Answers for Problems Assigned Week 6

A. Suppose that the following functions represent marginal cost functions.
(i) Calculate the total variable cost functions associated with these marginal cost functions.
(ii) Calculate the additional total cost associated with producing 6 rather than 3 units of output.

First find all the total variable cost functions by integrating the respective MC functions.

i. $MC = .5Q + 5$

a. $\int MC = \int .5Q + 5\, dQ = 0.25\, Q^2 + 5Q + c$
where c is an unknown constant

ii. $MC = Q - (K+Q-6)^2$

a. $\int MC = \int [Q - (K+Q-6)^2]\, dQ = \int\ Q - (Q^2 + 2\,(K-6)Q + (K-6)^2)\, dQ$

b. so $\int MC = Q^2 - (1/3)Q^3 - (K-6)Q^2 - (K-6)^2\, Q - c$

c. or $\int MC = -(1/3)Q^3 - (K-5)Q^2 - (K-6)^2\, Q - c$
where c is an unknown constant

iii. $MC = e^{.5Q}$

a. $\int MC = \int e^{.5Q}\, dQ$

b. $\int MC = 2\, e^{.5Q} + c$ using formula "v" from Lecture 6
and c is, again, an unknown constant

iv. $MC = A/K$ (with A, K constants)

a. $\int MC = \int A/K\, dQ$

b. so $\int MC = AQ/K + c$ where c is, again, an unknown constant

[Note that in the context of this problem, c = 0. Since the function characterized is a total variable cost function, we know that total variable cost at Q=0, is 0. If we were attempting to characterize a total cost function, then c would be interpreted as total fixed cost.]

Next find the definite integrals of marginal cost functions between 3 and 6. Note that this calculation requires subtracting the total variable cost of 3 from that of 6

- $TVC = 0.25\, Q^2 + 5Q + c$ yields $\Delta TVC = .25(6)^2 + 30 - .25(3)^2 - 15 = 21.75$
- $TVC = -(1/3)Q^3 - (K-5)Q^2 - (K-6)^2\, Q - c$ yields
  $\Delta TVC = -(1/3)6^3 - (K-5)6^2 - (K-6)^2\, 6 + (1/3)Q^3 + (K-5)3^2 + (K-6)^2\, 3$
  $\Delta TVC = -72 - 36K + 180 - 6K^2 + 72K - 216 + 3 + 9K - 45 + 3K^2 - 36K + 108$
  $\Delta TVC = -3K^2 + 9K - 42$
- $TVC = 2\, e^{.5Q}$ yields $\Delta TVC = 2\, e^{.5(6)} - 2\, e^{.5(3)} = 2\,(e^{3} - e^{1.5})$
  $\Delta TVC = (31.207)$
- $TVC = = AQ/K + c$ implies $\Delta TVC = 6A/K - 3A/K = 3A/K$

B. Suppose that Al can purchase insurance to eliminate the down-side risk of fire in his home. Suppose further that in its current state, the house is worth 200k and that after the fire it would be worth between 50K and 150k. The probability of a fire is 0.1. If there is a fire, the probable range of damages is uniformly distributed between 50K and 150K. (That is to say, given that there is a fire, the probability of a particular house value is f(V) = 0.01 within the 50-150 k range, and zero outside that range.)

i. Although not asked, it is useful to note that the expected **value of the house** without insurance would be
$E(V) = (0.90)\,(200K) + (0.1)[\,\int_{50k}^{150K} (0.01)\, dV\,]$

ii. If Al is risk averse with $U = V^{.75}$, where V is the value of the house, what is the highest price that he would be willing to pay for insurance?

a. The highest price makes him indifferent between having the insurance and not having it so. $U^e\,(\text{with}) = U^e\,(\text{without})$ at C^{max}

b. U^e (with insurance) will be $(200K - C^{max})^{.75}$

c. U^e (without) will be $(0.90)\,(200)^{.75} + (0.10)[\,\int_{50k}^{150K} (0.01)V^{.75}\, dV\,]$

d. (Remember that you have to calculate the expected utility along every value of the house that the owner may realize.)

