Applications of Game Theory

Much of economics deals with circumstances where there are large numbers of persons and firms so that the effects of an single individual, firm, or small group on any other are very small. However, there are a wide variety of choice settings in economics where the payoffs of one person, firm, or group depends in part upon what some other person, firm, or group is doing. This is the domain of models based on game theoretic concepts.

I. Stackeberg Duopoly

A. In the **Stackelberg** model, the first mover (leader) tries to take account of the likely output decision of the other firm (the second mover or follower). In effect it chooses its profit maximizing output given the "profit maximizing output schedule" of its competitor.

i. Let "a" be the leader. Now: $\Pi^a = PQ^a - C = p(Q^a+Q^b, Y)Q^a - c(Q^a, w, r)$ but Qb rather than being taken as given is anticipated to be the quantity required by firm b's reaction function, $Q^b = q^b(Q^a, w, r, Y)$.

ii. The first order condition for the leader becomes: $P + Q\ (P_Q (1 + Q^b_{Qa}) - C_{Qa} = 0$

iii. Note that the marginal revenue function now includes the effect of firm b's change in output on the prevailing price, as well as the firm a's own effect on price.

B. The Stackelburg model can also be used to model decisions in cases where there is a "natural order" to the decision setting of interest.

i. For example in crime control, normally the legal system "moves first" by imposing a schedule of fines and allocating resources to the police and courts. The criminal moves second by choosing his crime rate.

ii. Clearly the best decision for the "legal system" (median voter) is to take account of how the typical criminal will respond to its decision regarding penalties and policing effort

II. Imperfect - Monopolistic Competition: an Extention of Cournot

A. The Cournot duopoly model provides a very natural method of modelling the effects of entry. One can easily extend the model to include 3, 4, 5 ... N firms. The result will be market prices and output that more and more conform to the perfectly competitive result.

B. Consider an extension of the linear Cournot Duopoly problem in which there are N identical firms rather than just 2. Suppose each firm has cost function C = cQ and let the inverse market demand be: $P = XY - eQ$ where market production is $Q = [Q^a + (N-1)Q^o]$

i. The first order condition that characterizes maximal profits for firm "a" is now

$$(XY - e(N-1)Q^o - 2eQ^a) = c$$

ii. Firm a's reaction function is thus: $Q^a = [XY - e(N-1)Q^0 - c\ ]/\ 2e$

iii. and the Cournot Nash equilibrium output for a typical firm at the symmetric equilibrium is : $Q^* = [XY - c] / (N+1)e$ and total output is N times as large: $Q^* = \{[XY - c] / e\} \{N/(N+1)\}$

iv. As N approaches infinity, the total output approaches the competitive equilibrium. *Perfect Competition, thus, is a Limiting Case of entry in a Cournot/Nash type model.*

C. In general there are a wide variety of models of imperfect competition, which vary mainly with respect to the manner in which players anticipate or fail to anticipate reactions of other players in the game. [The so-called conjectural variation.]

III. Externalities and Pigovian Taxation

A. In cases where an externality is generated by an activity, it will often be the case that the privately optimal activity levels will ones that are **not Pareto efficient**.

B. The easiest way to demonstrate this mathematically is with a two person non cooperative game (group) illustration.

i. Suppose that Al and Bob are neighbors. Both own barbecues, and that neither enjoys the smell of smoke and such associated with the other use of their barbecue. Let us refer to Al as Mr. 1 and Bob as Mr. 2.

ii. Let $Ui = u_i(Ci, Bi, Bj)$ for each person i (here: i = 1, 2) with Ci being food cooked indoors and Bi being food cooked outdoors by i, and Bj being food cooked outdoors by the neighbor ($i \neq j$). To make the model tractable, assume that Mr. I allocates his "kitchen time" Ti between cooking and barbecuing so that Ti = Ci + Bi for all i.

iii. Mr I's barbecuing time can be determined by maximizing U subject to the time constraint. Substituting, the constraint into the objective function to eliminate Ci yields: $Ui = u_i(Ti - Bi, Bi, Bj)$.

