

## I. Introduction: Voting Rules and Collective Choice

**A.** Whenever a group attempts to solve public goods, coordination, and other related problems, requires a method of making collective choices.

- i. Most methods of collective choice for groups involves casting votes, counting them, and using the results to make decisions or recommendations.
- ii. In most polities, other processes are also important, as with lobbying by interest groups, delegation, federalism, and so forth. (These other influences on collective action are taken up later in the course.)

**B.** Voting rules can be placed along a continuum that specifies the minimum number of persons within the group that can make policies for the group as a whole. For example:

- i. Unanimity: 100% approval is required to pass new laws. Unanimity allows any voter to veto a new law. (The European Union requires unanimity at the level of national governments to adopt significant reforms of the founding treaties.)
- ii. Super Majority: Significantly more than 50% approval is required to pass new laws. (Supermajorities are required for constitutional amendments and impeachment under the US constitution.)
- iii. Majority Rule: Any group or policy receiving more than 50% of all votes cast can make a decision for everyone. This is the most widely used voting rule in democracies.
- iv. Plurality Rule: The policy/rule/candidate/party with the most votes is adopted. Note that this rule allows a variety of minority decisions to be adopted if there are more than two parties or options voted on.
- v. Committee rule (A relatively small elite makes decisions, possibly by majority rule within the committee.)
- vi. Dictatorship/Chief (One person independently chooses the policies.)

**C.** There are also other voting rules that might be used, but rarely are. For example:

- i. Approval Voting ( Individual voters can cast "yes" votes for as many options as they wish. The outcome is determined by the option with the maximum number of yes votes.) [Stephen Brahm, Department of Political Science, New York University]
- ii. Weighted voting: give some "worthy" individuals "more" votes than others.

**D.** In practice, a wide range of voting procedures are used around the world, and even within a single government.

- i. For example, the EU uses unanimous agreement (at the level of national governments) to make major policy decisions.

- It uses supermajority rules for important decisions.
  - It uses majority rule for ordinary policy decisions.
  - It uses committees (the Commission) for many others.
- ii. The US uses plurality rule to select Presidents and members of the House of Representatives and Senate.
    - It uses majority rule to pass legislation in the House and Senate.
    - It uses super majority rule for overriding Presidential vetoes and for constitutional amendments.
- E.** The selection of voting rules is a topic in Constitutional Political Economy and is analyzed at greater length in my various Constitutional Design courses.
- i. It bears noting that choosing a government's voting rule is often done via procedures that give those currently empowered by the *status quo* rules the power to determine the new rules.
    - The power to set the electoral agenda may be set by a leadership of some sort, rather than by direct majority vote.
  - ii. Moreover, the process of writing a constitution may itself be subject to various interest group and political pressures.
    - For example, originally the U. S. Senate was composed of representatives who were appointed by state governments.
    - It bears noting that the US constitution was written by representatives of the original 13, more or less, independent states.
    - (The rest of what is now the United States was in the hands of other governments and tribes with their own forms of government at that time.)
  - iii. For an important early rational choice based examination of issues in constitutional design see: *The Calculus of Consent*, by James M. Buchanan and Gordon Tullock.
  - iv. To broaden support for a constitutional regime, minority rights are often protected.
    - For example, the government of the US cannot make laws that reduce freedom to assemble or debate public policy under its First Amendment.
    - The fifth and sixth amendments guarantee certain legal rights, guarantees a jury trial for many types of crimes, the right to a speedy trial, and the assistance of a lawyer.
    - The "takings clause" of the fifth amendment makes government pay for goods and services taken from individual citizens.
    - The fifth also guarantees that no persons shall be deprived of life, liberty, or property without due process of law, nor required to be a witness against himself.