e. or $(0.90)\,(200)^{.75} + (0.10)[\,(0.01/1.75)(150)^{1.75} - (0.01/1.75)(50)^{1.75}\,]$

f. so C^{max} is such that: "b" equal "e" which after grouping a few terms is:

g. $(200 - C^{max})^{.75} = (0.90)\,(200)^{.75} + (0.001/1.75)[\,(150)^{1.75} - (50)^{1.75}\,]$

h. Taking the .75 root (or raising both sides to the 1.33 power) yields:

i. $(200 - C^{max}) = \{(0.90)(200)^{.75} + (0.001/1.75)[(150)^{1.75} - (50)^{1.75}]\}^{1.3333}$

j. so $C^{max} = 200 - \{(0.90)(200)^{.75} + (0.001/1.75)[(150)^{1.75} - (50)^{1.75}]\}^{1.33}$ (in K)

iii. If Bob is risk averse with $U = V^{.50}$, what is the highest price that Bob is willing to pay for insurance?

a. This problem is identical to the one above except that whereever 0.75 was an exponent there will now be a 0.50

b. U^e (with) will now be $(200 - C^{max})^{.50}$

c. U^e (without) will now be

$$(0.90)(200K)^{.50} + (0.1)[\,{}_{50k}\!\int^{150K} (0.01)V^{.50}\,dV\,]$$

d. or $(0.90)(200)^{.50} + (0.001/1.50)[(150)^{1.50} - (50)^{1.50}]$

e. Setting U^e (with) = U^e (without), then squaring both sides, and gathering terms yields:

f. So $C^{max} = 200 - \{(0.90)(200)^{.50} + (0.001/1.50)[(150)^{1.50} - (50)^{1.50}]\}^2$

iv. Who is more risk averse, Bob or Al? Explain.

a. The person that is most risk averse is the one willing to pay the most for insurance.

b. In the problem this boils down to which of

- $200K - \{(0.90)(200)^{.75} + (0.001/1.75)[(150)^{1.75} - (50)^{1.75}]\}^{1.3333}$

  or

- $200K - \{(0.90)(200)^{.50} + (0.001/1.50)[(150)^{1.50} - (50)^{1.50}]\}^2$

  is greatest.

c. Since these are both real numbers they can simply be computed and compared.

d. The first is $200 - \{47.865 + (0.00057)(5489.09)\}^{1.3333}$

- which is $200 - \{47.865 + (0.00057)(5489.09)\}^{1.3333} = 200 - 189.1115$
- or \$10.889K

e. The second is $200 - \{12.728 + (0.000666)(1483.56)\}^2$

- which is 200 - 188.155 = \$11.845K
- The second person, Bob, is *more* risk averse than the first, Al. Bob is willing to pay around \$956.37 more per year for insurance than Al to avoid the same risk.

v. An alternative way to determine which person is most risk averse is to use the **Arrow-Pratt** measure of risk aversion (from lecture 5)

- $r(Y) = -U''(Y)/U'(Y)$

a. For Al, $U' = (0.75)V^{-0.25}$, and $U'' = -(0.25)(0.75)V^{-1.25}$

- so the Arrow-Pratt measure is $(0.25)(0.75)V^{-1.25} / (1.75)V^{-0.25}$
- or $0.25/V$

b. For Bob, $U' = .5V^{-0.5}$ and $U'' = -0.25V^{-1.5}$

- so Bob's Arrow-Pratt measure of risk aversion is $0.25V^{-1.5} / .5V^{-0.5}$
- or $0.5/V$

c. Since a higher number indicates greater risk aversion, *Bob is more risk averse than Al.* That is to say, for a given level of V, $0.5/V > 0.25/V$.

- (Notice that the Bob and Al both exhibit less risk aversion as V goes up. That is to say, exponential utility functions exhibit diminishing "absolute risk aversion.")