Answers for Problems Assigned Lecture 9

I. Review Problems

A. Let R be the "reward from mutual cooperation," T be the "temptation of defecting from mutual cooperation," S be the "suckers payoff" if a cooperator is exploited by a defector, and P be the "Punishment from mutual defection." Show that in a two person game, relative payoffs of the ordinal ranking T > R > P > S is sufficient to generate a prisoner's dilemma with mutual defection as the Nash equilibrium.

		Player B: Cooperate	Player B: Defect
Player A	Cooperate	R, R	S, T
	Defect	T, S	P, P

ii. First, note that T> R and P > S is sufficient for each player to have a dominant strategy of "defection."

iii. The dominate strategies imply a Nash equilibrium of Mutual Defection, which yields a payoff of (P, P).

iv. Second, note that there is a dilemma at the Nash equilibrium in that a Pareto Superior move exists. Since R>P, both players prefer the equilibrium of mutual cooperation to that of mutual defection.

v. These are the essential features of a Prisoner's Dilemma problem. QED

B. Suppose that the inverse demand curve for a good is P = 100 - Q and that there are two producers. Acme has a total cost curve equal to C = 5Q and Apex has a total cost curve of C =10 Q. Each firm controls its own output. Prices are determined by their combined production. *Characterize the Cournot-Nash equilibrium to this game.*

i. Characterize the profit function for one of the firms:

a. $\Pi = PQ_1 - C_1 = (100 - Q_1 - Q_2)Q_1 - 5Q_1$ (for Acme)

b. Differentiating with respect to Q yields:

c. $100 - Q_2 - 2Q_1 - 5 = 0$ which implies that $Q_1^* = (95 - Q_2)/2$

ii. Repeat this for the other firm:

a. $\Pi = PQ_1 - C_1 = (100 - Q_1 - Q_2)Q_1 - 10Q_1$ (for Apex)

b. Differentiating with respect to Q yields:

c. $100 - Q_2 - 2Q_1 - 10 = 0$ which implies that $Q_2^* = (90 - Q_1)/2$

d. (Note that this is not a symmetric game so their best reply functions are a bit different from one another.)

iii. To find the Cournot-Nash Equilibrium, find where the two best reply functions can be satisfied simultaneously, e. g. where they intersect.

a. Substituting iic into ic yields:

b. $Q_1^{**} = (95 - [(90 - Q_1^{**})/2])/2$ or cross multiplying by 2 and gathering terms

c. $(50 + Q_1^{**}/2) = 2 Q_1^{**}$ or

d. $(3/2)Q_1^{**} = 50$ which implies that

e. $Q_1^{**} = 33.33$

f. And, since $Q_2^{**} = (90 - Q_1^{**})/2$, $Q_2^{**} = (90-33.33)/2 = 28.33$

C. Suppose that there are two neighbors each of whom enjoy playing their own music loudly enough to annoy the other. Each maximizes a utility function defined over other consumption, C, the volume of their own noise, and that of their neighbor's (a bad), $U_1 = u(C_1, N_1, N_2)$. Each has a budget constraint of the form , Yi = Ci + Ni.

i. Characterize each neighbor's reaction function, and determine its slope.

a. Use the substitution method to imbed the constraint into the utility function

b. $U_1 = u(Y_1 - N_1, N_1, N_2)$

c. Differentiate with respect to control variable N_1 . (Remember that Y and N_2 are exogenous for Mr. 1)

d. $dU/dC_1 = U_{C1}(-1) + U_{N1} = 0 \quad \equiv H$

e. so $N_1^* = n_1(Y, N_2)$ (This is Mr. 1's best reply or reaction function.)

f. The reaction function for Mr 2 can be derived in the same way:
$N_2^* = n_2(Y, N_1)$

g. At the Nash equilibrium, $N_1^{**} = c_1(Y, N_2^{**})$ and $N_2^{**} = c_2(Y, N_1^{**})$ where $C_1^{**} = Y - N_1^{**}$ and $C_2^{**} = Y - N_2^{**}$

h. That is to say at the Nash equilibria, both perons 1 and 2 will be simultaneously maximizing their utility, given equilibrium behavior of the other.

ii. The slope of Mr 1's reaction function is the effect that an increase in N2 has on his own optimal noise production.

a. This is can be determined using the implicit function differentiation rule (on ie and id).

b. $N_1{}^*{}_{N2} = H_{N2}/-H_C = [\, U_{C1N2}(-1) + U_{N1N2}\,] \,/\, -[\, U_{C1C1} - 2U_{N1C1} + U_{N1N1}] > 0$

if
$U_{C1N1} > 0$, $U_{N1N1} < 0$ and $U_{C1C1} < 0$

and $U_{C1N2} < 0$ and $U_{N1N2} > 0$.

c. (Only if the cross partials of the numerator have opposite signs can this slope be signed. It is likely that $U_{N1N2} > 0$ since the value of one's own noise (music) tends to increase its marginal value as a mask for other noises when the nieghbors are being loud. $U_{C1N2} < 0$ seems natural if you think of "eating" while obnoxious music is playing, e. g. the obnoxious music would reduce the marginal utility of eating.)

d. Given this geometry, the reaction curve of Mr 1 is upward sloping (e. g. he produces more noise) as his income increases.

iii. What happens to neighbor 1's reaction function (N_1^*) if his income increases?

a. This is can be determined using the implicit function differentiation rule (on ie and id).

b. $N_1{}^*{}_Y = H_Y/-H_C = [\, U_{C1C1}(-1) + U_{N1C1}\,] \,/\, -[\, U_{C1C1} - 2U_{N1C1} + U_{N1N1}] > 0$

if $U_{C1N1} > 0$, $U_{N1N1} < 0$ and $U_{C1C1} < 0$.

c. So Mr 1's reaction curve shifts up (e. g. he produces more noise) as his income increases, given any level of noise from Mr 2.

iv. Show the effect that a simultaneous increase in each neighbor's income has on the Nash equilibrium of this game.

a. Graphically, both reaction curves shift up as both recieve larger incomes.

b. This tends to increase total noise production as each buys more powerful stereos, partly to drown out the other.