Suggestions for your review. We have covered a lot of ground in the first part of the course. The most important material is the "substitution" method of optimization and the implicit function differentiation rule. Focus most of your attention on those two tools (60%). Focus half of the remainder on the envelop theorem, present value and expected value lectures (20%), and most of the rest on the various definitions and concepts (20%).

1. Identify and/or Define the following:

a. compact set	j. Kuhn-Tucker condition
b. convex set	k. Arrow-Enthoven theorem
c. strictly concave function	l. implicit function theorem
d. continuous function	m. envelop theorem
e. homothetic function	n. integral
f. partial derivative	o. expected value
p. Lagrangian	p. risk aversion
h. objective function	q. present value
i. second order conditions	r. quadratic formula

2. Find the "extrema" of the following functions.
   a. $Y = 10 + 27X - X^3$
   b. $C = Q^3 - 6Q + 1.5Q^2 + 100$
   c. $G = (1 - X)^3$
   d. $N = 15Q - 0.75Q^{.5}$
   e. $U = (10-PQ)^{.7}Q^{.3}$
   f. $\Pi = R(Q) - C(Q)$

3. Find the second order conditions for problem 2. Are your previously characterized stationary points, maxima or minima? Are they global or local extremal?

4. Suppose that *ACME* is a monopoly firm that sells its output at a single price. ACME faces the following demand curve for its product. **Qd = 10000 - 5.0P**. Find ACME's profit maximizing output if its total cost function is **C = (0.5Q + 120)w** where **w** is the wage rate. Show how would an increase in **w** affects *ACME*'s output and profit level.

5. Construct an abstract version of 4 making the usual economic assumptions of a downward sloping demand curve and diminishing marginal returns in production (in which case total cost increases at an increasing rate as output and also as the prices of inputs increase.)
   a. What partial derivatives are necessary to represent the assumptions of your model mathematically?
   b. Characterize the profit maximizing output of this firm.
   c. What additional assumptions do you need to assume (if any) to demonstrate that the first order condition characterizes a profit maximum?
   d. Use the implicit function theorem to characterize the profit maximizing output as a function of input prices.
   e. Determine who an increase in the price of an input affects the firm's output.
   f. Use the envelop theorem to characterize the change in profits associated with an increase in an input's price.
   g. Characterize the change in profits associated with an *decrease* in wage rates.
   h. Intuitively, we expect an increase in input prices to cause an increase in the monopolist's price. Can this properties be demonstrated from your initial assumptions?
      i. Are any other assumptions necessary to demonstrate this point?
      ii. If so what are their economic meaning?

6. Use the Lagrangian technique to characterize the utility maximizing combination (bundle) of goods X and Z for a consumer with the utility function, U = u(X) + v(Z), and budget constraint, W = X + PZ.

7. Repeat 6 using the substitution method instead. Your answer should not change.

8. How can it be determined whether the utility function in 6 is strictly concave?

9. Use an abstract, **nonseperable** and strictly concave utility function (defined over at least two goods) and an abstract linear budget constraint to derive the demand function for good **z**.

10. Discuss how you can prove that the first order conditions of 7 and 9 characterize utility *maximizing* combination(s) of X and Z.

11. Use the implicit function theorem to determine (i) whether the demand curve found in problem 9 is downward sloping, and (ii) whether the implied Engel curve is upward sloping. (An Engel curve characters demand as a function of wealth for given prices.)

12. Suppose that Mr. Workaholic's utility function is $\mathbf{U = Y^2}$ and that he faces the following constraints: **Y = (24-L)20, L ≥ 6** and **Y ≥ 10**. Find Mr. W's utility maximizing level of leisure, denoted L. Verify that your solution satisfies the Kuhn Tucker conditions.

13. Suppose that Mr. Moderation's utility function is **U = u(Y, L)** and that Y = w(16-L) where 16 is the number of hours that Mr. Moderation is awake during at typical day.
    a. Characterize Mr. M's utility maximizing level of leisure?
    b. What happens Mr. M's leisure if wage rate w increases?
    c. What happens to Mr. M's utility level if wage rate w increases?
    d. Would it ever make sense to compare Mr. W's utility with Mr. M's?
    e. What would be a plausible utility function for "Mr. Goofoff?" Maximize this utility function subject to the constraints in 13.

14. How does the Arrow-Enthoven theorem reduce the work required for problems 12, and 13?

15. Draw diagrams of the problems solved in problems 12, and 13 in the UxL plane.

16. Analyze a voter's preferred level of public services (G). Suppose that the voter has a monotone increasing, strictly concave utility function defined over private good X and

government service G, U = u(X,G). The private consumption good (X) costs P$/unit. The government service (G) is funded using tax receipts. Consequently, the voter's budget constraint is Y = PX + T, where Y is personal income and T is the tax paid to fund the public service. Assume that the voter's tax payment (T) increases with the level of public service provided and with personal income level, T = t(G,Y).

a. Use the Lagrangian method to find the first order conditions that characterize the individual's utility maximizing combination (bundle) of goods X and G.
b. Use the substitution method to find the first order conditions for the utility maximizing level of G.
c. Determine how the voter's ideal level of the publicly provided good changes if (i) the price of private consumption (P) increases, and (ii) if his income increases.

