

Mathematical Economics

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Office Hours: Wednesday 2:00 - 3:30, Thursday 2:00 - 3:30 and by appointment

Main Texts and References:

De La Fuente, A. (1999) *Mathematical Methods and Models for Economists*.
New York: Cambridge Univ Press (Paper Edition, Trd); ISBN:
0521585295

Hirschliefer, J. (2001) *The Dark Side of the Force: Economic Foundations of Conflict
Theory* Cambridge: Cambridge University Press, 2001, 366 pp (paper).

Class Notes, distributed weekly in class.

(PDF versions of the class notes are also available at
<http://rdc1.net/class/MathEcon/index.htm>)

Optional Text and References:

Dixit, A. K. and Nalebuff, B. J. (1993) *Thinking Strategically : The Competitive
Edge in Business, Politics, and Everyday Life*. New York: W.W. Norton &
Company; ISBN: 0393310353

Debreu, Gerard (1975) *Theory of Value* New Haven: Yale University Press.
(paper)

Other Useful Texts:

Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995) *MicroEconomic
Theory*. New York: Oxford University Press.

Varian, Hal R. *Microeconomic Analysis* 3rd Ed. New York: W. W. Norton
and Co., 1992. ISBN 0 393 95735 7.

Chiang, A. C., *Fundamental Methods of Mathematical Economics*, 3rd Edition,
New York: McGraw Hill, 1984. ISBN 0 07 01081307

Chiang, A. C., *Elements of Dynamic Optimization*, McGraw Hill, 1992.

The purpose of this course is to introduce the students to "model building" as practiced by the economics profession and rational choice strands of political science and sociology. To this end, students are introduced to the mathematical tools and concepts that are most frequently used in economic models of the firm and consumer behavior. The course focuses on mathematical representations of optimizing individuals in a wide variety of "choice settings." The methods

developed can be used to analyze traditional economic decisions by firms and consumers, and other choice settings in politics and law. The models can be used to analyze decision making in any setting in which individuals have reasonably clear goals (objective functions) and confront reasonably clear constraints.

The course is lecture and note based. All of the core material is covered in class and in the lecture handouts. The textbooks provide additional material and explanations that may help students better understand the concepts and tools developed in class. The textbooks also serve as a reference library for those interested in making contributions to economic theory or in better understanding published work in the leading economic journals.

As in any mathematics or economics course, successfully mastering the material means that the student should be able to apply the mathematical tools and concepts to model settings beyond those explicitly covered in class. For example, the term paper requires students to analyze a choice setting or economic problem of their own design using the tools developed in class.

TENTATIVE COURSE OUTLINE

Date	Topic	Chapter Readings
9/2	I. Introduction: The Mathematical Analysis of Rational Choice Scope of Course, Usefulness and limitations of deductive methodology. Mathematical Concepts: weak ordering, compact sets, convexity, continuity, functions. Economic Applications: consumer theory, the theory of the firm Problems: Handout, V1.1, 1.6, 1.11 (See also C12;V7; M1)	LF: 1.1-4, 2.4-8, 7.1, H:Int-1, 14
9/09	II. Derivatives and Optimization Mathematical Concepts: partial derivative, chain rule, first and second order conditions, concavity, quasi-concavity, homothetic functions, objective function, constraints, substitution method. Economic Applications: the profit maximizing firm, cost-benefit analysis Problems: Handout, V3.1, 3.4, 3.5, 3.3, 4.1 (See also C9; V27)	LF: 4, 6.2 -3
9/16	III. Allocating Scarce Resources: Constrained Optimization Mathematical Concepts: Lagrangian multipliers, Lagrangian method, Kuhn-Tucker method, Kuhn Tucker Sufficiency Theorem, Arrow-Enthoven Sufficiency Theorem Economic Applications: Consumer Theory, Social Welfare Functions Problems: Handout, C12.2 1,3,4; C12.5 1,2; V: 7.2, 7.5 (See also C12,21; V9,10)	LF: 7.1
9/23	IV. Comparative Statics in Abstract Models: The Implicit Function Theorem Mathematical Concepts: derivatives of implicit functions, neighborhood, continuity, duality.	LF: 5.2, 7.1

Economic Applications: Supply and Demand, Reaction functions

Problems: C8.5 1,2,3 C8.6 1,2,4 ; V 5.1, 5.2, 5.3, 5.4, 6.1,6.3 (See also C8;V6, Mme)

9/30 V. Time and Uncertainty in Economic Choices LF: 4.4

Mathematical Concepts: Probability Function, Series, Expected Value, Present Value, Convergence, Limit Point, Taylor Series,

Economic Applications: Intertemporal choice, Decision making under uncertainty, Stability analysis.

Problems: Handout, C6.6 1,2,3 C6.7 4 C13.5 4,5; V11.5, 11.9, 11.11 (See also C6.6, 13.5; V11, Mmf)

10/07 VI. More on Decision Making under Uncertainty

Mathematical Concepts: Probability Density Function, Integrand, definite and indefinite integrals, risk aversion, subjective rate of time discounting.