iv. Differentiating with Bi yields: $Ui_{Ci}(-1) + Ui_{Bi} = 0$. Each person will use the barbecue up to the point where the marginal cost in terms of reduced satisfaction from indoor cooking equals the marginal utility of further outdoor cooking.

v. The implicit function theorem implies that $B_1^* = b_1(B_2, T_1)$. This can be interpreted as Mr I's best reply function.

vi. In a Nash game between the two neighbors, equilibrium will occur when:
$B_1^{**} = b_1(B_2^{**}, T_1)$ and $B_2^{**} = b_2(B_1^{**}, T_2)$

C. The matter of whether this Nash equilibrium is Pareto Efficient or not is intuitively fairly obvious. Since each imposes costs on the other that are neglected, odds are they wind up in a setting where both would be better off if they produced less smoke.

D. That Al or Bob could be made better off by coordinating their behavior or not can be demonstrated in a number of ways.

i. One way to determine this is to show that a general **social welfare function**. $W = w(U^A, U^B)$ is maximized at by the relevant choices, given the same constraints. This can be ascertained by determining whether the first order conditions for maximizing "social welfare" are the same as those which maximize individual welfare.

ii. Another method of determining this without using a social welfare function (taken from Baumol) is to consider whether one person could be made better off at the Nash equilibrium without making the other worse off.

a. For example: maximize $L = u_1(T_1-B_1^*, B_1^*, B_2^*) - \lambda(U_2 - u_2(T_2-B_2^*, B_2^*, B_2))$ by varying B_1 and B_2

b. Differentiating with respect to B1 and B2, and appealing to the envelop theorem (to eliminate effects of B1* on B2* and vice versa) yields:

$$U1_{C1}(-1) + U1_{B1} - \lambda U2_{B1} = 0$$

and $$U1_{B2} - \lambda (U2_{C2}(1) + U2_{B2}) = 0$$

E. Note that these first order conditions are different than those met for either person insofar as they imply that the externality will be internalized at the margin for both parties..

IV. Electoral Competition in a Representative Democracy

A. If two candidates can choose policy positions, and voters will vote for the candidate closest to their preferred policy, it turns out that the candidate who is closest to the median voter will win the election.

i. (This "distance based" model of voter behavior is sometimes called the spatial voting model.)

ii. If the candidates may freely choose policy positions, there is a tendency for electoral competition to cause them to select essentially identical policy positions which maximize the median voter's welfare.

iii. At this Nash equibrium, the strong form of the median voter theorem results.

B. To see this consider the case where v(G) is the distribution of voter preferences and V^0 is the median voter's ideal point, that is to say the voter whose preferences lies exactly in the middle of the distribution, with half the voters preferring larger and half smaller values of G.

i. The votes received by a candidate 1 can be written as $V_1(G_1) = \int_{(-)}^{(G1+G2)/2} v(G)\, dG$ and that of candidate 2 as $V_2(G_2) = \int_{(G1+G2)/2}^{(+)} v(G)\, dG$.

ii. Each candidate wants to adopt the platform that maximize their own votes, given that chosen by the other candidate.

iii. The optimal decision of candidate 1 can be found by differentiating his vote function with respect to G_1, which yields: $V_{G1} = v((G_1+G_2)/2)\ /2$ which is not zero except where $G_1 = G_2$.

a. (The above is positive for $G_1 < G_2$, and is negative, $- v((G_1+G_2)/2)\ /2$, for $G_1 > G_2$.)

iv. This implies that a candidate should adopt the same platform as his competitor.

v. However, this is not really an equilibrium except at $G_1 = G_2 = V_0$. At every other place where the candidates adopt the same position, one of the candidates can always do better than the other (rather than accept a tie) by moving closer to the median voter.

vi. Thus, at the Nash equilibrium both candidates take the same position, the platforms maximize both candidate's votes (given what the other candidate has done), and also **maximize the welfare of the median voter.**

V. Contest Functions and Rent Seeking

A. The rent-seeking literature has used a game theoretic frame of analysis, which like that of the Chicago models, has been more focused on interests than on elections.

i. The core rent seeking model regards the process to be analogous to a lottery.