17. Suppose that a firm in a **perfectly competitive** market faces an exogenous price (P) for its output (Q) and exogenous prices for its inputs (labor and capital). Labor costs w $/unit and capital costs r $/unit. Assume that the firm uses a technologically efficient method of production which implies the following strictly convex long run total cost function:

$$C = c(Q, w, r)$$

a. Characterize the profit maximizing output for the firm.
b. Verify that the first order condition in part "a" characterizes the profit maximizing output. (Is the profit function strictly concave?)
c. Determine whether the firm's supply curve is upward or downward sloping.
d. Determine the effect that an increase in the *wage rate, w,* has on the firms profit-maximizing *output*.
e. Determine the effect that an increase in the *rental cost of capital, r,* has on the firm's *profit*.

18. Suppose that Margaret is thinking about purchasing a service contract for her new car. The service contract will *pay for all* repairs costs incurred during a five year period. The probability of a break down in each year is P(t) = .2 + .1t and the average cost of repair in case of a break down is C(t) = 100 + 50t. The interest rate is 5%/yr.

a. If Margeret is risk neutral, what is the highest price that she is willing to pay for the four year service contract? (An algebraic representation of this is sufficient.)
b. Suppose instead that M's utility function is $U = \Sigma(Y-R_t)^{.5}/(1+r)^t$ (for t = 1 to 4) where 4 is M's planning horizon, Y is her annual personal income, R_t is the repair expenses in year t, and r is her subjective rate of time discount. Suppose further that the distribution of actual repair costs are uniformly distributed over the range $75.00 on either side of the mean in each year.
   i. What is the highest price that M would be willing to pay for the four year service contract? (An algebraic representation of this is, again, sufficient.)
   ii. Is M risk averse? Explain.
   iii. Write down a mathematical expression for M's expected utility in the case where she *does* buy insurance.

19. Suppose that a lottery is to be played where winners collect $50,000 each year for 30 years. Tickets cost $1/each.

a. What is the expected net value of a ticket if the probability of winning is 1/50,000 and r=.10 ?
b. If the probability of winning is 1/100,000 and r = .05?

20. Explain the difference between total and partial differentiation. How is setting the first partial derivatives equal to zero different from setting the (first) total derivative equal to zero? Are these conditions ever the same?

21. Suppose that ACME is a paper mill that sells its output in a competitive market at price P. Suppose that it can choose more or less environmentally friendly methods of producing paper, but that more "friendly" methods are also more costly, C = c(Q, w, E) with $C_Q>0, C_w>0$ and $C_E>0$.

a. Characterize ACME's optimal output and environmental decision. How environmentally friendly will ACME's production method be?
b. Suppose that ACME faces a fine for pollution imposed (with certainty) with F = f(E). Characterize ACME's preferred production method for a given output Q^o. Demonstrate that ACME's preferred production method changes as the effluent fee/fine increases.
c. Suppose now that the fine is not always be imposed on ACME. Rather, the probability that the fine is imposed varies with the production method used and output level, L = l(E, Q). Characterize ACME's production and output decision.
d. Determine the effect of an increase in wage rates on ACME's profit levels.

22. Suppose that Pamela lives in Canton where there is substantial air pollution. Pam's utility function is an increasing function of private consumption and air quality, U = u(Z,A). Air quality in Canton varies with wind speed, which is a random variable, and the intensity of environmental regulation. Suppose that the wind speed is uniformly distributed between 0 mph and 20 mph, and that air quality, A, rises with wind speed, S, and with regulation, E. However, Pam's private consumption falls as environmental regulations become more rigorous, Z = Y - L(R), where Y is personal income.

a. Characterize Pam's ideal level of environmental regulation.
b. Characterize how her desired level of regulation changes as Y increases.
c. Is air quality an ordinary economic good?

23. Make up a (simple) economic model that analyzes a story from today's Washington Post or Wall Street Journal. (i) How often should a vote maximizing candidate lie if voters are fooled some, but not all of the time? (ii) What is the optimal intensity of anti-terrorist efforts if those efforts increase taxes and the costs of some private goods (such as air flight). (iii) How much internet stock should a risk averse person purchase? What does your model imply about the phenomena modelled which differs from a newspaper story?

24. A theorist (model builder?) always confronts the the problem of choosing features of the world to include in his model. Are there any reasonable rules of thumb to guide such choices? Discuss. (If your answer is no, be sure to explain why all conceivable rules fail in the social sciences but not in the physical sciences.)