Economic Applications: Continuous Discounting, Economic Growth, Risk and Uncertainty, Totals from Marginals, Measures of Risk Aversion

Problems: Handout, C13.2 1,2,4 C13.5 1,2 C13.6 2 ; V11.7; K3:3,4,5,8 (See also C13, V11, M6)

10/14 VII. Applications of the Rational Choice Model LF 8.1-2, 7.3

To Crime: The Wealth Maximizing Criminal (Becker, *JPE* 1968: 169-217)

To Politics: Tax Revenue Maximizing Leviathan (Buchanan and Brennan, *JPubE*, 1977: 255-73)

To the selection of a Central Banker (Waller, C. J., *AER*, 1992: 1006-12)

To the assignment of liability to lenders (Lewis and Sappington, *AER*, 2001:724-730.)

10/21 VIII. More Applications and Review for Midterm (study guide handout)

10/28 **Midterm Exam**

11/04 Exams returned and Reviewed

IX. The Existence of General Equilibrium (short lecture) LF 6.3

Mathematical Concepts: weak preference ordering, excess demand correspondence, Walras' law, upper semicontinuity, Brouwer's fixed point theorem, Kakutani fixed point theorem

Economic Application: Walrasian Equilibrium

Problems: Handout, V 17 1-6, 11,13, (See also V17, 21; Debreu 5,7; M 15-17)

11/11 X. Interdependent Decisionmaking in Small Groups: LF: 6.4-5, H:8
An Introduction to Game Theory

Mathematical Concepts: strategy, payoff, non-cooperative games, pure and mixed strategies, Kakutani fixed point theorem, Existence of Nash equilibrium, prisoner's dilemma game, zero sum game

Economic Applications: Reaction (best reply) Functions, Duopoly, Monopolistic Competition, Tragedy of the Commons

Problems Handouts: V15.1, 15.3,16.10, V15.2, 15.7 (See also V15, M7)

11/18 XI. Applications of Non-Cooperative Game Theory H: 5, 6, 7

Economic Applications: contest success functions, Stackelberg games, rent-seeking, anarchy, and the courts--also if time permits: contract enforcement, coalition formation, majority rule cycles Externality Theory, Signalling and Screening (Varian, Chapter 25), Contract Design and Enforcement: Moral Hazards, Separating and Pooling Equilibria.

Problems V15.2, 15.7, M9:B5, B7, B11, B14 (See also V15; K8,9.)

11/25 THANKSGIVING BREAK *no class*

12/02 XII. More Applications of Non-Cooperative Game Theory H11-13

Static and dynamic competition, Applications from Hirschliefer and, also, if time permits:

R&D in Duopoly (D'Aspermont and Jacquemin, *AER*, 1988: 1133-1137.)

Theory of Anarchy (Skaperdas, *AER*, 1992:720-739.)

Political Influence and Dynamic Consistency (Garfinkel and Lee, *AER*, 2000: 649-666)

Problems M: 13B3, 13B4, 13B8, 14B1, 14B3, 14C4, 14C8

12/09 **Papers Due**

12/09 XIII. Review for Final Study Guide/Class Notes

12/16 **Final Exam: 7:30-10:15** (necessarily comprehensive, but oriented toward material covered in second half of the semester)

Grades:

70% 2 exams

25% 1 Short Paper using Math-econ tools on a topic of interest

5% Homework (from handouts)/Class Participation Bonus

100%

Grades are based on the following set of minimums *A* >88 *B* >77% *C* >66%

For Additional or Extended Study Consider

Read Mas-Collel and or Varian in other areas of interest

More Game Theory

Roger B. Myerson (1997) *Game Theory : Analysis of Conflict*, Harvard Univ Press.

Evolutionary Game Theory

Jorgen W. Weibull, 1996, MIT Press.

Matrix Mechanics

LF: 3, C: 4, 5, 11

Matrix as a notational convention, addition and multiplication, singular matrices, transpose and inverse, matrix algebra and optimization, Cramer's rule, the bordered Hessian and positive definiteness

Economic applications: solution to linear demand system, second order conditions, comparative statics

Economic Growth and Stability

LF: 11.3, C:14,16

Mathematical Concepts: first order difference and differential equations, phase diagram, iterative method, chaos

Economic Applications: price expectations, dynamic stability, growth models

Problems: C 14.1: 1, 2; 14.2: 1, 4, 14.3: 1, 2, 3: 14.6: 1, 14: 2, 3, 4

Dynamic Optimization

LF: 11-13,V19; C² 1,2,6

Mathematical Concepts: optimal time path, initial state, functional, transversality conditions, control path, equation of motion, the Euler equations

Economic Applications: Optimal Investment and Savings Rules, Environmental policy

Problems: C²: 2.41,4,6