

1. Identify and/or Define the following:
 

<ol style="list-style-type: none"> <li>a. compact set</li> <li>b. convex set</li> <li>c. unit square</li> <li>d. initial endowment</li> <li>e. excess demand function</li> <li>f. Walrasian equilibrium</li> <li>g. Brouwer's fixed point theorem</li> <li>h. first order condition</li> <li>i. second order condition</li> </ol>	<ol style="list-style-type: none"> <li>j. dominant strategy</li> <li>k. mixed strategy</li> <li>l. extended form</li> <li>m. symmetric game</li> <li>n. zero sum game</li> <li>o. coordination game</li> <li>p. Nash equilibrium</li> <li>q. sub-perfect equilibrium</li> <li>r. Stackelberg duopoly</li> </ol>
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2. Determine whether the following functions have "fixed" points within the unit square and their values if a fixed point exists.
 

<ol style="list-style-type: none"> <li>a. <math>Y = X^2</math></li> <li>b. <math>C = .6Q</math></li> <li>c. <math>G = (1 - X)^2</math></li> </ol>	<ol style="list-style-type: none"> <li>d. <math>N = 10Q - 0.75Q^{.5}</math></li> <li>e. <math>U = 100 - (Q-10)^2</math></li> <li>f. <math>Y = (.1X)^2 + X + .001</math></li> </ol>
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3. Is the utility function,  $U = X^a(1-X)^{(b-a)}$  with  $b > a > 0$  strictly concave? Demonstrate.
4. Use an abstract strictly concave utility function (defined over at least two goods) and an abstract linear budget constraint to derive the demand function for some good.
  - a. Is your demand curve downward sloping?
  - b. Is the good represented a normal good?
5. Suppose that a criminal allocates time between legal and illegal activities to maximize his expected income. Assume that "his" opportunity cost wage is  $W$  and that criminal income increases with time devoted to crime at a diminishing rate. Suppose that both the probability of conviction and the penalty received *increase* with the time devoted to criminal activity.
  - a. Characterize the optimal amount of time this person "invests" in the criminal activity.
  - b. Describe and discuss the conditions under which corner solutions are chosen.
  - c. Suppose that the probability that a criminal is caught rises with public expenditures on law enforcement and that damage rises with the level of criminal activity. What level of expenditure would minimize the expected cost of crime?
6. Suppose that *ACME* is a monopoly firm which sells its output at a single price. *ACME* faces the following inverse demand curve for its product.  $P = p(P, Y)$ .
  - a. Find *ACME*'s profit maximizing output if its total cost function is  $C = c(Q, w, r)$  where  $w$  is the wage rate and  $r$  is the rental cost of capital.
  - b. What partial derivatives are necessary to assure that your first order conditions characterize the profit maximizing output?
  - c. What happens to *Acme*'s output level if consumer income increases?
  - d. What happens to *Acme*'s output level if the cost of capital increases?
  - e. What happens to *Acme*'s profit if the cost of capital increases?
  - f. How can you modify this model to analyze the effect of an exogenous change in production technology on *Acme*'s output?
  - g. How would you alter your model to characterize the effect of entry by a new firm on *Acme*'s output level and profits?
7. Suppose that *Apex* enters *Acme*'s market. To simplify, assume that both *Acme* and *Apex* have cost function,  $C = Qw$ . and face inverse demand function  $P = Z - aQ + bY$ .
  - a. Find the Cournot-Nash equilibrium outputs and profits of the two firms.
  - b. Suppose that *Acme* anticipates *Apex*'s entry, and adjusts its output beforehand to account for *Apex*'s likely production. How much does each firm produce?
  - c. Compare the Stackelberg equilibrium with the original Nash equilibrium. Are firms better or worse off with the Stackelberg solution? Consumers?
8. Use a two person two strategy matrix of payoffs to characterize the following situations:
  - a. The Prisoner's Dilemma
  - b. The gains from a mutually beneficial exchange
  - c. The Pork Barrel Dilemma: where coalitions favoring two negative net benefit projects in Nash equilibrium manage to get both projects adopted, although each would have been better off if neither had been built.
  - d. The Tragedy of the Commons: where output from a common property resource falls once use exceeds  $M$ . Once  $M$  is reached, further use reduces total output.
  - e. Free Riding: where a pure public good is to be provided. Assume that the subjective opportunity cost of producing a single unit of the public good (worth \$5 to each party) is \$6.
  - f. The Samaritan's dilemma
9. Repeat number 8 but,
  - a. specify games with (concrete) continuous strategies and payoff functions.
  - b. abstract continuous strategies and payoffs.