## II. The Median Voter Model

- A.** The next several lectures will focus on the policy consequences of making decisions via majority rule.
- We will examine two models of majoritarian electoral equilibrium: one based on non-stochastic voting, the Median Voter Model. This is the most widely used model in economic applications.
  - We will also spend a little time on the stochastic voting model. This model may be somewhat more widely used by political scientists.
- B.** The median voter model is based on some very straightforward mathematical properties of majority rule.
- C.** For example, suppose that three individuals: Al, Bob and Cathy are to make a decision about how much to spend on lunch based on majority rule.
- Al prefers to spend \$5.00, Bob wants to spend around \$10.00 and Cathy around \$20.00.
  - For convenience assume that, given any two options, each will prefer the lunch that is closest to their preferred expenditure.
  - (This "spatial voting" can be shown to be the result when their marginal benefit and marginal cost curves are straight lines.)
  - Consider some votes on various alternative spending levels:
- | Options         | Votes Cast |       |       | Outcome  |
|-----------------|------------|-------|-------|----------|
| • \$10 vs. 20\$ | A: 10      | B: 10 | C: 20 | 10 MP 20 |
| • \$5 vs. \$20  | A: 5       | B: 5  | C: 20 | 5 MP 20  |
| • \$5 vs. \$16  | A: 5       | B: 5  | C: 16 | 5 MP 16  |
| • \$10 vs \$5   | A: 5       | B: 10 | C: 10 | 10 MP 5  |
| • \$12 vs. 10   | A: 10      | B:10  | C: 12 | 10 MP 12 |
| • \$9 vs. 10    | A: 9       | B: 10 | C:10  | 10 MP 9  |
- Note that Bob always votes in favor of the outcome that wins the election. (The B column and the Outcome column are the same.)
  - Note also that exactly the same number of individuals prefer a more expensive dinner as prefer a less expensive dinner than Bob. (This is the definition of a median ideal point or "preference.")
    - Bob is the median voter.

- He is the voter with the median ideal point.
- Note that the median voter's ideal point can beat every other possible alternative in pair-wise voting.
- D.** The **Weak Form** of the *median voter theorem* says that the median voter always casts his vote for the policy that is adopted via majority rule and the candidate that receives a majority of the votes.
- E.** The **Strong Form** of the *median voter theorem* say the median voter always gets his most preferred policy.
- [For example, in the example above Bob's preferred expenditure level, \$10, will defeat any other policy.]
- F.** Summary and Extensions: Direct and Representative Democracy
- The previous illustration shows why the median voter has important effects on electoral outcome under majority rule.
    - The analysis suggests (1) that policies adopted within direct democracies will be those favored by the median voter, and (2) that if the median voter's preferred policy is ever adopted, that policy will be a stable political equilibrium. It will continue in force until the median voter's preferred policy change.
  - Illustrate the relationship between the MB-MC representation of rational choice and the spatial voting model.
    - Draw a diagram of a voter's choice of an ideal policy using straight-line MB and MC curves.
    - The intersection of the MB and MC is  $Q^*$ , the person's ideal point.
    - Now consider two points, one to the left and one to the right of  $Q^*$  that are equal distances from  $Q^*$ .
    - Use geometry to prove that both generate the same level of net benefits.
    - Note that if one of these points is slightly closer to  $Q^*$  than the other, that the policy closest to  $Q^*$  will generate higher net benefits for this voter than the one that is further away.
  - We next analyze the importance of the median voter for representative democracy.

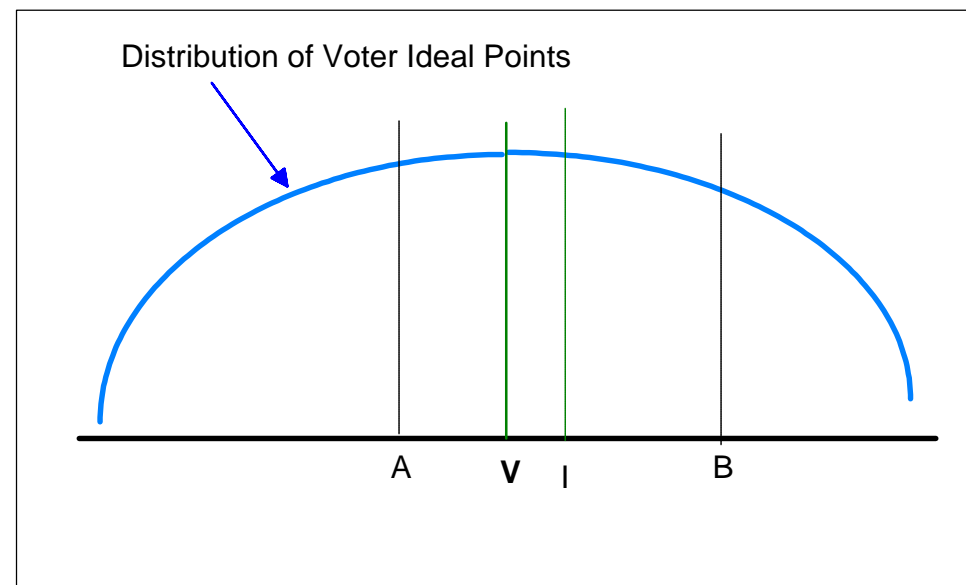
## III. The Median Voter and Representative Democracy

