## EC 812 -- L2: Introduction: Production Alone and for Markets

## VIII. Review

Last week, we examined a variety of models of human decision making, and applied them to some fairly straightforward settings (allocating one's spending money between two goods) and to more complex ones (shopping for characteristics, household production, altruism). All the models exhibited two characteristics of rational choice: coherent (consistant/ transitive) objectives and a set of feasible actions that can potentially further the individual's objectives.

In order to model, decision making we generally need both an objective function and an opportunity set. In the usual economic model where non-satiety is assumed, decisions can not be made without constraints. That is to say if utility is monotone increasing in all goods, more is always better, so individuals would "take it all." With an unbounded opportunity set, such individuals do not really have to choose.

If objectives are limited, as in the case where tastes exhibit satiety (e. g. have a "bliss point") an individual may be said to choose even in a setting without constraints in the sense that he/ she attempts to find the unconstrained utility maximizing combination of goods.

Today we will take a look at decisions to produce goods and services. We will show that it is possible to ground the theory of production and the firm on the same sort of optimizing model choice that we use for consumer theory. (This is not so often done in text books, but, as will be shown below, the firm need not be a black box.)

## IX. Some Fundamentals of Production

A. Production takes place when a person or group combines resources in a manner which yields something of greater value than they began with.
B. Even an individual alone on an island, as in the famous story of Robinson Crusoe, has to make a variety of economic decisions. These are not economic decisions in the sense that they involve purchases or other market transactions, but they are economic decisions insofar as individuals allocate a scarce resource (time) among alternative activities.
C. In fact, decisions regarding production may be said to be considerably older than what we think of as markets insofar as production is much older than money.
i. Many of the principle uses of human time involve production: hunting and gathering, planting and harvesting. Indeed, production with tools if often said to be one of the distinguishing features of our species.
ii. Primitive stone tools became common about 2.5 million years ago. 50,000 years ago sophisticated stone tools, weapons, and art emerged. The earliest form of money (cattle?) dates just a few thousand years, the earliest surviving coins to about 700bc (in Lydia).
iii. Thus it is clear that desisins abat wheretowark (hunt) andhownumtimeto wark predatemongbosed markes by thosands of years
D. Production is a valueincesingtransfamation of existingresarces(e. g. building a house, planting and harvesting crops, making a peanut butter and jelly sandwich, etc.)
i. The classic model of production is often told in terms of the famous novel by D aniel D efoe, RdinsonCrusee Crusoe on the island spends a good deal of time building a house, fishing, and clearing fields.
ii. Because various forms of leisure were also of value to this hard working pioneer, he did not spend his entire time in productive activities.
iii. Productive activities had an opportunity cost, namely activities that were valued for their own sake rather than for what they manufactured.
iv. (Such directly satisfying activities are generally called leisure. For example, playing golf may be enjoyed in and of itself, but putting a small white ball into 18 holes (the "output" of a round of golf) is not.)
E. Of course, some productive activities may be valued for their own sake as well as for their tangible outputs, as seeing a task through to a successful conclusion may be enjoyed for a number of reason other than its tangible output.
i. For example, productive activities may generate self esteem or physical fitness.
ii. Generally in economics, productive uses of time are modeled as being of only indirect, instrumental, value to the person engaging in them.
a. The fish caught are valued by the individual is valued, not the process of fishing itself.
b. Obviously this assumption is not true for all people in all jobs, but in most cases, it is the output of a productive activity that explains why it is undertaken.
F. Engaging in prourdion is genally represetedas a menns rather than anend

## X. Producing for Oneself: H ousehold Production

A. Many essential features of production can be analyzed by modeling Crusoe's time allocation problem between fishing and leisure. Suppose that Crusoe enjoys both fishing and leisure, $\mathrm{U}=\mathrm{u}(\mathrm{F}, \mathrm{L})$, and has 16 hours to allocate between them. The number of fish caught tends to increase as more time is spent fishing, T , so $\mathrm{F}=$ $f(T)$. [ $f(T)$ is Crusoe's production function for fish. A productionfundion daradeizes outputs as a fundion of inputs]
i. Note that one can use the time constraint and the production function to characterize Crusoe's choice in terms of either time spent fishing or time used for leisure. For example $\mathrm{U}=\mathrm{u}(\mathrm{f}(\mathrm{T}), 16-\mathrm{T})$
ii. Differentiating with respect to T , and setting the result equal to zero characterizes Crusoe's utility maximizing level of time spent producing fish, $\mathrm{U}_{\mathrm{F}} \mathrm{F}_{\mathrm{T}}-\mathrm{U}_{\mathrm{L}}=0$
iii. Essentially the first order condition says that Crusoe will fish up to the point where the marginal utility produced by fishing, $\mathrm{U}_{\mathrm{F}} \mathrm{F}_{\mathrm{T}}$, equals the marginal value of leisure sacrificed to fish, $\mathrm{U}_{\mathrm{L}}$.
iv. [One can diagram this equilibrium using a production possibility frontier and indifference curves, or using marginal subjective cost and marginal benefit curves.]
v. The first order condition developed in "b" is sufficient to characterize Crusoe's optimal level of fishing if the second order condition is negative at $\mathrm{T}^{*}$, where

