I. Introduction to stochasatic voting models:

A. The only widely used alternative to the median voter model of electoral equilibrium is the stochastic voting model.

B. This model uses a stochastic (random) representation of the voter's behavior that is not entirely consistent with the usual economic representations of choice--or at least assumes that candidates for elective office believe that voters act in this somewhat non-rational manner.

C. The stochastic voting model is widely used in the rational politics literature in Political Science and is widely *referred to* by many pieces in public economics that want to model government as maximizing a social welfare function.

II. The Logic of Stochastic Voting

A. As a Consequence of the Voter's Trembling Hand

   i. The behavior of a stochastic voter is represented with a probability function. One common and simple way of representing the probability that a particular voter chooses candidate X over candidate Y assumes that this probability is simply the ration of the expected utility generated by candidate X's policies divided by the sum of the expected utilities of both (all) candidates. $P(X|Y) =$
   $U^e(X)/[U^e(X) + U^e(Y)]$

   ii. The most straightforward interpretation of voting in a stochastic voting model is that voters make mistakes. That is to say, voters do not always maximize utility, but rather *more often maximize utility than not.*

B. As a Consequence of Candidate Ignorance

   i. Another, and now more common interpretation of the model, is that candidates do no fully understand the preferences of voters and so try to imagine the consequences of their policies on the welfare of individual voters. These estimates are assumed to resemble the $P(X|Y) = f(U(X), U(Y))$

   ii. Note that the non-stochastic voter model developed previously is a special case of this more general representation of the voter's behavior. $P(X|Y) = 1$ if $u(X) > u(Y)$ and $P(X|Y) = 0$ if $U(X) < u(Y)$ and $P(X|Y) = 0.5$ if $U(X) = U(Y)$.

   iii. However, the stochastic voting literature generally assumes that function f is continuous and differentiable, which the former is not.

C. Candidates Choose Policies to Maximize Expected Votes

   i. As in the median voter model, candidates choose policies to maximize their prospects for election. Here they can either maximize expected voters--the usual assumption--or the probability of being elected (which is not always the same as the former, but more difficult to represent.)

III. The Positive Properties of Electoral Equilibrium in a Stochastic Voting Model

A. Given the position of the other candidate, and the probability functions for each voter, the expected voter for candidate X taking policy G^X is

$$V^e = \Sigma\ fi(U_i(G^X),\ U_i(G^Y)\ )$$

   i. Note that candidates take account of the welfare of EVERY voter, and that the extent to which a voter's interest is taken account of by is a combination of the marginal utility to "voter i" of the policy variable of interest and the extent to which "voter i" will be *more likely* to vote for X as his utility increases.

   ii. Another contrast with the median voter model, there is, in principle, a specific ideal policy position for each policy position a candidate's opponent(s) might take.

B. Note that in cases where the joint (across all voters) probability distribution is symmetric that the median voter outcome can be an electoral equilibrium in this model as well, but more generally the policy positions taken will reflect a "weighted average" of all voter interests.

   i. The comparative statics of electoral equilibrium reflects changes in the (unstated) constraints of voters and/or changes in the technology of delivering government services.

   ii. Give the assumptions made, an electoral equilibrium always exists. That is to say there are no electoral cycles in the classic stochastic voting model.

IV. An Introduction to Social Welfare Functions

A. Work that examines **normative** properties of electoral equilibrium relies either upon the Pareto Criteria or some social welfare function.

   i. **Pareto Optimal**/Efficient: A state of the world is Pareto Optimal if there is no change possible which would make at least one person better off and no one is worse off.

   ii. **Pareto Superior**. State A is Pareto Superior to state B, iff at least one person prefers A to B and no one prefers B to A.

   iii. (Note that a state C is Pareto Optimal is there are no feasible states which are Pareto superior to C.)

B. A general individualist social welfare function has the property that social welfare increases in any case where one person gains and no one loses from a change in resource allocation. That is to say, a social welfare function is monotone increasing in the utility level of every individual in the society of interest.

$$W = f(U_1, U_2, ... U_N)$$

C. Interesting special cases of social welfare function include the utilitarian (Benthamite) social welfare function and the Nash social welfare function.

　i.　Benthamite:　$W = \Sigma\, Ui$

　ii.　Nash:　　　　$W = \Pi\, Ui$

D. All individualistic social welfare functions have the property that constrained maximization yields a Pareto Optimal state. (Why?)

　i.　For example, if a fixed stock of a pure private good were to be allocated to maximize social welfare, one would maximize W subject to the resource constraint $X = \Sigma Xi$. The result would be a unique social welfare maximum.

　ii.　Note that many other states would also be Pareto optimal. In fact any distribution of private good X among non-altruists would be Pareto optimal including the one that maximized the social welfare function of interest.

　iii.　From this illustration, you should be able to see that the weights of the social welfare function (consider the weighted Benthamite social welfare function, $W = \Sigma\alpha iUi$ where $\alpha i > 0$) are very important determinants of the particular conclusions that emerge from maximizing social welfare.

　iv.　If all individuals had the same utility functions, and all the weights were the same, than this divide the pie problem would have recommended dividing the pie up equally. However other weights would yield different recommendations/"social welfare" maximizing policies.

