Chapter 9: Intertemporal Choice

I. Introduction

Most neoclassical models are timeless in the sense that "time" is left out of the model. That is not because time is never important, but because for some purposes leaving time out of a model or analysis does not undermine our understanding of the puzzle or phenomena being addressed. Economic choices and consequences often in the present or near future. If a consumer decides that he or she will this month's wages in a particular way, the fact that the actions associated with that decision take place during the month (or perhaps in the next month) does not materially influence the optimization process that led to that decision or the kinds of choices reached.

However, there are cases in which time matters. Time cannot be ignored when actions are taken today that affects one's possibilities in the future—if one is rational and forward looking. Indeed, the phrase "forward looking" implies that rational beings takes some account of the effects of one's present actions on one's future possibilities. For example, a consumer's decision to spend a certain amount of money in the future may affect the extent to which he or she works (and how they work) today. One may save up for a vacation, an automobile, a house, a college education, or retirement. Such long-erm plans simultaneously affect consumption decisions today and in the future. If one decides to save money today, then less money is spent today than otherwise would have been spent, and more will be spent in the future.

Or, a consumer may engage in the opposite type of behavior. He or she may borrow against future income to pay for capital goods (computer, automobile, house, etc.) or for ordinary consumption today. Such choices increase current expenditures and reduce future ones (apart from loan payments). In the average course of a life, wage rates rise during the first 20-30 years of one's career, so an individual's future wages are very likely to be higher in the future than in the present. Borrowing allows one to shift a bit of future income to the present.

On the other hand, a theory that models such choices has to account for the fact that there is generally a somewhat greater reduction in future consumption than gained in the present because interest payments will have to be paid until the loan is paid off.

The same logic applies to decisions made by economic organizations. Firms and other organizations may also divert current income into future enhancing uses. For example, they may invest in new equipment or attempt to build a cash reserve today that can be used to address nasty surprises that may arise in the future, or simply to smooth out predictable fluctuations in their cash flow during the course of their planning horizon. Firms may also borrow against their assets or future profits to pay for capital goods that will be used in production. Those capital goods must, in a sense, pay for themselves, which is to say that they must increase profit flows through time sufficiently to be used to pay off the loan—unless the decision was unwise, or bad luck (uncertainty) reduces the demand for their products.

Many businesses have sales patterns that are connected with the seasons, and firms try to hold onto their employees for the entire year. The routines required for efficient team production benefit from the stability of the team members. Workers, on the other hand benefit from more predictable cash-flow which means that they will accept a lower wage than they otherwise would have (otherwise a risk premium would have been required). Most firms use "a wages fund to keep their teams on the payroll. They save some net income in the most profitable times in the year to pay their employees in the least profitable times. Examples of markets with cyclic demands include the market for toys, holiday foods and beverages, and markets for housing (because of school year effects).

This chapter develops models of choice and markets that characterize such intertemporal decision making.

To do so requires somewhat "stronger" assumptions about how individuals and firms make intertemporal decisions than needed in part 1 of the course. The focus remains on net-benefit maximization. However, as in the previous chapter, a specific assumption is added to analyze how individuals think about the future. Namely, it is assumed that consumers and firms use the **present discounted value** method to think about future benefits, costs, and net benefits—the algebra of which emerges naturally when it is recognized that both borrowing and saving have opportunity costs.

However, as with the assumption that individuals use "expected values" to make decision in risky settings, the assumption that individuals use "present discounted values" for intertemporal choices is reasonable and a useful "first approximation," but it is not as general as the basic "maximize net benefit" assumption used in the first half of this course. Some individuals will take account of time and risks in other ways, but most will behave in a manner that is more or less consistent with these models of forward looking choices in more or less risky settings.

II. Markets for Savings and Borrowing

In the chapter on capital accumulation and growth, we briefly modelled markets for savings and loans. For this chapter, we'll need to dig a bit deeper into how such markets operate.

Borrowing is impossible unless there are savers, because borrowing requires the current purchasing power (income) of some consumers and firms to be shifted to other consumers and firms. If no one wants to save, no one will be able to borrow. On the other hand, the interest and other returns that one can realize by saving is possible only because there are borrowers that are willing to pay a premium to have money now, rather than the future.

