

**XVI. Income and Substitution Effects: the Slutsky Equation**

- A.** As most of you remember from your undergraduate courses in economics, the effects of any price change can be divided into income and substitution effects. A price reduction for good X reduces the marginal cost of good X in terms of other goods which implies that consumers will tend to substitute good X for all others.
- A price reduction also expands the size of the opportunity set, making every net buyer of X wealthier. This increase in wealth also tends to affect behavior, although it does not always imply that individuals will purchase more of the good in question.
  - (Substitution effects are always positive, if indifference curves are downward sloping, tending to increase the amount purchased of the good whose price has fallen. Income effects may have positive or negative effects on the amounts that consumers will purchase according to whether the goods are "normal" or "inferior" goods.)
- B.** Two versions of the geometry of these two effects has been developed and are familiar for the two good case.
- First, assume an initial budget constraint, and find the initial utility maximizing bundle of goods. [Call this bundle point 0.]
  - Second, find the new budget constraint at the new ( usually lower) price for one of the goods, and find the new utility maximizing bundle of goods for your assumed (typical) individual. [Call this bundle point 2.]
  - Third, to decompose the overall change from "1" to "2" into relative price and income effects, construct a new budget line by drawing a line parallel to the new budget line through point 0. Find the combination of goods that would maximize utility along this "new" constructed budget line. [Call this bundle, point 1.]
  - The move from 0 to 1 is the "substitution effect;" it is caused by the change in relative prices alone. (Well almost alone, note the higher indifference curve that one winds up on with the "Friedman" derivation.) The change from 1 to 2 is the "income effect."
  - [The alternative Hicksian method of finding the substitution effect is to construct a budget line parallel to the new budget line that is *tangent* to the indifference curve passing through bundle 0. Continue as before with iii and iv. *This is the geometry implicit in the Slutsky equation.*]
  - [Note that the Friedman method could be calculated experimentally entirely by manipulating objective variables: wealth and observed choices, whereas the Hicksian method requires direct knowledge (or good estimates) of utility functions.]
- C.** The Slutsky equation is the mathematical counter part to this geometry. There are a number of alternative derivations, but to a large extent they may be thought of as methods for working out mathematical notation for the geometry discussed above.
- Nicholson derives the Slutsky relationship using a "duality trick."
  - Let  $h_x$  be the compensated (Hicksian) demand function for x  $H_{x^*} = h_x(P_x, P_y, U)$  where U is the initial utility level.
  - Let E be the minimum expenditure that allows one to achieve utility level U at price  $P_x, P_y$ .  $E = e(P_x, P_y, U)$

- If the consumers income were adjusted to hold income at this level as the price of X changed, one could use the ordinary demand function,  $D_x = dx(P_x, P_y, Y)$ , to characterize the compensated demand curve:  $h_x(P_x, P_y, U) = dx(P_x, P_y, e(P_x, P_y, U))$
- Differentiating with respect to  $P_x$  we get:  $H_{x_{P_x}} = D_{x_{P_x}} + D_{x_E} E_{P_x}$
- Rearranging we have:  $D_{x_{P_x}} = H_{x_{P_x}} - D_{x_E} E_{P_x}$
- The change in ordinary demand is composed of two effects: the change in compensated demand for a change in price and the change in purchases induced by a change in expenditures.
- By definition we can write the first term as  $X_{P_x} U = \text{constant}$  since this is how we determine the Hicksian compensated change in X for a change in price.
- Since we have used expenditure E to represent income, the second term can be written ( in my notation) as  $X_Y Y_{P_x}$  (where X is again desired consumption of X)
- Recall that  $Y = P_x X^* + P_y Y^*$  so  $Y_{P_x}$  is just  $X^*$ , so  $X_Y Y_{P_x} = X_Y X$
- Thus:
 
$$D_{x_{P_x}} = X_{P_x} U = \text{constant} + X_Y X$$
- E. G. The change in demand induced by a change in price is the sum of its income and substitution effects. (Nicholson's equation 5.27)

**XVII. Factor Income and Profit Under Constant Returns: Euler's Theorem**

- A.** One problem confronting the theory of competitive markets is whether firms generate sufficient revenues to pay for their factors of production in equilibrium, and whether or not firms might earn an economic profit in equilibrium. Economists use one of the many Euler theorems to demonstrate that *in a constant returns industries, factor payments exactly exhaust firm revenues.*
- B.** Euler's theorem states that if  $f(X_1, X_2, \dots)$  is homogeneous of degree  $m$ , that is if  $f(kX_1, kX_2, \dots) = k^m f(X_1, X_2, \dots)$ , then
- $$df/dX_1 (X_1) + df/dX_2 (X_2) + \dots = m f(X_1, X_2, \dots). \quad (\text{N: pg 208})$$
- C.** In the case where the function of interest is a production function that is homogeneous of degree 1,
- i.  $q(kX_1, kX_2, \dots) = k q(X_1, X_2, \dots) = kQ$
  - ii. Differentiating with respect to  $k$  yields  $Q_{X_1} X_1 + Q_{X_2} X_2 + \dots = Q$  : the sum of the marginal products of each input times the amounts used equals total output.
  - iii. Multiplying by *output price*,  $P$ , implies that:  $PQ_{X_1} X_1 + PQ_{X_2} X_2 + \dots = PQ$
  - iv. That is to say, in the case where each input is paid its marginal revenue product ( $PQ_{X_i} X_i$ ) the sum of the factor payments exactly equals total revenue. There are no economic profits in this sort of firm in long run equilibrium.
  - v. (Firm owners only earn the marginal revenue product of factors that they themselves provide to the firm!)