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T* is a time spent fishing that satisfies the first order condition. The second order condition is: $\mathrm{U}_{\mathrm{FF}}\left(\mathrm{F}_{\mathrm{T}}\right)^{2}-2 \mathrm{U}_{\mathrm{FL}} \mathrm{F}_{\mathrm{T}}+\mathrm{U}_{\mathrm{LL}}+\mathrm{U}_{\mathrm{F}} \mathrm{TT}>0$.
vi. Notice that if the utility and production functions are both subject to diminishing marginal returns, and the cross partial derivatives are positive, that all the terms will be negative. This implies that the Utility function is strictly concave in the region of interest. [Are these mathematical assumptions reasonable?]
vii. $T^{*}$ together with the time constraints and production function can be used to characterize Crusoe's output of fish and time spent in leisure. $\quad \mathrm{F}^{*}=\mathrm{f}\left(\mathrm{T}^{*}\right)$ and $\mathrm{L}^{*}=16-\mathrm{T}^{*}$.
B. Fishing and many other production activities are actually not as cut and dry as this representation suggests. The model could be made a bit more realistic by making production of fish a probability function conditioned on time and having Crusoe maximize his expected utility. $U^{e}=\int u(F, L-T) f(F \mid T) d F$ where $f(F \mid T)$ is the probability of catching $F$ fish if one fishes for $T$ hours.
i. Naturally the average catch rises with time spent fishing, but Crusoe might not catch anything even if he spent all day fishing.
ii. Other more round about methods of fishing might also be used, Crusoe might first build a boat or collection of fishing gear, etc.

## XI. Production for Market

A. As markets develop, one can imagine changes in the production choice. Now instead of producing for one's own consumption, the individual may produce goods to be sold in a competitive market and use the proceeds to purchase consumption goods for his/ her own use.
B. For example, $U=u(C, L)$ where $P c C=f(T) P f$ (Pc is the price of the final production good and Pf is the price of the good sold in the market.)
C. Here, substitution implies that $\mathrm{U}=\mathrm{u}(\mathrm{f}(\mathrm{T})(\mathrm{Pf} / \mathrm{Pc}), \mathrm{L}-\mathrm{T})$ can be used to characterize the individuals decision to engage in the productive activity.
D. The first order condition is $\mathrm{U}_{\mathrm{C}} \mathrm{f}_{\mathrm{T}}(\mathrm{Pf} / \mathrm{Pc})-\mathrm{U}_{\mathrm{L}}=0$
E. The implicit function theorem implies that $T^{*}=t(\mathrm{Pf}, \mathrm{Pc})-$ - the amount worked varies with the prices of the goods sold and purchased.

## XII. Inputs for Hire

A. As markets develop still further, rather than engage in home production, or produce for sale in markets, some individuals will find it to their advantage to trade their labor time for money and use the proceeds to purchase final goods (consumer goods) that are of direct value.
i. Again the principle change in the optimization problem is in the constraint faced.
ii. Now Pc C $=w T$ where $w$ is the wage rate.
iii. And the first order condition is $\mathrm{U}_{\mathrm{C}}(\mathrm{w} / \mathrm{Pc})-\mathrm{U}_{\mathrm{L}}=0 . \quad(\mathrm{w} / \mathrm{Pc}$ is sometimes called an individual's real wage.)
B. Here: $T^{*}=t(w, P c)$ is the individuals labor supply curve. [D erive the comparative statics.]
C. Note that persons who can engage in production or hire themselves out for labor, tend to speeializein one or the other as a method of maximizing their income and thereby utility.

## XIII. Specialization and the Firm

A. In cases where more than one good can be sold at market, or one can work in similar jobs at various wages, the individual will choose the "career" that generates the greatest income.
B. In effect, income maximization (and comparative advantage) leads producers to specialize in specific forms of production, e. g. to comer solutions in production
C. The same economic incentive to maximize the return from selling one's labor also leads to specialization insofar as talent and human capital makes people more productive at some jobs than others. Bothlabores and produres tend to speialize
D. (This incentive for specialization may be incomplete in cases where the income generated is risky -- budding actors and singers are often told not to quit their "day" job. In such cases, , some diversification may pay.)
E. This income maximizing tendency of those who sell services or produce for sale in markets is the rational choice foundation of the theory of the firm. Producers have clear incentives to maximize the profit realized from economic activity.