10. Use backward induction to demonstrate that the "rational" method of playing a finite-repeated prisoners dilemma game ( say 2, 3, or T times) is defection. How would an uncertain ending point change your answer?
11. Use a two by two game matrix to illustrate the logic of the free rider problem. Suppose that the public good of interest provides benefits of \$5.00 to each "player" and costs a total of \$6.00 to produce. The cost is shared if both contribute, but must be paid entirely by one person if that person provides the public good while the other free rides.
- Label all payoffs, and explain the logic of the game.
  - Now suppose that provision of the good is subsidized. How much would the cost of the public good have to fall to eliminate the free rider problem?
  - How high would a "shirking tax" have to be to solve the problem?
12. Construct a general N-firm Nash-Cournot model of duopoly in a market where the demand curve is linear. Show that as the number, N, of firms increases, profits fall to zero, and price falls until it equals marginal cost.
13. Develop an extended form (two period) game in which a contract between two individuals is self-enforcing. Explain the key properties of your game that make your contract self enforcing.
14. Show that in a two strategy zero sum game, a mixed strategy is the only possible Nash equilibrium. Find a Nash equilibrium mixed strategy pair for your game. Demonstrate that it is an equilibrium.
15. Suppose that Al and Bob have Cobb-Douglas utility functions defined over personal income Y and leisure L. Assume that team production takes place via a Cobb-Douglas production function defined over Al and Bob's labor. Al and Bob are each be paid their full marginal product (which is partly determined by the other's effort) and can allocate 16 hours between work and leisure. (To simplify assume that all of the exponents take the value 0.5.)
- Find the Nash equilibrium of this team production game.
  - Compare this equilibrium with the Pareto optimal level of effort and production the maximizes the sum of their utilities.
  - Draw diagrams of the solutions in the  $L^A \times L^B$  plane.
  - Can these results be represented as a two by two game matrix?
  - What is the problem here?
  - What side payment from Al to Bob and/or from Bob to Al can solve the problem?
16. Suppose that Al and Bob are in the running for a new position at their firm. The position brings with it a change in salary in the amount of R dollars. The individuals can spend their time producing unobservable output, Q, for the firm, or in transmitting evidence of their productivity, E, to their boss. The latter does not, itself, increase the firm's output. Initially assume that the employees are indifferent between "signalling" and "working." Consider two cases.
- In case one, the signals,  $E^A$  and  $E^B$ , are all that the firm's owner bases his decision on with the probability that Al gets the new salary being  $P^A = p(E^A, E^B)$  and the probability that Bob gets the new salary being  $P^B = 1 - P^A$ . Characterize the work/signaling allocation decision of each employee on a typical work day, and the Nash equilibrium to the game.
  - In case two, firm owners can independently estimate actual output, but can not do so precisely. Overall, the evidence received by the boss now increases with both personal output and signaling in a multiplicative fashion, that is  $P^A = p(Q^A E^A, Q^B E^B)$ . Again characterize the individual employee time allocation decision and the Nash equilibrium of the game.
  - Now analyze how the firm owner can manipulate the size of R in these two cases to maximize the profits generated by these two employees in a setting where the firm's output can be sold at price M. (Assume that initially both employees are paid the same salary, Y.) Discuss your results.
17. Suppose that a dictator's expected return to holding power is  $R^e = P ( tY - G ) + (1-P) (L)$  where P is the probability of continuing in office and L is the income realized if overthrown. Suppose that national income Y increases with G but falls with t, and that P increases with national income.
- Characterize the dictator's optimal tax rate for a given level of government service G. Explain why this tax level does not maximize national income.
  - Characterize the dictator's optimal service level for a given tax rate,  $T = T^o$ . Determine whether this tax rate maximizes national income.
  - Does the dictator's ideal tax and service combination differ from the combination characterized when  $G^o$  is the solution to "b" and  $T^o$  is the solution to a?. Explain.
  - Now consider the case where national income, Y, is produced in two regions "a" and "b" so that  $Y = Y_a + Y_b$ . The dictator's probability of staying in office varies with regional incomes such that  $P = p(Y_a, Y_b)$  where  $P_{Y_a} > 0$  and  $P_{Y_b} < 0$ . Suppose that income in both regions increases with government services, G, but diminishes with the tax rate, t.
18. Discuss the generality of the sufficient conditions for the existence of a Walrasian equilibrium and a Nash equilibrium in a finite game. Are these conditions reasonable? Explain your reasoning.