

I. The last lecture noted a variety of implications of majority rule voting. In particular, we developed the simple median voter model of government policy formation (some of which we will explore in more detail this week).

A. For example,

- i. Policies will tend to be moderate, e. g. drawn from the middle part of the political spectrum. (The middle can be regarded as "moderate" essentially by definition.)
- ii. Most people will be at least partially displeased with the policies chosen insofar as they have different ideal point, even in a perfectly functioning democracy, as long as peoples tastes, circumstances, or expectations differ.
 - (However, it is still possible that most people are dissatisfied with government policy yet still prefer the use of majoritarian decision rules to any other they are aware of.)
- iii. At the Nash equilibrium in many two party or two coalitions election contests, government policies will maximize the median voter's expected utility, given her constraints, expectations, and goals.
 - An implication of this is that **any change in circumstance that changes the constraints of the median voter, or the identity of the median voter, will have systematic effects on the size and composition of government programs.**
 - Another implication is that increases dispersion of the distribution of voter preferences (increased radicalism) tends to have little, if any, effect on public policies unless it also affects the median of the distribution of voter ideal points.
 - *This implies that median voter policies will be more stable than average voter policies.*

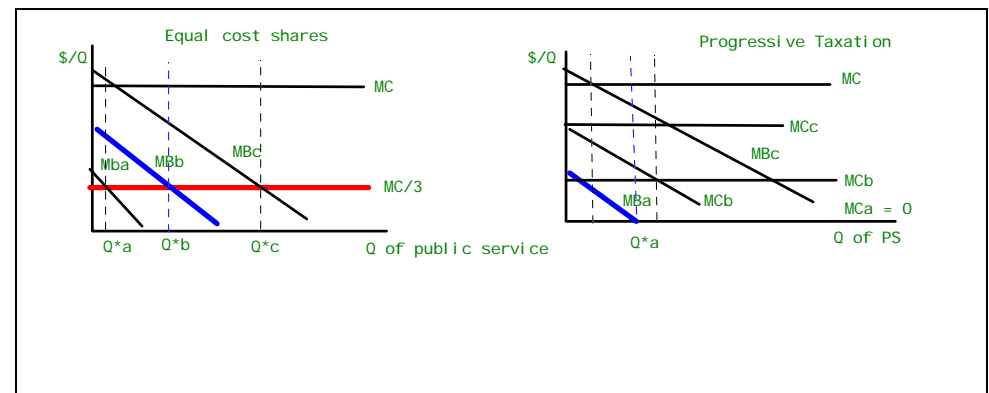
B. Today we will use the strong form of the median voter model to analyze some public policy issues and begin exploring weaknesses of the model.

- i. The median voter model is widely used to analyze the level and growth of government service levels.
- ii. It also plays a significant role in both the theoretical and empirical public finance literature dealing with taxes and expenditure levels.
- iii. And, it can be used as test of the "competitiveness" of existing democracies.

- iv. However, from a theoretical perspective, it has a number of serious weaknesses.

II. The strong form of the median voter theorem, implies that particular policies can be modeled as the solution to one person's political optimization problem.

- A. Such optimization problems are often very straightforward to characterize and perform comparative statics on.
- B. Median voter models of public policies can be developed both geometrically and mathematically.
 - i. Consider first a geometric representation of three voters (below), who are trying to decide the level of a public service, given particular tax institutions.
 - A pure public good or service is one that is equally available to all because of its technical "sharability" properties (Samuelson 1954).
 - A pseudo public good or service is one that equally available to all because of the manner in which it is provided (Buchanan and Congleton 1998).
 - ii. For the problem at hand, both the nature of the service and the tax system have significant effects on voter demands.
 - For example in the figures above, the tax code determines how the cost of public services will be divided among the voters, which affects the level of services demanded by each of the three voters modeled, Al, Bob, and Cathy.



