

Selected Answers for Study Guide I

3. Suppose that Al has the utility function $U = Y^2 L$, defined over labor income Y , and leisure, L . Al can allocate 16 hours a day between paid labor and leisure.

A. Determine Al's supply of labor function if he earns w \$/hr while at work. (Hint: note that $Y = wW$ and $W+L = K$.)

i. Substituting yields:

ii. $U = (wW)^2 (K-W)$

iii. Differentiating with respect to W yields:

iv. $2w(wW)(K-W) + (wW)^2 (-1) = 0$

v. Simplifying yields: $2(K-W) = W$ or $2K = 3W$

vi. which implies that $W^* = 2K/3$

B. Is Al's supply of labor function monotone increasing in the wage rate?

i. In this case, Al's labor supply is surprisingly not affected by the wage rate at all!

ii. This is evidently a peculiar feature of Cobb-Douglas utility functions.

4. Suppose that automobile size (weight) and gasoline are inputs into the household production of transport services, $T = t(S, G)$, with T increasing as G increases but decreasing as S increases. Suppose also that travel is a bit risky, and that the probability of an accident increases with travel, $P = p(T)$, while the damage generated falls as automobile size increases, $H = H^0 - d(S)$, other things being equal. If no accidents occur, damages equal zero, e. g. $H = H^0$. Assume that individuals value only health, H , transport services, T , and other consumption, C .

A. Characterize a typical person's (Al's) expected utility maximizing automobile size. (Assume that Al has W dollars to allocate between C , S , and G which are purchased in competitive markets.) Explain the economics behind the mathematics that characterizes Al's optimum.

i. We are indirectly told that $U = u(C, T, H)$, and that $W = C + S + G$ (The later assumes that the units are such that the prices for each are \$1.00. The problem does not ask anything about prices so keep it simple.)

ii. so $U^e = p(T) U(W - S - G, t(S, G), H^0 - d(S))$

iii. $+ (1-p(T)) U(W - S - G, t(S,G), H^0)$

iv. There are two control variables left, G , and S .

v. Differentiating with respect to G and S yields:

vi. $P_T T_S U^a + P (U_C^a (-1) + U_T^a T_S + U_H^a H_S) - P_T T_S U^a + (1-P) (U_C^b (-1) + U_T^b T_S + U_H^b H_S) = 0$

vii. and $P_T T_G U^a + P (U_C^a (-1) + U_T^a T_G) - P_T T_G U^a + (1-P) (U_C^b (-1) + U_T^b T_G) = 0$

viii. where U^a is the utility realized in an accident and U^b is that realized if no accident occurs.

ix. These two equations must be satisfied simultaneously at G^* and S^*

B. Characterize Al's demand function for automobile size.

i. The first order conditions imply that $S^* = s(W, H^0)$ in this case

ii. (The prices of size, gas and consumption would also be variables in S^* had I not assumed all the prices to be constant and equal to 1)

C. Does Al's *short run* demand for gasoline necessarily slope downward when Al's utility function is separable and strictly concave?

i. In the short run capital (S) can not be varied, so only the second of the first order conditions is relevant (vii) and $G^* = g(W, H^0, S^0)$ where S^0 is the given size of the car. To derive the slope of the demand curve we will have to take account of the price of gasoline, P_g . We could do this by for example assuming that $W = C + P_g G + S^0$, doing the substitution and taking the derivative with respect to S .

ii. This would give us:

iii. $P_T T_G U^a + P (U_C^a (-P_g) + U_T^a T_G) - P_T T_G U^a + (1-P) (U_C^b (-1) + U_T^b T_G) = 0$

- iv. which yields $G^* = g(P_g, W, H^0, S^0)$
- v. Recall that the assumption that U is separable is equivalent to assuming that the cross partials of U are all zero. Unfortunately, the result is still a bit of a mess because of effects of G on the probability of an accident. Using the implicit function differentiation rule we would obtain:
- vi. $H_{Pg} / -H_G = \{ [P_{TT}(T_G)^2 U^a + P_T T_G U^a(-G) +$
- vii. $P (U^a_{CC} (PgC) - U^a_C + U^a_T T_{GG}) + P_T T_G (U^a_C (-Pg) + U^a_T T_G)]$
- viii. $+ [- P_{TT}(T_G)^2 U^a - P_T T_G U^b_C(-G) +$
- ix. $(1-P) (U^b_{CC} (PgC) - U^b_C + U^b_T T_{GG}) - P_T T_G (U^b_C (-Pg) + U^b_T T_G)] \} / -H_G$
- x. (Note that P_g only occurs in the "C" argument of U and the partial derivatives of U , but since cross partials are all zero we need only account for the effect of G on U_C , U_T , and T)
- xii. Assuming diminishing marginal utility and affects of T on P , all the terms in the first bracket term are negative and most of those in the second bracketed term are as well. The positive terms arise from all the $-P_T$ and P_{TT} terms. The denominator ($-H_G$) is the just the negative of the second order condition so that will be positive assuming the objective function is strictly concave.
- xiii. The problem here is how to handle the positive terms in the numerator.
- xiii. Note that *separability* implies that the marginal utilities and size of all the partial derivatives in both bracketed terms are the same except for the ones on health! So *the positive terms in the second brackets will simply cancel out against the negative ones in the first bracket*.
- xiv. Consequently we can conclude that the short run demand for gasoline is *downward sloping!*

5. Consider the plight of a husband. His utility function is $U(c, a)$, where c is his annual consumption and a is the amount of "female attention" he receives in a given year. Assume $U_c > 0$ and $U_a > 0$. Female attention, a , is bestowed by the husband's wife, with $a = f(g, t)$, where y is the annual amount of gifts (of consumption goods) the husband gives to his wife, and y is how long they have been married. The husband earns a fixed income W (of consumption goods) each year.
- A. Write out the optimization problem faced by the husband each year. Clearly label his choice variables, his objective, and any constraints he faces. Use calculus to solve for the husband's optimal choices, and verbally interpret any first-order conditions.
- $U = u(c, a)$ with $a = f(g, t)$ and $y = c + g$ implies that $U = u(y - g, f(g, t))$
 - $U_g = u_c(-1) + u_a f_g = 0$ at g^* and $c^* = y - g^*$
 - Gifts are given up to the point where the marginal utility of additional female attention generated equals its marginal cost in terms of reduced consumption.
 - The gift giving function can be written as $g^* = g(y, t)$
- B. Suppose $f_g > 0$ and $f_y < 0$. Interpret these conditions verbally. How would the evolution of the marriage depend on the sign and magnitude of f_{gy} ? Use the logic of income and substitution effects to explain what happens to the husband's gift giving as the marriage matures.
- The cross partial will determine how the value of marginal value of gifts is affected by time (at the margin).
 - The time effect may increase or decrease the marginal cost of obtaining female attention.
- C. Mathematically show how the duration of the marriage, y , affects gift giving by the husband. Explain your results.
- Using the implicit function differentiation rule we obtain:
 - $g^*_t = H_t / -H_g = [u_{ca}(-1) f_t + u_{aa} f_t f_g + u_a f_{gt}] / (-H_g)$
 - The denominator is just the negative of the second order condition(--), so the qualitative effect of t on gift giving is determined by the numerator.
 - The first two terms are clearly positive, but the last may be positive or negative according to the sign of f_{gt} , if positive, gift giving increases with time!