An Introduction to Non-Cooperative Game Theory

It is probably fair to say that the application of game theory to economic problems is the most active area of theory in modern economics. A quick look at any economics journal published in the past decade will reveal a host of articles which rely upon elementary game theory to analyze economic behavior of theoretical and policy interest.

To some extent, the tradition of game theory in economics is an old one. The Cournot duopoly model (1838) is an example of a non-cooperative game with a Nash equilibrium. Analysis of Stackelberg duopoly and monopolistic competition have always been based on intuitions very much like those of game theorists, and recent analysis has been based on the Nash equilibria of non-cooperative games.

Other modern work on: the self-enforcing properties of contracts, credible commitments, the private production of public goods, externalities, time inconsistency problems, and models of negotiation have also relied upon game theoretic models.

I. The Prisoners' Dilemma Game: A Simple Nash game.

A. Game theory is used to model a wide variety of economic settings. The choice settings in which economists apply game theory are generally small number settings in which individual decisions and welfare are interdependent. That is to say, in economic games each person's welfare depends, in part, on the decisions of other individuals "in the game."

- i. In Cournot duopoly, each firm's profits depend upon its own output decision and that of the other firm in the market.
- ii. In a setting where pure public goods are consumed, one's own consumption of the public good depends in part on one's own production level of the good, and, in part, on that of all others. [For example, after a snow fall, the amount of snow on neighborhood sidewalks depends partly on your own efforts at shoveling and partly that of all others in the neighborhood.]
- iii. In an election, each candidate's vote maximizing policy position depends in part on the positions of the other candidate(s).
- iv. Game theoretic treatments are less useful in cases where there are no interdepencies. A case where there is *no interdependence* it that of a producer or consumer in perfect competition. Here a consumer (or firm) is able to buy (or sell) as much as they wish without affecting market prices. Game theory can still be used in such cases, but with little if any advantage over conventional tools.

B. The *Prisoners' Dilemma game* is probably the most widely used game in economics.

- i. The "original" prisoners dilemma game goes something like the following. Two individuals are arrested under suspicion of a serious crime (murder or theft). Each is known to be guilty of a minor crime (say jay walking), but it is not possible to convict either of the serious crime unless one or both of them confesses.
- ii. The prisoners are separated. Each is told that if he testifies about the other's guilt that he will receive a reduced sentence for the crime that he is known to be guilty of.
- iii. The equilibrium of this game is that BOTH TESTIFY (CONFESS).

C. To see this consider the following game matrix representing the payoffs to each of the prisoners:

- i. Each cell of the game matrix contains payoffs, for A and B, in years in jail (a bad).
- ii. Each individual will rationally attempt to minimize his jail sentence.
- iii. Note that regardless of what Prisoner B does, Prisoner A is better off testifying. 10 < 12 and 1 <2. Testifying (also called confessing in other discussion of PD games) is the *dominant* strategy.
- iv. Note that the same strategy yields the lowest sentence for Prisoner B. If A testifies, then by also testifying B can reduce his sentence from 12 to 10 years. If A does not testify, than B can reduce his sentence from 2 to 1 year by testifying. The dominant strategy is a *pure* strategy in that *only one* of the strategy options is ever used.

		Prisoner B	
		Testify	Don't
Prisoner A	Testify	(10,10)	(1, 12)
	Don't	(12,1)	(2,2)

- v. The (testify, testify) strategy pair yields 10 years in jail for each. This is said to be the *Nash equilibrium* to this game because given that the other player has testified, each individual regards his own choice (testifying) as optimal. *No player has an incentive to independently change his own strategy at a Nash equilibrium.*
- vi. It is a dilemma because each prisoner would have been better off if neither had testified. (2 < 10). Independent rational choices do not always achieve Pareto optimal results.
- vii. [Of course, society at large may regard this particular dilemma as optimal insofar as two dangerous criminals are punished for real crimes.]

D. The prisioner's dilemma game can be used to model a wide range of social dilemmas.

- i. Competition between Bertrand (price setting) duopolists.
- ii. Decisions to engage in externality generating activities. (Pollution)
- iii. Competition over grades vs leisure in graduate school
- iv. Contract Breach/Fraud
- v. The free rider problem of revolution and other more mundane free rider problems.