- A.** Competition between candidates for government office can be analyzed with a diagram that shows the distribution of voter ideal points over public policies.

- i. The distribution of voter ideal points can be generated by plotting the number of voters that have each possible "ideal point."
  - To make drawing such distributions possible, it is often assumed that voter ideal policies can be mapped into a single dimension--that is sometimes considered to be the ideological left-right spectrum.
- ii. It is also generally assumed that voters will all vote for the candidate that is "closest" to them in the policy dimension.
- iii. The area under the resulting curve gives you a number of voters.
- iv. A policy receives a majority if the area representing the number of votes in favor of a policy or candidate is larger than that representing those opposed to that policy or candidate.

**B.** A model of two-party electoral competition can be developed from such a diagram.

- i. Assume that distance from each voter's ideal point can be used to each voter's preferences, rank order of policies, and thereby candidates policy positions.
  - (In the discussion below, we assume that the candidates are individuals, but the same logic applies to parties and to stable left of center and right of center coalitions.)
- ii. Characterize the distribution of voter ideal points on the policy that you are interested in with a frequency distribution (or a probability density distribution).
- iii. The *median* voter's position, V, will be that position such that the area under the frequency distribution to the left of V will be exactly equal to the area under the frequency distribution to the right of V.
  - (Recall that those areas are the number of voters to the left or right of V.)
- iv. *The Candidate (Party, or Coalition) that is closest to the median voter's ideal point will always get the most votes.*
  - To see this, find the "indifferent voter." The indifferent voter (I) is exactly half way between the two candidate positions, A and B.
  - Every voter that is to the left of the "indifferent voter's" ideal point votes for the candidate on the left (A). Every voter to the right of the "indifferent" vote will voter for the candidate on the right (B).
  - **Note that the candidate that gets the median voter's vote always gets at least half of the votes!**



- v. Now, assume that candidates care more (or only) about being elected than about policy.
  - Note that both candidates can *increase* their votes, *if the other candidate does not move*, by moving towards the median voter.
  - When Candidate "C" moves towards the median voter, the "indifferent voter" moves toward the other candidate, which implies that "C" will get *more votes* than before and candidate "D," will get fewer.
  - **EACH CANDIDATE therefore has an incentive to move towards the median voter's position in order to win the election.**

**C. In the limit**, each candidate (A and B party or coalition) takes exactly the same position, namely the median voter's ideal point, and the strong form of the median voter theorem holds.

- i. Extreme policy positions (green/libertarian) positions will *never* win a two candidate election.
  - (Unless, of course, the political view of most of the electorate changes, so that those positions stop being extreme.)

- ii. The strong form of the median voter theorem is the unique **Nash equilibrium** of the two party election "games" (if a median voter exists).
  - Note also that the median voter's ideal point is the *only* policy position that *can never* be defeated outright in a two candidate election.