Together savers and borrowers create a market for both savings and loans. The price paid for savings is normally called "interest" or "interest rate." The higher the return on savings (the higher is the interest rate, adjusted for risk and expected inflation) the more inclined savers are to put aside money. On the other hand, for borrowers, the interest rate is the cost of borrowing, the higher interest rates are, the less money they are inclined to borrow.

Because there are many places where one can save and many places where one can borrow, ordinary supply and demand curves can be used to characterize this market. Figure 9.1 characterizes a simple "direct" market for savings and loans.

Figure 9.1: Market Clearing Interest Rate w/o Intermediary

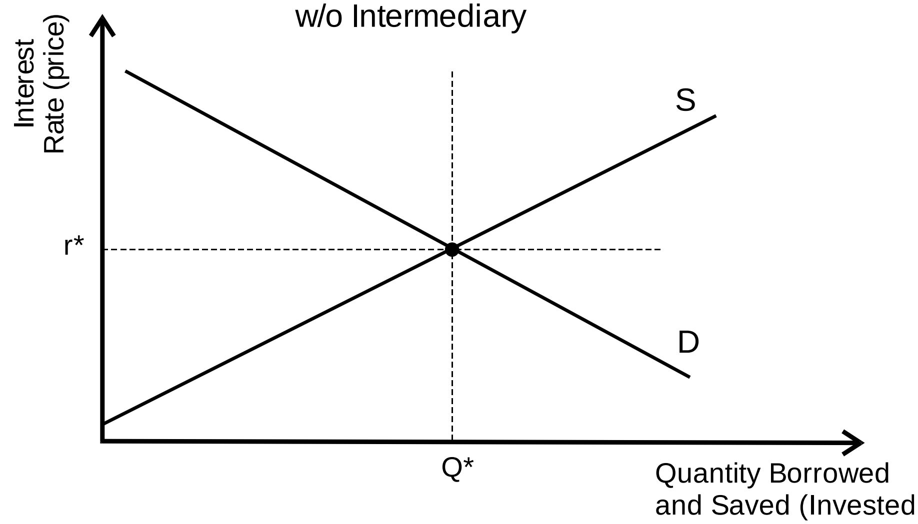


Figure 9.1 represents the equilibrium that emerges for a given class of loans. If this is the imaginary "risk free" market for savings and loans than the interest rate that emerges is called the risk-free interest rate. It simply represents the value of having "a dollar" today rather than a dollar in the future. That difference is sometimes called the subjective rate of time discount or discount rate. In the "direct" case, the interest rate earned on savings is the same as that paid by borrowers and the interest rate is the discount rate of the marginal saver and borrower. (Other savers have lower discount rates and other borrowers have higher discount rates.) If the loans are risky because a significant fraction of borrowers will not pay back their loans, then a risk

premium is added to the risk-free interest rate to take account of the risk of default.

Direct saving and loans take place in bond markets where savers buy bonds from seller of bonds. However, not all saving and loans are done directly. Often there are third parties involved that serve as intermediaries between savers (lenders) and borrowers. For most consumers, banks are (or were) them most common intermediary between savers and borrowers.

Banks provide a variety of services to both lenders (savers) and borrowers. For example, banks often "guarantee" a fixed interest rate for savers, which reduces their risks. They can do so, because they make lots of loans and can accurately judge to risk of default on loans that they make. In this market, banks are like insurance companies, and their expectations about losses from defaults have a much lower variance than a direct lender would tend to have. Banks also provide "liquidity" for their depositors (saver=lenders), which is to say, rather than having all of their money tied up in loans to borrowers, banks keep a cash reserve that can be used to pay off savers that want to withdrawal their savings as cash or use it to pay bills (as with checking accounts).

Similarly, those borrowing can borrow at lower rates (especially if they have assets that can be used to guarantee the loans [collateral]) for the same reason. If banks can judge risks accurately, their overall return from their entire loan portfolio is quite stable and on average, the expected value of the loans made. Banks, thus, require a smaller risk premium than would be required in direct lending from savers.