E. The PD game's principal limitations as a model of social dilemmas are its assumptions about the number of players (2), the number of strategies (2), the the period of play (1 round). However, these assumptions can be dropped without changing the basic thrust of the analysis because essentially the same conclusions can be reached for N-person games where the players have an infinite number of strategies (along a continuum) and play any *finite* number of rounds.

F. Note that the mathematical requirements for completely specifying a game are met in the Prisoner's Dilemma game.

i. The possible strategies are completely enumerated
ii. The payoffs for each player are completely described for all possible combinations of strategies.
iii. The information set is (implicitly) characterized. (A player is said to have perfect information if he knows all details of the game. A perfectly informed player knows the payoffs for each party, the range of strategies possible, and whether the other players are fully informed or not.)

II. A Few Other "Named Games"

A. Other games can be represented using the 2x2 payoff matrix.

i. A *zero sum* game is a game where the sum of the payoffs in each cell is zero. In this game, every advantage realized by a player is at the expense of other players in the game. (Individuals with no training in economics seem to regard all economic activities as zero sum games. Of course, in most cases, exchange creates value for each player. Trade is a *positive sum* game.)
ii. *Coordination games* are games where the "diagonal" cells (top left or bottom right) have the essentially identical payoffs which are greater than those of the off diagonal payoffs. Here it is important that some norm be followed, and either "on diagonal corner" is an equilibrium. (All drive on the left side of the road or all on the right have higher payoffs than some drive on each side of the road.)
iii. In games of "chicken," the scores of the (Don't Chicken Out, Don't Chicken Out) cell are the worst of the game. ("Chicken Out,""Chicken Out,") yields a higher score although both players are embarrassed. The off diagonal scores (one player chickens out but not the other) yield the highest payoff for the "non-chicken" strategy and generally the lowest for the "chicken out" strategy. The scores for mutual non-chicken are so low that the off diagonal cells (bottom left or top right) are potential equilibria.

B. As in two person models of exchange, 2 person - 2 strategy games are very useful models of interaction and interdependency between individuals because the capture essential features of the choice settings of interest.

III. PD-like Games with Continuous Strategy Options

A. There are many settings in which players strategies lie along a continuum of some sort. Players on a team may work more or less. More or less of a public good may be provided. Such two person game can be represented mathematically by specifying a payoff (or utility) function that characterizes each player's payoffs as a function of the strategy choices of the players in the game of interest.

B. Example: consider a *symmetric game where each player has the same strategy set and the same payoff function.* Suppose there are just two players in the game, Al and Bob.

i. Let the payoff of player A be G1 = g(X1, X2) and that of player B be G2 = g(X2, X1) where X1 is the strategy to be chosen by player 1 and X2 is the strategy chosen by player 2.
ii. *Each player in a Nash game attempts to maximize his payoff, given the strategy chosen by the other.* To find payoff maximizing strategy for player A, differentiate his payoff function with respect to X1 and set the result equal to zero. The implicit function theorem implies that his or her best strategy X1* is a function of the strategies of the other player X2, that is that X1* = x1(X2).
iii. A similar reaction (or best reply) function can be found for the other player.
iv. At the Nash equilibrium, both reaction curves intersect, so that
X1** = x1(X2**) and X2** = x2(X1**)

IV. Review Problems

A. Let R be the "reward from mutual cooperation," T be the "temptation of defecting from mutual cooperation," S be the "suckers payoff" if a cooperator is exploited by a defector, and P be the "Punishment from mutual defection." Show that in a two person game, relative payoffs of the ordinal ranking T > R > P > S are sufficient to generate a prisoner's dilemma with mutual defection as the Nash equilibrium.

B. Suppose that the inverse demand curve for a good is P = 100 - Q and that there are two producers. Acme has a total cost curve equal to C = 5Q and Apex has a total cost curve of C =10 Q. Each firm controls its own output. Prices are determined by their combined production. Characterize the Cournot-Nash equilibrium to this game.

C. Suppose that there are two neighbors each of whom enjoy playing their own music loudly enough to annoy the other. Each maximizes a utility function defined over other consumption, C, the volume of their own noise, and that of their neighbor's (a bad), $U_1 = u(C_1, N_1, N_2)$. Each has a budget constraint of the form , Yi = Ci + Ni.

i. Characterize each neighbor's reaction function, and determine its slope.
ii. What happens to neighbor 1's reaction function if his income rises?
iii. Show the effect that a simultaneous increase in each neighbor's income has on the Nash equilibrium of this game.