- The "publicness" of the service implies that everyone will have the same amount of the service whether they have a high demand for it or not.
 - iii. The geometry of rational choice implies that the median voter's preferred service level depends upon his or her tax price.
 - A voter's demand for the service partly depends on the cost of the service, and partly on the way that cost is divided up among tax payers.
 - And, it partly depends on his or her own marginal benefits from the service.
 - iv. Note that insofar as demand (marginal benefits) for government services reflect income differences, and the service is a normal good, then it is even possible that a progressive taxation will cause high income "high demand" tax payers to vote in favor of smaller service levels than "low demand" voters, because of their relatively higher price for those services.
 - v. Note also that a suitably designed progressive tax system can reduce the dispersion among voter ideal service levels.
 - Indeed, a tax system can be designed, at least in principle, so that everyone prefers the same public service level (Lindahl 1919).
 - vi. The above model can be modified to represent voter preferences for regulations of various kinds by replacing a "tax price" with a "regulatory price."
 - A "regulatory price" is simply the extent to which prices rise (and real income falls) as a consequence of the regulation of interest.
 - In some cases, regulations may use taxes, as with the proposed carbon taxes (a Pigovian tax on CO2 emissions), which also implies higher prices for many products that voters purchase.
- C. Mathematical representations** of a median voter model generally focus on the choice made by a single "typical" voter, and then use parameters of the model to "identify" the particular voter that is the median.
- In general, the comparative statics of the median voter in such models is essentially the same as that of other voters, although her preferred service level differs from that of other voters because her tastes, income, age, etc. differs somewhat from that of other voters.
- D. First, consider the mathematics of net benefit maximizing choice.**
- i. Suppose that the net benefits for a typical voter "i" can be represented as:
 - a. $N_i(G) = B(G, Y_i, A_i) - C(G, Y_i)$

- b. where N is net benefits, B is total benefits, and C is total cost.
- c. and Y_i is voter i's income and A_i is voter i's age.
- d. Assume that every voter gets the same service level G
- ii. The G that maximizes N for voter "i" can be found by differentiating the N equation with respect to G
 - This yields
 - $dN/dG = dB/dG - dC/dG = 0$
 - or
 - $dB/dG = dC/dG$
 - Since dB/dG is marginal benefit and dC/dG is the marginal cost of the government service, this means that to find G^* , one should look for the G that sets marginal benefit equal to marginal cost
- iii. One can get specific algebraic or numerical answers if you assume specific functions for B and C.
 - a. For example let $N = bG^{1/2} Y_i A_i - CY_i$
 - b. Differentiating N with respect to G yields
 - $\frac{1}{2} b G^{-1/2} Y_i A_i - CY_i = 0$
 - rearranging yields
 - $G^{-1/2} = 2CY_i / bY_i A_i = 2C/bA_i$
 - inverting and squaring to find G^* yields
 - $G^* = [bA_i/2C]^2$
- iv. To use this mathematics to think about median voter choices, rather than general voter ideal points, one simply substitutes values of Y and A that characterize the median voter.
 - This implicitly assumes that voters are all pretty similar.
 - why?
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- E. A widely used alternative to the net benefit maximizing model of rational choice is the **utility maximizing model of rational choice**.
- i. Its main advantage is that it allows one to see how changes affect several areas of choice simultaneously.
 - ii. For example $U = u(Q_{\text{spargel}}, Q_{\text{brat}})$
 - iii. lets you analyze how combinations of spargel and bratworst are affected by changing prices.
 - iv. Normally, the utility maximizing framework is combined with the idea of an opportunity set to characterize a choice.
 - v. For example, a consumer may maximize utility subject to a budget constraint $W = P_S S + P_B B$ where P_S is the price of spargel, P_B is the price of bratworst, S is the quantity of Spargel purchased and B is the quantity of bratworst purchased, and W is the amount of money that the consumer can spend.
 - vi. See class notes for a quick review of the geometry of this sort of representation of rational choice.
- F. To use this framework to think about public policy choices, one focuses on the median voter and his or her tastes (utility function) and budget constraints (normally, a public and a private constraint).
- i. Consider electoral selection of a public services that is funded with a non-distorting "head tax."
 - ii. Each voter in his capacity as a policy "maker" looks very much like the standard consumer in a grocery store, except that in addition to private budget constraints, he has a "public" budget constraint to deal with.
 - iii. Suppose:
 - a. that voter's have the same utility function defined over private consumption (C) and some public service (G).
 - b. that each voter has a different amount of money, W_i , to allocate between private consumption, C , and public services, G , and that there are N tax payers in the polity of interest.
 - c. And, to simplify a bit, assume also that the government faces a balanced budget constraint, and that all expenditures are paid for with a head tax, T .
 - iv. This allows the typical voter's decision to be represented as:
 - a. maximize: $U = u(C, G)$