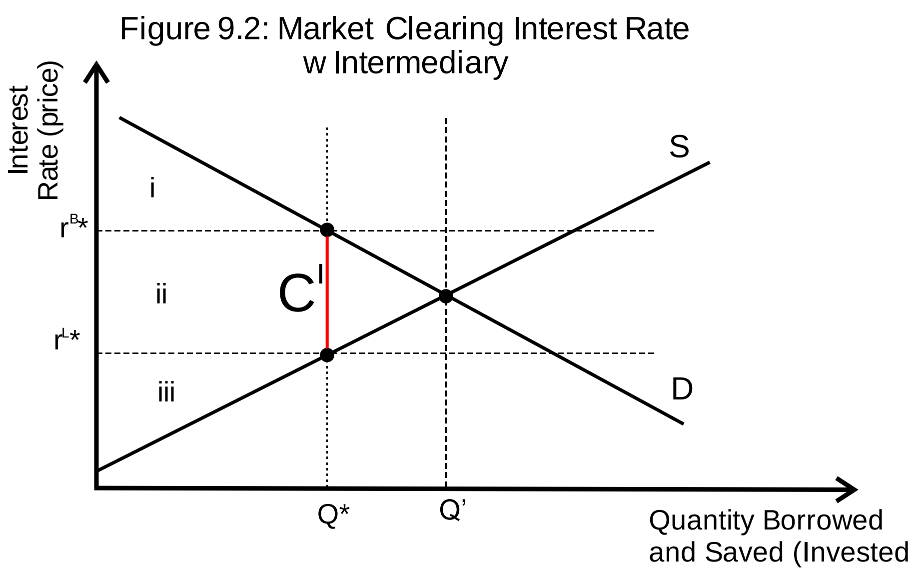
(Here we are ignoring other risks associated with government policies and business cycles, topics taken up in macroeconomics.)

Intermediaries have costs and expect at least an average return on their investments. These markets are competitive, and market forces tend to limit their returns as in the markets studied in the first part of the course.

Nonetheless, the costs of their services come between the interest rate paid to depositors (saver-lenders) and borrowers. A diagram of the savings and loan markets when intermediaries are used (as they usually are) resembles that of the tax diagrams developed in chapter 5—there are three parties to the transactions in this market—buyers (borrowers), suppliers (savers), and intermediaries. Figure 9.2 illustrates such a saving and loan market in equilibrium.

The marginal cost of intermediary services is denoted as C^I. Area "i" is the net benefit or profits of those taking out loans and "iii" is the net benefit or profit of those saving/investing through the intermediary (here banks). C^I reduces both savings and loans over what they would be in a world without transactions costs, but given real-world transactions costs and risks, they actually tend to increase the size of saving and loan markets over what they otherwise would have been if each saver-investor had to find borrowers, assess the risks, and write up contracts.

"Middlemen" in financial markets normally take on at least some of the risk from such intertemporal transactions and reduce transactions costs for both borrowers and lenders. It is their risk reduction and liquidity services that induce both savers and borrowers to use their services.



If all loans were risk free, few borrowers or lenders would use the services of an intermediary. However, when risk are non-trivial and liquidity is useful, their services can be very useful to both.

If the risks of default were higher, the supply curve would be lower (be to the left of the supply curve depicted) which would cause interest rates to be higher for both lenders and borrowers. The rates of return and cost associated with loans would reflect the risk premium demanded by savers (and intermediaries) associated with different kinds and numbers of loans and the costs of attracting depositors and providing them with liquidity. (For example, house and automobile loans are pretty similar, but loans to startups firms are all quite different and generally far riskier.)

Shifts in the demand and supply of loans and savings affect market interest rates in much the usual way. But the shifts in those two curves are driven by somewhat different considerations than in the previous cases. Borrowing by firms reflects expected net benefits from capital investments. Savings by investors and savers reflect expectations about alternative rates of return and risks associated with different investments, and also future consumption plans. For example, if average longevity increases, the supply of savings tends to increase because forward looking individual will put aside more funds for retirement. Changes in the risk of default will also shift supply curves, as would increases in personal income.

It turns out that interest rates play a role in determining both the cost and benefits of decisions with respect to savings and loans—partly through effects on the value placed on future consumer surplus and profits.

Most of the rest of this chapter focuses on the use of the **present discounted value** formulae for intertemporal calculations of benefits and costs.

