing output level.
    - $\Pi = PQ - C$  so here  $\Pi = p(Q, Y)Q - c(Q, w, r)$
    - Differentiating with respect to  $Q$  and setting the result equal to zero yields:
    - $P_Q Q + P - C_Q = 0$  (= H for ii b below)
    - which according to the implicit function theorem implies that  $Q^* = q(Y, w, r)$
    - [Note that the second order condition is  $\Pi_{QQ} = P_{QQ}Q + 2P_Q - C_{QQ} < 0$ , and is satisfied given the conventional assumptions about demand curves and production,  $P_Q < 0$  and  $C_{QQ} > 0$ , if  $P_{QQ}$  is less than zero or "small." This conditions would assure that the profit function is strictly concave.]
  - Characterize how Acme's output will increase as household income increases.
    - Use the implicit function differentiation rule. (on "i c" above)
    - $Q^*_Y = H_Y / (-H_Q) = (P_{QY}Q + P_Y) / -(P_{QQ}Q + 2P_Q - C_{QQ}) <> 0$
    - The denominator is just the negative of the second order condition, and will be positive ( $-\Pi_{QQ} > 0$ ) if the s. o. c. is satisfied. The numerator will be positive if  $P_Y > 0$  and  $P_{QY} > 0$ .
    - Remember that  $P$  is the inverse demand curve, and  $P_Y > 0$  implies that the good of interest is a normal or superior good.  $P_{QY}$  is the effect that income has on the slope of the demand curve.
      - If an increase in income makes the demand curve less negatively sloped, then  $P_{QY} > 0$ .
      - (On the other hand, if income makes the demand curve more steeply downward sloping (less price sensitive) then  $P_{QY} < 0$  and the sign of "b" is ambiguous. That is to say the sign of  $Q^*_Y$  will depend on the relative size of  $P_{QY}Q$  and  $P_Y$ .

- Characterize how Acme's profits change as interest rates increase.
    - The profit observed will tend to be  $\Pi^*$  if the firm of interest is a well informed profit maximizer, where  $\Pi^* = p(Q^*, Y)Q^* - c(Q^*, w, r)$ .
    - Differentiating with respect to  $r$  yields:  
 $\Pi^*_r = P_Q Q^*_r Q^* + P Q^*_r - C_Q Q^*_r - C_r$ .
    - Note that this can be rewritten as:  
 $Q^*_r (P_Q Q + P - C_Q) - C_r$
    - The term inside the parentheses is the first order condition *which is zero* at  $Q^*$ --that is after all how we defined  $Q^*$ . So the result simplifies to:  
 $\Pi^*_r = -C_r$
  - Characterize how Acme's profits change as household income increases.
    - Again, the profit observed will tend to be  $\Pi^*$  if the firm of interest is a profit maximizer, where  $\Pi^* = p(Q^*, Y)Q^* - c(Q^*, w, r)$ .
    - Differentiating with respect to  $Y$  yields:  
 $\Pi^*_Y = (P_Q Q^*_Y + P_Y)Q^* + P Q^*_Y - C_Q Q^*_Y$ .
    - Note that this can be rewritten as:  
 $Q^*_Y (P_Q Q + P - C_Q) + P_Y Q^*$
    - The term inside the parentheses is just the first order condition *which is zero* at  $Q^*$ --that is after all how we defined  $Q^*$ . So the result simplifies to:  
 $\Pi^*_Y = P_Y Q^* > 0$  (if the good in question is a normal good)
- B. Suppose that Al wins the lottery and will receive \$100,000/year for the next twenty five years.
- What is the present value of his winnings if the interest rate is 4%/year?
    - Use the present value of an annuity formula  $P = F [(1+r)^T - 1] / [(1+r)^T r]$
    - $100,000 [(1+.04)^{25} - 1] / [(1+.04)^{25}(0.04)] = 100K(15.622) = 1,562,200$
    - (Note this is significantly less than  $20 \times 100k = 2$  million!)
  - How much more would a prize that promised \$100,000/year forever be worth? (Hint: find the limit of formula IIB as  $T$  approaches infinity.)