- b. subject to a private budget constraint: $W_i = C + T$
 - c. and a public budget constraint: $gG = NT$, where g is the marginal cost of government service G , N is the population size of tax payers and T their "head tax."
 - d. Note that a bit of algebra allows T to be written as $T = gG/N$
- G. There are two ways to proceed with the math. First, we can choose one of the conventional mathematical forms for U . Second, we can use more general methods to optimize with abstract utility functions like that above, after make some assumptions about the derivatives of U .
- i. The less general method would, for example, assume that $U = C^a G^b$.
 - ii. Substituted T into the private budget allows a single unified budget constraint to be derived:
 - $W_i = C + gG/N$
 - a. This in turn can be solved for C and substituted into the utility function:
 - Note that C can now be written as: $C = W_i - gG/N$
 - and U as: $U = (W_i - gG/N)^a (G)^b$
 - b. (Note that the substitutions imply that the voter has in fact only a single degree of freedom. Once G is chosen, tax rates and personal consumption levels are also determined.)
 - c. Differentiating with respect to G yields:
 - $a(-g/N)(W_i - gG/N)^{a-1} (G)^b + b(W_i - gG/N)^a (G)^{b-1} = 0$
 - dividing by $(W_i - gG/N)^{a-1} (G)^{b-1}$ yields
 - $a(-g/N) (G) + b(W_i - gG/N) = 0$
 - gathering all the G terms on the left yields:
 - $a(g/N) (G) + b(gG/N) = b(W_i)$
 - so $G^* = b(W_i) / [(a+b) g/N] = [b / (a+b)] W (N/g)$

III. On the Normative Properties of Median Voter Policies

A. Although the median voter model implies that the median voter gets what "he or she wants," it does not imply that public policies will be efficient in the usual Paretian sense.

i. This can be seen mathematically by comparing the service level in the above model with that which would be Pareto efficient in a society of three individuals with different tastes or wealth.

- [Recall that the Pareto Efficient level can be characterized with a social welfare function, or by maximizing one person's utility while holding the other's constant. See lecture notes.]

ii. This can also be seen by developing a graphical illustration that contrasts the median voter's preferred output of a public service or regulation with a Pareto efficient one.

- Generally, the median voter's preferred policy is Pareto inefficient whenever the median and "average" voter have different ideal points.
- (A **Pareto efficient policy** is one that cannot be changed to make at least one person better off without making at least one person worse off.)

B. **Rational Ignorance and Fiscal Illusion.** The median voter model developed to this point has ignored information problems.

- For example, information costs faced by voters will generally cause voters to be less than perfectly informed about their tax burdens or the benefits of public programs.
- That is to say they may "rationally" choose to remain ignorant of many policy details and also to economize on their "political research" by using small samples.

i. In cases in which the median voter's expectations are **unbiased** estimates of the consequences of public policies, he/she will still **on average** get what he/she wants.

ii. However, in cases in which rational ignorance implies **biased** expectations about the consequences of policies (as for example when one remains entirely ignorant of some policy detail or implication) then the median voter **may not get** what he/she *truly* wants.

C. Information problems open the door to interest groups and the bureaucracy who may manipulate voters by strategically subsidizing particular kinds of

information. It also allows malfeasance (agency costs, corruption) on the part of elected and unelected government officials.

i. Such problems would not exist if voters were completely informed about government policies and electoral competition was "perfect" in the sense that it lead to median voter policies.

ii. **[Student puzzle: explain why?]**

iii. Indeed, it can be argued that essentially the whole special interest group/rent-seeking literature is predicated on informational problems of these kinds in open democratic societies.