III. Intertemporal Choice: Time Discounting and Present Discounted Values

The most general way to think about "present discounted value" is to think about the amount in the present (PV) that you would be indifferent to having rather than some other value (F) in in T years. One way to estimate this, is to think in money terms. That is, one can calculate the amount of money (PV) that one would have to invest today to have F dollars T years in the future.

We'll initially use two letters to indicate present discounted value (PV) and normally write "present value" rather than present discounted value from now on.

- If the interest rate or rate of return is r, one can just apply the compound interest formula to determine how much money an investment of PV at interest rate r will generate in T years—its future value denoted as F_T. $\mathbf{PV\,(1+r)^T = F_T}$
- Solving for PV yields $\mathbf{PV = F_T/(1+r)^T}$ which is the basic formula for calculating the present discounted value of some value F that is realized T years in the future. Notices that the present value of F_T is always smaller than the actual value in future dollars because interest rates are greater than zero, r>0.
- To make the calculation more concrete, suppose that F is $20,000 that T=2 and R=3% or 0.03. In that case,

  $$PV = (20{,}000)/(1.03)^2 = \$18{,}851.92$$

- Notice that the PV of future value F goes down when the interest increases and/or when the time period increases.
- The PV of $20,000 in two years at an interest rate of 5% is

  $$PV = (20{,}000)/(1.05)^2 = \$18{,}140.59$$

- The PV of $20,000 in ten years at an interest rate of 5% is

  $$PV = (20{,}000)/(1.05)^{10} = \$12{,}278.27$$

If one thinks in purely financial or money terms, one would be indifferent between $12,278.27 today and $20,000 in 20 years if the 20-year interest rate is 5%.. This assumes that no inflation occurs (or that F_T is stated in inflation adjusted terms in current dollars) and that there is no risk involved about whether the future amount will be paid or not.

When one takes account of inflation either everything should be in inflation adjusted (real) terms (including the interest rate and future values). The real interest rate is the nominal or state rate of interest less the average annual inflation rate over the period of interest. Or everything should be in nominal (ordinary dollar) terms. When there is the risk that amount F will not be paid, then one needs to also take account of the risk using the expected value methods that were developed in chapter 8.

Many decisions involve long term flows of costs and benefits that need to be evaluated by a decision maker or group of decisionmakers. These flows are easiest to compare if one can construct a common measure for the purposes of comparison. The present value of a series of benefits and/or costs through time is one such measure. It is the amount, **P**, that you could deposit in a bank at interest rate **r** and used to replicate the entire stream of benefits or costs, F_1, F_2, F_3, ... F_T. That is to say, you could go to the bank in year 1, and withdraw the amount (B_1) for that year, return in year 2, pull out the relevant amount for that year (B_2) and so on ... . The present discounted value of a series of future amounts is simply the sum of the present values of each element of the series—which is calculated as above.

DEF: Let **Ft** be the value of some asset or income flow **"t"** time periods from the present date. Let **r** be the interest rate per time period over this interval.

- **The present discounted value** of F_t, now written as $P(F_t)$ is

$$P(F_t) = F_t/(1+r)^t$$

- **The present value (now written as P to reduce notation) of a series of future income flows** (which may be positive or negative) over **T** years when the interest rate is **r** (as a fraction) per period is simply the sum of the present values of each element of the series of future amounts:

$$\boldsymbol{P = \sum_{t=1}^{T} F_t/(1+r)^t}$$

The present discounted value of any series of values is the sum of the individual present values of each element of the series.

This formula always "works" but it is somewhat cumbersome to use as the planning period, T, becomes relatively large. Another useful formula is one that characterizes the present discounted value of a **steady flow** of values on off into the future for the next T years. In cases where a **constant value** is received through time, e.g. V_1 =

$V_2 \ldots = V_t \ldots = V_T = v$, a bit of algebra allows the above present value formula to be reduced to:

$$P = v\,[\,((1+r)^T - 1)/r\,(1+r)^T]$$

Derivation of the Above Formula (optional)

This formula can be derived as follows:

- Multiply $P = \sum_{t=1}^{T} V_t/(1+r)^t$ by (1+r) which yields

  $(1+r)P = \sum_{t=0}^{T-1} V_t/(1+r)^t$

- Subtract P from (1+r) P which yields:

  $rP = v\,[\,1/(1+r)^0 - 1/(1+r)^T)]$

  (Note that all the terms in the two sums are the same except for the first and last one, so the middle terms all cancel out.)

