EC630 **Mathematical Economics** Fall 2000
R. Congleton **Exam 1** GMU

1. (5 points each) Carefully, but briefly, define the following:
   a. convex set
   b. transitive relationship
   c. homogeneous function
   d. Arrow Enthoven sufficiency theorem
   e. expected value

2. (5 points each) Find the "stationary" points of the following functions and determine whether the stationary points found are maxima or minima.
   a. $U = 24 - 36X + X^3$
   b. $Y = w H (1-H)$ (with H the sole control variable)
   c. $R = p(Q)Q - Q^{1.5}$ (with Q the sole control variable)

3. (25 points) Suppose that a firm in the competitive market for widgets faces an exogenous price (P) for its output (Q) and exogenous prices for its inputs (labor and capital). Labor costs w $/unit and capital costs r $/unit. The widget producers all engages in technologically efficient production in regulatory environment R, and have the following long run total cost function, C = c(Q, w, r, R)
   a. Characterize the profit maximizing output for a typical firm in the widget market.
   b. Verify that the first order condition in part "a" characterizes the profit maximizing output.
   c. What qualitative effect would an increase in wage rates have on the firm's profit maximizing output? (Be sure to note the method used, and briefly discuss your answer.)
   d. Determine the effect that an increase in the rental cost of capital, r, has on the firm's profits. (Be sure to note the method used and briefly discuss your answer.)

4. (25 points) Suppose that Alice is attempting to determine her ideal level of environmental regulation (R) in order to make up her mind about who to vote for in the next election. Alice has a strictly concave utility function defined over personal consumption X and environmental quality E, U = u(X,E), where E is environmental quality. She has Y dollars to spend on personal consumption X. Environmental regulation increases environmental quality, E = e(R), but drives up the price of ordinary consumption good X. The price of the personal consumption good is P(R).
   a. Use the Lagrangian method to find the first order conditions that characterize Al's utility maximizing combination of consumption good X and environmental regulation R.
   b. Use the substitution method to characterize her utility maximizing combination of R and X.
   c. Determine whether the first order condition(s) developed in "b" characterizes a utility maximum. (What assumptions are necessary for your first order conditions to characterize a utility maxima?)
   d. What happens to Al's demand for environmental regulation if her income increases? (Demonstrate.)

5. (10 points) It is well known that increasing the width of a highway median strip systematically reduces the probability and severity of several kinds of automobile accidents. Show how the department of transportation can use cost-benefit analysis to determine the optimal width, W, of the median strip. Assume (i) that suitable land for the median strip can be purchased at C$/foot for the highway system of interest, (ii) that the probability of accidents in a given year is p(W), (iii) that the expected damage in the event of an accident is d(W). Assume further that the department of transportation has a 20 year planning horizon and that the interest rate on government securities is 5%.
   a. Characterize how wide the highway median strip should be to maximize *the present value of expected net benefits,* and briefly discuss your results.