#### IV. The Median Voter and Proportional Representation

- A.** The "plurality" voting system used in the United States and several other countries around the world--in which politicians compete for office within a geographical district of some kind and the person with the most votes wins the office--is not the only form of democracy.
- B.** There are also many "proportional representative" (PR) systems of government that assign seats to political parties in their legislatures based on the proportion of the total votes received by the parties in the electoral contest.
- i. In these "proportional representation" (PR) electoral systems, voters generally vote for parties rather than persons.
  - ii. Some systems, such as the German system, combine elements of PR with Plurality systems (mixed member systems).
  - iii. We will focus on the pure PR systems in this section.
- C.** PR systems normally include more than two significant political parties.
- i. In some cases, the parties are grouped in two more or less stable coalitions, in which case the plurality voting models will provide a very useful representation of politics.
    - (After all, the two dominant parties within plurality systems are also stable coalitions that represent a variety of voter interests.)
  - ii. The maximum number of parties that can be sustained in a PR system is determined by its "representative threshold," the fraction of votes that are required to get seats in parliaments.
    - The thresholds required to win a seat in the legislature vary in PR systems from around  $1/10$  to  $1/N$  (where  $N$  is the number of seats in the legislature).
    - Consequently, the number of viable parties varies from approximately 10 parties in systems with a relatively 10% threshold to about  $N$  parties in open PR systems with  $1/N$  thresholds.
  - iii. In effect PR systems create a distribution of preferences within the legislature that are similar to that of the electorate as a whole.
    - If legislators vote independently of one another on policies, you can see that the result will tend toward the same result as predicted by the median voter theorem.

- In this case, it will be the **median legislator** that determines the outcomes rather than the median voter, but the median legislator will have approximately the same preferences over policies as the median voter.
- (In plurality systems only more or less "median" positions are represented, although these may vary widely if there is geographical clustering of ideal points.)

**D.** The median voter theorem implies that in coalition governments, the **median legislator will be a member of the majority coalition**, (if one exists).

- i. In such cases, the **weak form of the median voter theorem** holds because the median voter will normally vote for the party at the median position and of which the median legislature is a member.
- ii. If two coalitions compete to form a majority coalitions (and thus determine the government: Prime Minister and cabinet), they will compete for the "vote" of the median legislator.
  - In this case, the **strong form of the median voter theorem** will apply to policies adopted in PR systems.
- iii. The median voter model provide several useful predictions about PR system, as long as a median legislature exists.
  - (A median legislature is least likely to exist in cases in which there is no median voter in the population at large.)

#### V. The Median Voter Model and Public Policy

- A.** Given these results, which can be generalized, within limits, a wide variety of public policies in democracies will be simply those which maximize the welfare of the median voter (as implied by the strong form of the median voter theorem).
- Because medians tend to be from the middle of the range of voter ideal points, democratic politics tends toward relatively moderate positions.
- B.** Changes in government policy will reflect changes in the median voter's circumstances: as "she" becomes richer or poorer, older or younger, more concerned about a particular voting issue etc., her ideal point will change and so will government policy.
- (Can you think of any examples of this?)

## VI. Geometric Illustration of the Median Voter's Preference for Government Service/ Regulation Levels

A. Some basic characteristics of "majoritarian demand" for public service or regulation can be demonstrated geometrically using a median voter model.

- i. Illustration: suppose there are three individual voters with differing appraisals of the value of some public good, say environmental quality.
  - (This makes their perceived marginal benefit curves of each voter different.)
  - (Differences in MB curves may be caused by differences in tastes, differences in information about perceived risks, location, and differences in income.)
- ii. To simplify a bit, assume that the cost of providing this public good will be shared equally by all three individuals.
  - (This makes the marginal cost of environmental quality the same for each person.)
  - (The main purpose of this assumption is to simplify the diagram a bit.)
- iii. Each person prefers the level of environmental quality that maximizes their own consumer surplus.
  - (So, each voter has a different ideal point, a different preferred level of environmental quality.)
- iv. The individual whose preference is in the exact middle of the distribution of preferences will be the median voter.
  - (See the figure drawn in class.)
- v. If the strong form of the median voter theorem holds, then the *median voter's ideal point* will be the level of environmental quality that is adopted.
  - Note that it *does not matter* how much more or less of the government service or regulation the other voters want.
  - That is to say, the degree of extremism (for or against) a policy will not affect the political outcome *as long as it does not affect the location of the median*.
  - (In this sense, the median voter model implies that democratic outcomes are very robust and stable. An increase in extremism on the right or left generally will not by itself affect public policy.)

