# Answers for Problems Assigned Lecture 8

**A.** Work through an existence proof for a two dimensional Edgeworth box. That is to say formally lay out your assumptions and work through a two dimensional version of the proof outlined above.

i. Proof of the Existence of a Walrasian Equilibrium (from Varian)

a. The main trick in most existence proofs is to construct a transformation of the problem so that a continuous function maps a set into itself.

b. Define map $g : S^{k-1} \rightarrow S^{k-1}$ by $g^j(P) = [P^j + \max(0, z^j(P)] / [1 + \Sigma_n \max(0, Z^n(P)]$ *(where in this case k, n = 2)*

c. The price of good one is normalized as: $P^1 = P^1/(P^1 + P^2)$ which bound $P^1$ to the 0-1 interval.

d. (This of course will not affect aggregate demand as we have already established above, but does assure that $P^1$ is in the 0-1 interval.)

ii. The map becomes map $g : S^1 \rightarrow S^1$ :

$$g^1(P^1) = [P^1 + \max(0, z^1(P^1)] / [1 + \max(0, Z^1(P^1)) + \max(0, Z^2(P^1))]$$

*(Remember that k = 2, and n = 2, so the aggregate excess demand functions $Z^1$ and $Z^2$ are for goods 1 and 2.)*

a. This mapping is continuous since z and $\max(0, z^n(p^1))$ are continuous.

b. It lies in the unit simplex since $0 \leq g^1 \leq 1$.

iii. By Browers fixed point theorem there is a P* such that $P^* = g(P^*)$ for good 1. (That is to say a fixed point exists.)

a. Thus, dropping the "1" supperscripts for the good:
$P^* = [P^* + \max(0, z^j(P^*)] / [1 + \Sigma_n \max(0, z^n(P^*)]$

iv. Now we have to show that P* is an Walrasian equilibrium.

v. Cross multiplying by the denominator yields

$$P^* [1 + \Sigma_n \max(0, z^n(P^*)] = [P^{j*} + \max(0, z^j(P^*)]$$

a. Then Multiplying both sides by $z(P^*)$

$$z(P^*)P^* [1 + \Sigma_n \max(0, z^n(P^*)] = z(P^*)[P^{j*} + \max(0, z^j(P^*)]$$

b. From Walras law we know that the left-hand side equals zero. (The first term before the brackets terms has to be zero, the money value of excess demand has to be zero in the aggregate.)

c. If the right hand side equals zero, then $z(P^*)$ *has to be zero* for j = 1, 2. (Otherwise, the product of $z(P^*)[P^{j*} + \max(0, z^j(P^*)]$ would exceed zero.

d. Thus the excess demand in both markets must equal zero at P*. Q. E. D.

**B.** Critique the Walrasian model. To some extent the above existence proof looks very general. Think a bit about the assumptions and see if you can find any implicit or explicit assumptions which are unbelievable.

i. Although there are not many assumptions used in the above proof, there are many mathematical and implicit empirical assumptions that one might hesitate to make.

ii. Among the mathematical ones:

a. All the goods are assumed to be available in infinitessimal amounts, otherwise the excess demand functions can not be continuous.

b. Preference orderings are assumed to be complete and transitive. This rules out cases where prefernces might exhibit some local intransitivity--perhaps because differences between alternative bundles can not be decerned until they become fairly large.

iii. A few of the conceptual ones are:

a. The characterization of the budget set implicitly assumes that all persons know all prices and all possible goods that can be purchased. (Personally, I am always finding new goods and services.)

b. All goods sell for a unique price, moreover the price at which transactions (finally) take place is the equilibrium price.

c. Persons know their preference ordering even in areas they have no experience in. (preferences are complete)

d. No one has any bargaining power.

e. Property rights are perfectly enforced. There is no crime.

f. There does not appear to be a government (although this can be incorporated into a GE model that includes production--at least to a limited extent).

g. Price movements appear to be instantaneous, or waiting for markets to clear is costless, otherwise trade would emerge at disequilibrium prices.