- Recall that $1/(1+r)^0 = 1$ so $rP = v\,[\,1 - 1/(1+r)^T)]$

  Putting the lefthand term over a common denominator yields $rP = v\,[\,(1+r)^T - 1]\,/\,[(1+r)^T]$

- Dividing both sides by r yields

  $P = v\,[(1+r)^T - 1]\,/\,[r\,(1+r)^T]$ QED.

A Useful Extension of the Formula for Calculating the PV of a Constant Flow of Benefits or Costs or Net Benefits

Note that this constant flow of benefits (or costs) formula **has a very simple limit** as T approaches infinity, namely:

$$P = v/r \quad .$$

This is another very convenient formula. There are many long-term investments and regulatory policies that have very long lives that can be thought of as infinitely lived investments as a "first approximation". The P=v/r formula allows the present values of such flows of cost or benefits to calculated very easily.

IV. Illustrative Applications

(1) These formulae can, for example be used for net revenue analysis. Suppose that a windmill can be built that cost $1,000,000 and will produce $50,000/year in electricity for 40 years. Is the windmill profitable to construct if the interest rate is 5%/year?

- Use the PV formula: $P = v\,[\,((1+r)^T - 1)/r\,(1+r)^T]$
- The PV of the future profits are $P = 50{,}000[((1.05)^{40} - 1)/(.05)(1.05)^{40}\,] = \$857{,}954.31$
- So, the answer in this case is NO
- What if the interest rate is 2%/year? In this case PV = $1,367,773.96
- In this case the answer is YES
- (Real) interest rates matter. Note that the net benefits in the distant future are worth far less when r = 0.05 than when r = 0.02
- Note that if the dam would provide electricity forever, then

  $P = v/r = \$50{,}000/0.05 = \$1{,}000{,}000$

- In that case the dam project exactly breaks even (ignoring any maintenance expenses)
- But, also note that the all the years after year 40 add relatively little to the present discounted value of the future benefits.

(2) Suppose that Al can afford to pay \$5000/year in car payments for 5 years toward a new automobile. If the bank's opportunity cost rate of return is 7%, what is the largest amount that the bank will loan Al given his budget?

- Use the PV formula: $P = v\ [\ ((1+r)^T - 1)/r\ (1+r)^T]$
- $P = 5000\ [\ ((1.07)^5 - 1)/r(1.07)^5] = \$20{,}500.99$
- That is the bank's opportunity cost of tying up P dollars during the 5 years the loan will be repaid.

V. Risky Intertemporal Choices: Combining Present Value and Expected Value Calculations

The present value and expected value formula can be combined to deal with uncertain flows of future benefits and costs or uncertain future income levels.

For example, consider the purchase of a lottery ticket in a "million dollar" lottery game. Suppose that the winner receives \$50,000/year for twenty years, the interest rate is 5%, the probability of winning is 1/1,000,000 and the lottery ticket costs 1 dollar. Suppose also that there are just two outcomes: winning and losing.

The present value of winning the lottery is the present value of \$50,000/year for twenty years. Substituting into the present value formula for a constant flow of future benefits yields:

$$(50{,}000)\ [\ (1.05)^{20} - 1)\ \ /\ (\ .05\ (1.05)^{20}\ )] =$$

$$(50{,}000)(12.4622) = \$623{,}110.52$$

when the current interest rate is 5%/year. This is, of course, much less than the \$1,000,000 value that lottery sponsors usually claim for the prize of such contests.

The expected present value of such a lottery ticket requires calculating the expected value of the ticket The probability of winning is 1/1000000 and the probability of losing is 999999/1000000, so the expected value of the ticket is:

$$[1/1000000][\ 623{,}109.52-1.00] + [999999/1000000][-1.00] = -\$0.37$$

Notice that this ticket is a "bad bet." It has a negative expected discounted value. (By the way, this hypothetical lottery is a better deal than most state lotteries, which have expected present values of less than -\$0.50)

VI. Applications to Normative Policy Analysis: Benefit-Cost Analysis

One of the most widely used tools of policy analysis is benefit-cost analysis. In principle, benefit-cost analysis attempts to determine whether a given policy or project will yield benefits sufficient to more than offset its costs.