B. The illustration also indicates that the median voter result may not be Pareto optimal.

- i. To see this redraw the diagram and do the following:

- Recall that the demand (or social marginal benefit) for a pure public good is the "vertical sum" of the individual's demand (marginal benefit) curves.
- ii. The Pareto efficient outcome/quantity occurs where the social marginal cost curve equals the marginal social cost curve.
  - In the illustration, the marginal cost of each person was assumed to be one-third of the total marginal cost, so the social marginal cost curve is just three times as high as the individual marginal cost curves.
- iii. Generally, the *Pareto efficient quantity of a pure public good is the same as the median voter's demand ONLY if the median voter is also the average voter*.
  - DEMONSTRATION: Note that in a three voter model, if the median voter is also the average voter than his  $MB = SMB/3$ , and the place where his MB equals his MC is the same as that where  $SMB = SMC$ .
  - Note that more generally, if  $Q^{**}$  is such that  $0 = SMB(Q^{**}) - SMC(Q^{**})$ , then  $0 = [SMB(Q^{**})/N - SMC(Q^{**})/N]$  where  $N$  is the number of voters.
  - The latter characterizes the median voter's ideal point if she/he is also the average voter.

C. Another demonstration of the median voter's policy preference can be worked out for the cases in which there is a given fixed budget to be allocated.

- i. In this case, however, another way of representing rational choice works better than the MB-MC representation that we have been using.
  - One can use indifference curves and the governmental budget constraint to show (see McCubbins and Schwartz) the median voter's ideal allocation of the budget.
- ii. To see this consider a budget allocation of a fixed budget between guns and butter.
  - As long as the budget is taken as given, all the voter policy preferences will lie along the budget constraint. (No other options are possible in this case.)
  - In that case, there will be a median voter. Moreover, the median along the "guns" axis will be the same as the median along the "butter" axis.
- iii. The result would be that some moderate result would *normally* obtain.

## VII. Discussion/Food for Thought

A. How empirically relevant is the Median Voter Model?

- i. Do candidate's converge to moderate positions?
- ii. Think about committees, clubs and other organizations that make decisions using majority rule.

- iii. Do you normally see moderate, middle of the road policies adopted?
- iv. Does it seem to work for American elections?

**B.** The median voter model is consistent with, and provides an explanation of, what George Stigler (1970, JLE) has called **Director's Law**.

- Namely, that "Public expenditures are made for the primary benefit of the middle classes financed with taxes which are borne in considerable part by the poor and rich."
- Discuss: does this seem to be true? (Is social security a good example?)

### VIII. Mathematical Appendix (optional for this class)

- A.** For example, consider the following model of the median voter's preferred level of environmental regulation. [For those interested, a mathematical version of the model is included at the end of this set of notes.]
- Let  $U = u(Y, E)$  where  $Y$  is material consumption (income) realized by the median voter, and  $E$  is the (perceived) level of environmental quality. Suppose that environmental quality is a function of regulatory stringency  $R$  and national income,  $E = e(R, Y)$ .
  - To simplify a bit, suppose that the median voter gets a constant fraction "a" of national income which is decreasing in regulatory stringency,  $Y = y(R)$  and  $Y_m = aY$
  - The constraints and definitions can be substituted into the median voter's utility function:  
 $U = (ay(R), e(R, y(R)))$
  - This can be differentiated with respect to  $R$  to characterize the median voter's ideal stringency of environmental regulation  $R^*$ .
  - $R^*$  will satisfy  $U_Y aY_R + U_E (E_R + E_Y Y_R) = 0$
  - The first term is the median voter's marginal cost and the last is his marginal benefit from more stringent environmental regulation. (Explain why.)
  - The implicit function theorem (see class notes) can be used to determine the comparative statics of environmental regulation with respect to parameters of the median voter's optimization problem. The results are (qualitative) forecasts of public policy in this area.
- H.** A similar model of the median voter's demand for public goods or transfers to the poor or elderly can be readily developed by changing the constraints a bit.
- For example, suppose that  $G$  is a public service received by the median voter (which may or may not be a pure public good). Again let his utility level be defined over other consumption, here  $C$ , a variable affected by the policy of interest, here the level of  $G$  provided.
  - Let the level of  $G$  be an increasing function of the taxes collected,  $G = g(T)$  and total tax revenue be a function of national income and the tax rate chosen,  $T = tY$ , where  $Y$  is itself negatively affected by the marginal tax rate  $t$ , here  $Y = y(t)$ .
  - Again assume that the median voter receives some constant fraction of national income  $Y$ , so that his personal private consumption is  $C = (1-t) aY$ .
  - Now substituting into the median voter's utility function again yields an optimization problem with one control variable (here  $t$ ) which implicitly controls another policy variable,  $G$ .  $U = u((1-t) aY, G(t, y(t)))$