E. Illustrations of Social Welfare Functions for two persons

　i.　in "utility space" (below)
　ii.　and in "goods space." (in class)
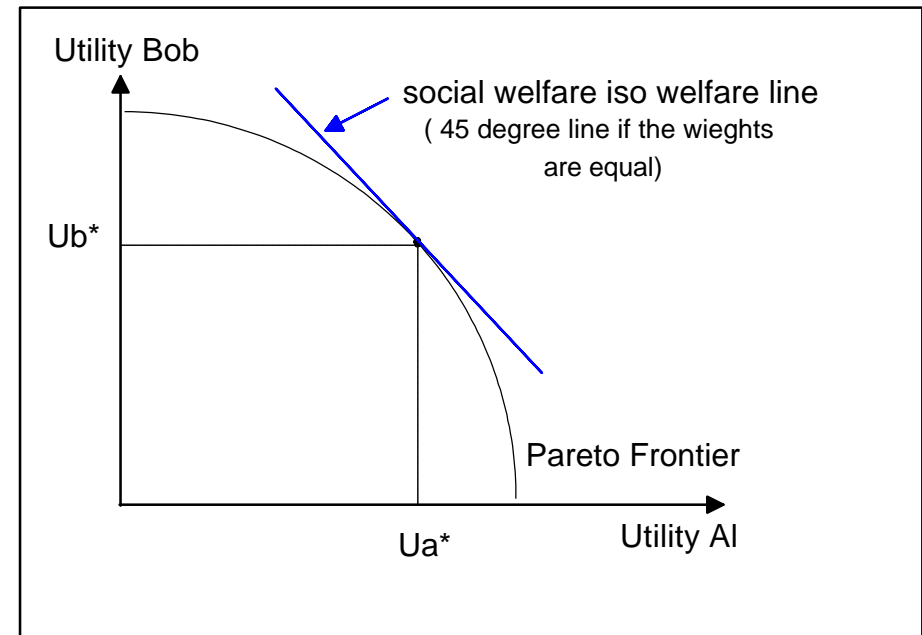
V. On the Normative Properties of Electoral Equilibrium

A. Note that the normative properties of the median voter model and the stochastic voter model differ.

　i.　The median voter model will not generally produce policies that are Pareto optimal.

　ii.　One case in which the median voter will prefer the Pareto efficient level of a public good is that in which he or she pays a Lindalh tax.

　iii.　(Of course, it is not clear why the median voter would do so, although some mildly progressive tax systems and benefit taxes may roughly approximate a Lindalh tax.)

B. Note that the functions being maximized by candidates in a stochastic voting model *are* somewhat odd special cases of a *social welfare function* where the weights are implicitly determined by voter responsiveness to policy variables.

　i.　That is to say, each candidate maximizes a social welfare function OVER THE VOTERS participating in the election of interest.

　ii.　Consequently, if every one votes, the electoral policy result in such a model is ALWAYS Pareto optimal.

VI. Normative Controversies.

A. In a stochastic voting model, the electoral result would fail to be Pareto optimal, only if some individual interests were entirely neglected, or possibly given negative weight.

  i. Some people may never vote, or may not vote when policy positions of candidates are close to each other or very far from their own ideal policy.

  ii. (For example, some voters may be more likely to support a candidate that promised to punish a subset of the electorate: perhaps racists, specific ethnic groups, or smokers.)

  iii. In these cases, the expected vote function is not a social welfare function and the electoral results would be unlikely to be Pareto optimal.

B. Note, however, that the "wieghts" in the stochastic voting "welfare function" are determined by a voter's flexibility--that is to say how much his or her vote is influenced by small changes in policy.

  i. Thus, the results may be Pareto effiecient, but incompatible with other social norms, such as fairness.

  ii. However, it is interesting to see that this particular "social welfare function" has to be estimated by every successful candidate.

  iii. And, thus, it can be said to avoid some of the usual conceptual problems of the social welfare functions.

    a. For example, the "weights" are determined by an candidate's interest in winning office, rather than philosophical debate.

    b. On the other hand, for utilitarians wieghts other than "1" are unfair or immoral.

    c. (Thought question: does it matter whether a normative theory achieve Pareto optimality? Is there always some Pareto efficient result that is consistent with a particular normative theory?)

VII. Other Controversies

A. Another area of controversy involves the cycle free nature of electoral equilibria in stochastic voting models. It turns out that the nature of the probability function describing voter behavior is critical. (See Feldman and Lee (1988)).

  i. A simple illustration may suffice, suppose that voters always voted for the candidate with the more favorable program, but occassionally made errors. Suppose that the errors are not affected by the other candidate's position.

  ii. In this case, one would have a stochastic voter who behaved in an expected value sense just like the non-stochastic voters of the median voter type model. Again cycles could and would tend to be endemic in a 2 or more dimensioned policy space.

VIII. Studying Advice:

A. In reviewing the stochastic voting model, it is most important for the purposes of this class that you understand the basic ideas: (1) What a stochastic voting model is, (2)How it differs from the non-stochastic voting model that forms the basis of the median voter model, (3) What its equilibrium is like in general terms.

B. Don't spend much time memorizing the equations, rather focus on the ideas.