Cost-benefit analysis, ideally, attempts to find policies that maximize social net benefits measured in dollars. (Every diagram that includes a dead weight loss triangle is implicitly using cost benefit analysis.) Economists use this approach to characterize externality and monopoly problems. It is also used to criticize ideal and less than ideal public policies and taxes. Unfortunately, the data do not always exist for these calculation to be made. The most widely used methods for dealing with uncertainty and time in Benefit-Cost analysis is to use various combinations of "Expected Value" and "Present Value" calculations.

Cost-benefit analysts carefully estimate the benefits, costs, and risks (probabilities) associated with of alternative policies through time. If several policies are possible, cost-benefit analysis allows one to pick the policy that adds most to social net benefits (in expected value and present value terms) or that has the highest social rate of return. If only a limited number of projects can be built or policies adopted, then one should invest government resources in the projects or regulations that generate the most net benefits (the highest rates of return in terms of social net benefits). One can also use cost-benefit analysis to evaluate alternative environmental policies.

When many projects can be adopted, the policy question is essentially a yes or no question is: Does the policy of interest generate sufficient benefits (improved air quality, health benefits, habitat improvements etc.) to more than offset the cost of the policy (the additional production costs borne by those regulated plus any dead weight losses and the administrative cost of implementing the policy)?

The *net-benefit maximizing* norm implies that both good projects, and good regulations, should have **benefit-cost ratios** that exceed one, B/C > 1. That is to say, the benefits of a project should exceed its costs to be worth undertaking. However, many of the goods and services generated by environmental regulations *are not sold in markets* and so *do not have prices* that can be used to approximate benefits or costs at the margin.

These "implicit prices" can be estimated, but the estimates may not be very accurate. Thus, a good deal of the policy controversy that exists among environmental economists is over the proper method of estimating non-market benefits and costs. For example, the recreational benefits of a national forest may be estimated using data on travel time. However, this estimate is biased downward. We know that the benefit must be somewhat greater than the opportunity cost of driving to the forest! Survey data can also be used, but people have no particular reason to answer truthfully (or carefully) to such questions as how much would you be willing to pay to access "this national forest," "to protect this wetland," or to "preserve this species?"

In cases where the benefits and costs are not entirely predictable, the probability of benefits and costs also have to be estimated. The probabilities assigned to the various outcomes also are often difficult to estimate.

Thus, although arguably better than nothing, benefit-cost analysis tends to be quite inaccurate (e.g., estimates of net benefits typically have high variance). So instead of attempting to find the best (social net benefit maximizing) policies, cost benefit analysis often simply attempts to determine whether the benefits of a policy exceed its costs. A policy is said to improves a situation if it generates Benefits greater then its Costs. This is, of course, a normative statement—one based loosely on the utilitarian school of philosophy.

In spite of all these difficulties, benefit-cost analysis has several advantages as method of policy analysis. It forces the consequences of policies to be systematically examined. It provides "ballpark" estimates of the relevant costs and benefits of regulations for everyone who is affected by a new regulation or program.

A Relatively Simple Illustration of an Environmental Cost-Benefit Analysis Suppose that Acme produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Acme saves $5,000,000/year by using this disposal method, rather than one which does not endanger the ground water. What is the present discounted value of Acme's savings (much of which is passed on to consumers) if the interest rate is 10% and Acme expects to use this method for 30 years?

The easiest method is to use the formula

$$\mathbf{P = v\ [(1+r)^T - 1]\ /\ [r\ (1+r)^T]}$$

Here: $P = (5{,}000{,}000)\ [\ ((1+.10)^{30} - 1)]/[(.10)\ (1+.10)^{30}] =$ \$49,574,072.44

One could also approximate the present value of Acme's cost savings using the present value of an infinite series formula (P=F/r) which yields (5,000,000/0.1 = $50,000,000.00. Note that this simpler calculation produces nearly the same answer, and so is often a good way to check one's math.