- Differentiating with respect to  $t$  characterizes his ideal tax level,  $t^*$  which satisfies:  $U_C [(-1)aY + (1-t)aY_t] + U_G G_T (Y + tY_t) = 0$
- His ideal public service level is thus  $G^* = g(t^* Y(t^*))$ .
- Again the implicit function differentiation rule can be used to characterize the comparative statics of the median voter's choice and to thereby make forecasts about the course of public policy in this area.
- [As an exercise you might construct a simpler model where there is a balanced budget constraint, and  $G$  is produced via constant returns to scale. Other income tax schedules could also be used, E. G. a linear one  $T = a + tY$ ]

- B.** Notice that, in practice, the median voter model is consistent with, and provides an explanation of, what George Stigler (1970, JLE) has called **Director's Law**. Namely, that "Public expenditures are made for the primary benefit of the middle classes financed with taxes which are borne in considerable part by the poor and rich."

### IX. Illustration 1: the demand for public services with a "head tax"

- The median voter in his capacity as a policy "maker" looks very much like the standard consumer in a grocery store, except that in addition to private budget constraints, he has a "public" budget constraint to deal with.
- Suppose that the median voter's utility function is defined over private consumption ( $C$ ) and some public service ( $G$ ). Suppose further that the median voter has  $W$  dollars to allocate between  $C$  and  $G$ , and that the government faces a balanced budget constraint, and that all expenditures are paid for with a head tax,  $T$ . Assume that there are  $N$  tax payers in the polity of interest.
- Thus:
  - $U = u(C, G)$
  - $W = C + T$
  - $g(G) = NT$
- Note that  $T$  can be written as  $T = g(G)/N$  and substituted into the private budget constraint to make a single unified budget constraint:
  - $W = C + g(G)/N$
  - This in turn can be solved for  $C$  and substituted into the utility function:
  - $U = u(W - g(G)/N, G)$
- Differentiating with respect to  $G$  yields a first order condition that characterizes the median voter's preferred government service level:

a.  $-U_C(g_G/N) + U_G = 0 = H$  or equivalently as  $U_C(g_G/N) = U_G$

**A.** The right hand side of the latter is the subjective marginal benefit (marginal utility) of the government service, the left-hand term is the subjective marginal opportunity cost of government services in terms of lost private consumption.

- i. Note that the subjective marginal cost of the service is determined by both preferences (marginal utility of the private good C) and objective production or financial considerations,  $cG/N$ . The latter can be called the median voter's marginal cost share, or price for the government service.
- ii. An implication of the first order condition together with the implicit function theorem is that the median voter's demand for public services can be written as:

- $G^* = \gamma(W, N)$  that is to say, as a function of his own wealth (holding of the taxable base) and the population of tax payers in the polity of interest.
- The implicit function differentiation rule allows one to characterize comparative statics of the median voter's demand for government services.

• Specifically  $G^*_W = H_W / -H_G$  and  $G^*_N = H_N / -H_G$  where H is the first order condition above.

• Recall that solving for these derivatives requires using the partial derivative version of the composite function rule and paying close attention to the location of all the variables in the various functions included in "H," the first order condition. We find that:

•  $G^*_W = [-U_{CC}(g_G/N) + U_{GW}] /$

a.  $[-U_{CC}(g_G/N)^2 - U_C(g_{GG}/N) - 2U_{CW}(g_G/N) + U_{GG}] > 0$

•  $G^*_N = [-U_{CC}(g_G/N)(g(G)/N^2) + U_C(g_G/N^2) + U_{GW}(g(G)/N^2)] /$

a.  $[-U_{CC}(g_G/N)^2 - U_C(g_{GG}/N) - 2U_{CW}(g_G/N) + U_{GG}] > 0$

- That is to say, with a head tax the demand for a pure public service rises with personal wealth and with population.