ii. The special favors which may be obtained through government--tax breaks, protection from foreign competition, contracts at above market rates etc.-- are the prize sought by rent seekers.

iii. The process by which these prizes are awarded is considered to be complex in that a wide variety of unpredictable personalities and events may ultimately determine who gets which prize. None the less, it is believed that the more resources are devoted to securing preferential treatment (e. g. the better prepared and more widely heard are the "rationalizations" for special preference) the more likely it is that a particular rent-seeker will be successful. Contrariwise, the greater the efforts of alternative rent-seekers, the less likely a particular rent-seeker is to succeed.

iv. As a first approximation of this political influence game, investments in political influence are often modeled as if they were purchases of lottery tickets.

B. The Basic Rent-Seeking Game

i. Suppose that N risk neutral competitors participate in a rent seeking game with a fixed prize, Π.

ii. Each player may invest as much as he wishes in the political contest.

iii. The prize is awarded to the player whose name is "drawn from a barrel" containing all of the political lottery "tickets." So, the expected prize for player i is $\Pi[R_i / (R_i + R_O)]$, where R is the value of the prize, R_i is the investment in rent seeking by player i, and t_O is the investment by all other players.

iv. If the rent seeking resource, R, cost C dollars each, the number of tickets that maximizes player 1's *expected reward* for a given purchase by all other players can be determined by differentiating the expected rent $\Pi^e = \Pi[R_i / (R_i + R_O)] - CR_i$ with respect to Ri and setting the result equal to zero.

v. $\Pi [1 / (R_i + R_O) - R_i / (R_i + R_O)^2] - C = 0$

vi. Which implies that: $\Pi [R_O / (R_i + R_O)^2] - C = 0$ or $\Pi R_O/C = (R_i + R_O)^2$

vii. So player 1's best reply function is $Ri^* = -Ro \pm \sqrt{\Pi R_O/C}$ Of course, only the positive root will be relevant in cases where Ri has to be greater than zero.

viii. In a symmetric game, each player's best reply function will be similar, and at least one equilibrium will exist where each player engages in the same strategy.

ix. Thus, if there are N-1 other players, at the Nash equilibrium, $R_o^{**} = (N-1)R_i^{**}$. which implies that $Ri^{**} = -(N-1)R_i^{**} \pm \sqrt{\Pi (N-1)R_i^{**}/C}$.

x. which implies that $NRi^{**} = \sqrt{\Pi (N-1)R_i^{**}/C}$ or squaring both sides, dividing by Ri** and N^2 and gathering terms, that: $Ri^{**} = [(N-1)/N^2] [\Pi/C] = [(1/N) - (1/N^2)] [\Pi/C]$

C. So for example with N = 2 and C = 1, $Ri^{**} = (\Pi/4)$

i. Total rent seeking effort is N times the amount that each player invests

ii. Thus in the two person unit cost case, $\mathbf{R} = \Pi/2$. Half of the value of the prize is consumed by the process of rent seeking. [Illustrating Figure]

iii. In the more general case, $\mathbf{R} = [(N-1)/N] [\Pi/C] = [1 - 1/N] [\Pi/C]$

D. The affect of entry on individual and total rent seeking expenditures can be determined by inspection or by differentiation Ciii and Bx above with respect to N. It is clear that individual contributions fall as the number of rent seekers increase, but also clear that the total amount of rent seeking "dissipation" increases.

i. In the limit, as $N \Rightarrow \infty$ the total rent seeking investment approaches the level where the value of those resources, RC, equals to the entire value of the prize, $\mathbf{R}^{**} C = [\Pi/C] C = \Pi$.

ii. The effect of increases in the cost of participating in the political influence game and/or changes in the value of the regulation to the rent-seeker can also be readily determined in this game.

E. The basic model can be generalized to cover cases where the prize is endogenous and where the probability of securing the prize varies, and to cases where the prize is shared rather than awarded to a single "winner take all" winner.

i. For example, $R_i^e = P(R_1, R_2, ...R_N)\Pi_i(\mathbf{R})$ encompasses many of these features.

F. The affects of economies of scale may also be examined in this general framework and in the earlier explicit one.