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D. **OPTIONAL APPENDIX:** The most general median voter models use abstract functions throughout their analysis. For example, they might assume that first derivatives of $U = u(C, G)$ are positive, second derivatives are negative and cross partials are positive.

i. Substituting for C in the utility function yields $U = u(W_i - g(G)/N, G)$

ii. Differentiating with respect to G yields a first order condition that characterizes the median voter's preferred government service level:

$$\bullet -U_C (gG/N) + U_G = 0 = H$$

$$\bullet \text{ or equivalently as } U_C (gG/N) = U_G$$

a. The right hand side represents the **subjective marginal benefit** (marginal utility) of the government service, the left-hand term represents the **subjective marginal opportunity cost** of government services in terms of lost private consumption.

b. Note that the subjective marginal cost of the service is determined by both preferences (marginal utility of the private good C) and objective production or financial considerations, cG/N .

- The latter can also be called the median voter's marginal cost share, or price for the government service.

iii. An implication of the first order condition together with the implicit function theorem is that each voter's demand for public services can be written as:

$$a. G_i^* = \gamma(W_i, N)$$

- That is to say, each voter's demand for the public service is a function of his own wealth (holding of the taxable base) and the population of tax payers in the polity of interest.

b. The implicit function differentiation rule allows one to characterize the comparative statics of a typical voter's demand for public services.

- How will changes in wealth, W_i , and number of tax payers, N , affect a voter's demand for government services?

c. Applying the implicit function differentiation rule implies that:

- $G^*W = H_W / -H_G$

- and $G^*N = H_N / -H_G$

- where H is the first order condition above.

a. Solving for these derivatives requires using the partial derivative version of the composite function rule and paying close attention to the location of all the variables in the various functions included in " H ," the first order condition.

- We find that::

$$G^*W = [-U_{CC}(gG/N) + U_{GW}] /$$

$$-[U_{CC}(gG/N)^2 - U_C(gGG/N) - 2U_{CW}(gG/N) + U_{GG}] > 0$$

and

$$G^*N = [-U_{CC}(gG/N)(g(G)/N^2) + U_C(gG/N^2) + U_{GW}(g(G)/N^2)] /$$

$$-[U_{CC}(gG/N)^2 - U_C(gGG/N) - 2U_{CW}(gG/N) + U_{GG}] > 0$$

a. That is to say, **with head tax finance**, each voter's demand for a pure public service rises with personal wealth and with population.

iv. Moreover, **since demand is strictly increasing in W , this implies that the median voter is the voter with median income** (in this particular model).

a. This voter's demand for public services will lie in the middle of the distribution.

- That is to say, the voter with median income has a preferred service level G^{**} such that the same number of voters prefer service levels greater than G^{**} as those who prefer service levels lower than G^{**} .

- b. The comparative statics of a voter with median income can, in this case, be used to characterize the course of government spending through time, as other variables change (here, exogenous shocks to W or N , changes in tastes, etc.).

E. Other, somewhat richer, mathematical models can be built to analyze such problems as:

- The effects of different tax instruments: proportional and progressive tax instruments
- The effects of varying degrees of publicness on demand for services: club goods
- Optimal redistribution motivated by narrow self interest and/or altruism.
 - For example, Meltzer and Richards (1981) provide a Spartan but sophisticated analysis of how the median voter model can be used to represent the equilibrium size of government in a pure transfer model of government policies.

F. It bears noting that **not every median voter model yields unambiguous predictions** about the effects of changes in the parameters of the median voter's choice problem on the median voter's demand for a given public policy.

- For example, when public services are financed with a progressive income tax, the tax price changes with income.
 - In this case, whether demand increases or decreases with income depends on whether a voter's marginal cost rises faster than willingness to pay as his or her income and/or wealth increases.
- However, useful insights may be obtained about the relationships between the median voter's own choice setting and the parameters of public policy formation are obtained even in those cases.
- In either case, the final test of the median voter model is empirical.
 - How well does the median voter representation of a policy formation explain real policies and real world data about such policies.
 - On this score, the median voter model does quite well.