## XIV. Production functions and the Cost of Production

A. Production by entrepreneurs generally involves combining a variety of inputs in order to produce desired outputs.
B. Tehndogicallyeffident production implies that outputs are achieved at minimal resource utilization.
C. [Illustration/ math]
D. Economicallyeffient production occurs when every output is produced with the combination of inputs that minimizes its cost.
E. [Illustration/ math]
F. Note that a "firm" or "entrepreneur" can be completely characterized by his production function because they all have the same objective function, e. g. maximize profits (net income and utility).

## XV. Economies and Diseconomies of Scale

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A. Neoclassical firms are often assumed to produce goods via production methods that exhibit constant reumstoscale That is to say via production methods where doubling all inputs exactly doubles outputs. (E. G. the production function is assumed to be homogeneous of degree 1.)
B. However, such functions do not provide a convincing explanation for the existence of firms (especially teams of unspecialized labor) insofar as there is no advantage associated with scale of operation. Very small firms (one person) would be just as efficient as larger firms.
C. An output range where economies of scale exist provides one very natural way of rationalizing the existence of multiple person firms. Over some range "barn raising" costs less as the number of laborers employed increases. There will be mutual gains to team production in such cases.
D. Moreover there will tend to be a minimum efficient scale of operation.

## XVI. Economic Development: Knowledge, Capital and Specialization

A. Economic progress, in the Paretian sense, occurs when people uniformly prefer a new setting to the previous one. The theory of rational decision making explored in the first lecture makes it clear that this tends to happen when opportunity sets expand.
B. What is relevant for the present lecture is that this may occur when individuals can produce more from the same natural resources than they could have initially. (It may also occur as new methods of consumption are invented or learned about.)
C. One source of economic progress via improved production arises through the adoption of more and more effective production methods. That is to say, as technological improvements in production take place, one is able to obtain more outputs from fewer inputs.
i. Progress, in this sense, reduces the resource cost of outputs.
ii. The evolution of production methods and tools used in production all arise because individuals gradually learn about the productive possibilities that nature provides them.
iii. Steel replaces Iron which replace Bronze which replace stone tools because they are discovered to be better at generating outputs than previous tools.
D. A good deal of technological growth involves the adoption of more complex and roundabout methods of production.
i. Much of the remaining coordination is done via managed teams or "firms" organized to produce a few particular products.
ii. Ronald Coase argues that transactions costs determine which activities are coordinated (managed) within the firm, and which ones are left to the decentralized coordination of markets.
E. A second source of economic progress much emphasized in theoretical work arises through the accumulation of capital goods. That is to say more tools, as well as more sophisticated tools, can make more ordinary persons more productive.
F. A third source of economic progress, emphasized by Adam Smith, and recently by James Buchanan is specialization. As markets expand, opportunities for additional specialization arise. The possibility of exchange, as we will see below, provides
and incentives for individuals and firms to specialize in their productive activities.
G. Legal and political institutions affect all three of these methods (capital accumulation, technological advance, and specialization) insofar as they facilitate or hinder trade, production and innovation.
i. Advanced production methods may require coordinating thousands of individuals through many years of effort transforming resources from the far ends of the earth in order to make both mundane and sophisticated products.
ii. Obviously, using markets to coordinate activities is "less expensive" when property rights are clearly defined, contracts are enforced and transgressions are punished.
iii. In areas where the government fails to deliver such services, firms themselves will have to perform them internally to the extent that they can.
H. A good deal of this coordination is done by market prices. "Price signals" direct resources out of relatively unproductive (low value added) to more productive activities (more value added).

## XVII. Problems

A. Consider an individual's labor supply decision in a model like the developed in lecture.
i. Derive an individual's labor supply function.
ii. Under what circumstances might an individual's labor supply curves be back ward bending.
iii. Show the effect of an proportional income tax on the supply of labor, when the individual's utility function is separable over leisure and purchased consumption goods.
B. D etermine:
i. the demand for Labor for an entrepreneur who uses Cobb-D ouglas Production function, $\mathrm{Q}=\mathrm{L}^{\mathrm{a}} \mathrm{K}^{1 . \mathrm{a}}$ with $0<\mathrm{a}<1$,
ii. and also whether or not a Cobb-D ouglas production function, exhibits constant returns to scale.