Suppose that an environmental law is passed which requires firms like Amex to adopt the more costly but safer technology. If the fine assessed is $10,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate (interest rate) is 10% ? The expected fine in a given year has to be greater than the expected cost savings, Thus, P*10,000,000 > 5,000,000 in order for the fine to affect Acme's choice. (In this case the interest rate is not necessary for finding the solution because it is assumed that violations would be detected and fines paid annually. Although, we could also use present values for both the penalties and cost savings.) The smallest probability of punishment that "works" is 0.5, because this makes the expected fine equal to the expected cost savings.

Suppose that administering the enforcement regime costs $1.000,000/year that produces a 0.75 probability of punishment. What is the smallest annual external damage that can justify the program? Given the fine and probability of being caught and punished, we know that this program will induce Acme to clean up, so the only important question is when the present value of the damages (net of administration costs) avoided are greater than the present value of the extra costs borne by Acme (and its consumers).

Intuitively, we can see that if the damage per year (D) less the administrative costs ($1,000,000/year) are greater than the cost imposed then the program is worthwhile in cost-benefit terms. (D - $1,000,000 > $5,000,000). This implies that the damages must be greater than $6,000,000 per year. If the damages vary a bit through time, then we would need to use present and expected values to figure this out.

In that case the present value of the damages avoided minus the present value of the administrative costs would have to be greater than the present value of the cost increase imposed on Acme (and its consumers). If the damages were random, perhaps because rainfall is random, then we would have to compare the expected damage reductions (net of administrative costs) with the cost of "cleaning up."

For example, suppose that on rainy days the "dirty" waste disposal system causes $20,000,000of damages and that on dry days, the "dirty" waste disposal causes no damages to the local ground water supply. Suppose that it rains one third of the time. In this case the expected damages from the "dirty" waste disposal system has expected damages, D^e = (.33) ($20,000,000) + (.67) (0) = $6,666,666 per year.

In this case the cost of eliminating the damage is the cost of the clean up (more expensive waste disposal system) plus the administrative costs ($5,000,000 +$1,000,000) while the benefits are the expected reduction in damages: ($6,666,666 per year). The **expected present value** of the social net benefits from the program over thirty years can be calculated with formula $P^e = v\ [\ ((1+r)^T - 1)/r$

$(1+r)^T]$ given a planning horizon (T) and discount rate (r). Let T= 30 and r = 10% again.

P^e = (\$666,666) $[((1+0.1)^{30} - 1)/(0.10)\ (1+0.1)^{30}]$ =

(\$666,666) (9.4269)

Thus, P^e = \$6,284,603.40

Given all these details, this program will produce a bit more than 6.28 million dollars of expected net benefits over a thirty year period (in present value terms).

VII. Some Practice Exercises

1. Suppose that Al wins the lottery and will receive \$100,000/year for the next twenty five years. What is the present value of his winnings if the interest rate is 6%/year?, 5%/year, 3%/year? How much more would a prize that promised \$100,000/year forever be worth?

2. Suppose that Al can purchase lottery tickets for \$5.00 each and that the probability of winning the lottery is P. If Al wins, he will receive \$100,000 dollars per year for 20 years. The twenty year interest rate is 3%/year.

   What is the highest price that Al will pay for a ticket if he is risk neutral? Determine how Al's willingness to pay for the ticket increases as P, the probability of winning, increases and as the interest rate diminishes.

3. Suppose that Amex produces a waste product that is water soluble and that its current disposal methods endanger the local ground water. Amex saves \$1,000,000/year by using this disposal method rather than one which does not endanger the ground water. What is the present discounted value of this waste disposal technology to Amex if the interest rate is 6%? if it is 4%?

4. Suppose that an environmental law is passed which requires firms like Amex to adopt themore costly but safer technology. If the fine assessed is \$2,000,000, what probability of detection and conviction will Amex adopt the safer technology if its discount rate is 5%? if it is 10% ?

5. Suppose that global warming is caused (at the margin) by CO_2 emissions and that to reduce CO_2 emissions enough to affect future temperatures requires policies that will reduce economic output by 5% per year. U. S. GNP is currently about 15 trillion dollars and is expected to grow by about 2.5% per year in the future. How large do expected damages have to be to justify such an aggressive environmental policy?