- a. Recall that  $P = F [(1+r)^T - 1]/[(1+r)^T r] = F [1/r - 1/((1+r)^T)]$   
(from the derivation of this formula last week)
- b. Consequently, as  $T$  approaches infinity,  $P$  approaches  $F/r$ .
- c. Substituting into  $P = F/r$  yields  $P = 100K/0.04 = 2,500,000$  (2.5 million)
- d. (Note that the entire flow after year 25 is worth less than a million dollars!)
- C.** Suppose that Al can purchase insurance to reduce the down side risk of fire in his home. Suppose that in its current state, the house is worth 200k and that after the fire it would be worth 100k. The probability of a fire is  $P$ . Insurance increases the after fire value of the house back to 200k.
- i. If Al is risk neutral, what is the highest price that he would pay for this insurance?
- a. The highest price that Al would pay for insurance is the one that would make him indifferent between having insurance and not having it.
- b. The present value of the house without insurance is  $P(100) + (1-P)(200)$  where  $P$  is the probability of a fire.
- c. Since the insurance provides full coverage, the expected value of the house with insurance is  $P(200-C) + (1-P)(200-C)$
- d. So the highest price that Al would be willing to pay is the one that sets  $P(200-C) + (1-P)(200-C) = P(100) + (1-P)(200)$
- e. i.e.  $200 - C = 100P - 200P + 200$ , so  $C^{\max} = 100P$
- f.  $100P$  is the expected damage which is avoided by buying full coverage insurance.
- ii. If Al is risk averse with  $U = V^{.75}$ , where  $V$  is the value of the house, what is the highest price that he would be willing to pay?
- a. The highest price that Al would be willing to pay for insurance is the one that makes him indifferent between having insurance and not having it, which now *has to be* characterized in terms of expected utility.
- b. Since  $U = V^{.75}$ , the expected utility of the house without insurance is  $U^c = P(100)^{.75} + (1-P)(200)^{.75}$
- c. The expected utility of the house with full replacement insurance is:  
 $U^c = P(200 - C)^{.75} + (1-P)(200 - C)^{.75} = (200 - C)^{.75}$
- d. Al is indifferent if  $P(200 - C)^{.75} + (1-P)(200 - C)^{.75} = P(100)^{.75} + (1-P)(200)^{.75}$
- e. or  $(200 - C)^{.75} = P(100)^{.75} + (1-P)(200)^{.75}$
- f. The implicit function theorem suggests that  $C^* = c(P)$
- g. In this case, we can solve for  $C^* = 200 - [P(100)^{.75} + (1-P)(200)^{.75}]^{1.33}$
- D.** Suppose that Al can purchase lottery tickets for \$5.00 each and that the probability of winning the lottery is  $P$ . If Al wins, he will receive \$50,000 dollars per year for 20 years. The twenty year interest rate is 3%/year.
- i. What is the highest price that Al will pay for a ticket if he is risk neutral?
- a. This problem combines present value and expected value calculations. First, note that the actual cost (\$5.00) of the ticket is irrelevant for the problem since we are trying to determine Al's reservation price for lottery tickets.
- b. The present value of losing is just the cost of the ticket,  $C$ .
- c. The present value of winning is  $50K [(1.03)^{20} - 1]/[(1.03)^{20} (0.03)] - C$  or  $50K(14.8775) - C$  or  $743.873K - C$
- d. The highest value that Al is willing to pay is the one that sets the expected value of the ticket equal to zero, e. g.  $PW + (1-P)L = 0$  where  $P$  is the probability of winning and  $W$  is the payoff of winning and  $L$  that of losing.
- e. In this problem:  $0 = P(743,873 - C) + (1-P)C$
- f. or gathering terms  $0 = 743,873P - C$ . Thus  $C^{\max} = P(\$743,873)$
- g. The highest price that a risk-neutral person will pay for such a lottery ticket is the *expected present value of the winnings*.
- ii. Determine how Al's willingness to pay for the ticket increases as  $P$ , the probability of winning, increases and as the interest rate diminishes.
- a. This is simply the derivative of  $C^{\max}$  with respect to  $P$  which here is just:
- b.  $C_p^{\max} = \$743,873 > 0$