   Hint 1: in this case, the future value of GNP is $Yt = 15*(1+.025)^t$, because of economic growth, which works like compound interest. The reduction in non-environmental income in year t is thus $Vt = (.05)15*(1+.025)^t$

   Hint 2: This implies that present values can be calculated using the summation formula P =□ $(Vt/(1+r)^t$ by substituting for $Vt = (.05)\ 15*(1+.025)^t$

   { That is to say, $P = \Sigma\ (\ (.05)\ (15 \text{ trillion})\ (1+0.025)^t/(1+0.05)^t$

   Hint 3: more generally one can write this expression as P = □ $(Vo\ (1+g)^t/(1+r)^t$ where g is the economic growth rate, r is the discount rate (interest rate), and Vo is the initial value of the "thing" that is growing at rate g.

   Hint 4: It turns out that in a present value problem with an infinite planning horizon, one canuse a relatively simple formula to calculate the present values of a series of values that grow by a constant percentage each year:

P = Vo / (r-g) where Vo is the initial value, r is the discount rate (or interest rate) and g is the long term growth rate.)

[**Now you can easily calculate** the present discounted value of the cost of reducing CO_2 emissions in this way, which is approximately 30 trillion dollars, given all the assumptions made.]

Appendix Excel Spreadsheet Calculations of Present Discounted Values and Expected Present Discounted Present Values

Present Value Calculations

Constant Values through time **P = v [(1+r)T- 1] / [r (1+r)T]**

	V	r	t	P	discounted by 4 years in future F/(1+r)4	
	100	0.1	3	248.69		Simple PV problem from end of recorded lecture 9A
Bene-fit						
	10,000.00	0.05	40	171,590.86	141,168.23	value of a college education that increase salary by 10K/year
	20,000.00	0.05	40	343,181.73	282,336.46	value of a college degree that increases salaries by 20K/year
	40,000.00	0.05	40	686,363.45	564,672.91	value of a college degree that increases salaries by 40K/year
Cost						
	24,000.00	0.05	4	85,102.81		cost of 4 year degree at WVU (in state)
	42,000.00	0.05	4	148,929.92		cost of 4-year degree at WVU (out of state)
	20,000.00	0.025	40	502,055.50	454,837.50	value of a college education that increase salary by 5K/year
	40,000.00	0.025	40	1,004,111.00	909,675.01	value of a college degree that increases salaries by 10K/year
	24,000.00	0.025	4	90,287.38		cost of 4 year degree at WVU (in state)
	42,000.00	0.025	4	158,002.92		cost of 4-year degree at WVU (out of state)

Present Values

V	r	t	P	discounted by 4 years in future F/(1+r)4	
10,000.00	0.05	40	171,590.86	141,168.23	value of a college education that increase salary by 10K/year
20,000.00	0.05	40	343,181.73	282,336.46	value of a college degree that increases salaries by 20K/year
40,000.00	0.05	40	686,363.45	564,672.91	value of a college degree that increases salaries by 40K/year
24,000.00	0.05	4	85,102.81		cost of 4 year degree at WVU (in state)
42,000.00	0.05	4	148,929.92		cost of 4-year degree at WVU (out of state)
10,000.00	0.025	40	251,027.75	227,418.75	value of a college education that increase salary by 10K/year
20,000.00	0.025	40	502,055.50	454,837.50	value of a college degree that increases salaries by 20K/year
40,000.00	0.025	40	1,004,111.00	909,675.01	
24,000.00	0.025	4	90,287.38		cost of 4-year degree at WVU (in state)
42,000.00	0.025	4	158,002.92		cost of 4-year degree at WVU (out of state)
30,000.00	0.05	4	106,378.52		opportunity cost wage at 15$/hr (30k/year)
30,000.00	0.025	4	112,859.23		opportunity cost wage at 15$/hr (30k/year)

Expected PV with equal chance of each salary increment (in-state)

Let P1=P2=P3 = 0.33	**PV of Expected Net Profit**	
	134,617.28	**expected net value if each salary increase is equally probably, r**
	322,190.71	**expected value if each salary increase is equally probably, r = 0. state)**

Expected PV lottery ticket with 50K per year winning for 20 years, with a 1/mil chance of winning, and a ticket cost of 1 dollar

v	**r**	**t**	PV of Prize
50,000.00	0.05	20	623,110.52

Expected value of this lottery ticket, given 1/1000000 of winning

-0.377 **expected pv of lottery ticket**