EC 741: Handout 6-- On the Microeconomics of Public Goods Problems and Solutions

I. Introduction: Welfare Economics and Public Policy

A. During the first part of the course, we demonstrated that the NB maximizing model can be used as a normative theory by using SNB as to characterize better and worse public policies.

i. It showed that competitive markets for private goods tend to produce the service levels that maximize social net benefits (SNB).

ii. It also showed that this conclusion does not apply when there are externalities (uninternalized marginal costs and benefits).

a. When positive externalities exist, services tend to be under-provided.
- Public goods, as seen below, tend to have positive externalities associated with them.

b. When negative externalities exist, services tend to be over produced.
- Production processes that produce pollution tend to have negative externalities.

B. The second part of the course examined various productive and extractive theories of the state.

i. Rational choice - based analysis of governments [the Public Choice, Rational Politics, and New Political Economy approaches] implies that normative arguments may not determine actual policies.

ii. Rather public policies are determined by political institutions that affect how interests influence public policies.

iii. These approaches partly explain why public policies to address externality and public goods problems tend to be suboptimal from the perspective of the most widely used normative theories.

iv. Its important to remember--which most text books do not--that the existing policies to regulate or otherwise address externality, public goods, and monopoly problems are all products of public policy.

C. The latter suggests that to improve public policy (using any normative theory that you want) may require changing institutions in some way--in order to change the political equilibrium.

i. As we will see later in the course, the net benefit maximizing approach can also be used to analyze institutional choice.

ii. That is not to say that one cannot critically examine public policies one at a time.

iii. It is both necessary and useful for citizen-voters and scholars to learn a bit about what the issues are and how to systematically evaluate them.

a. Note that in such cases, economists may become an interest group that advocates SNB improving policies.

b. Sometimes, such efforts are successful, although the coalitions that get reforms adopted are never composed of entirely economists or utillitarians.

c. Many of the groups that caused industries to be deregulated and various price controls to be eliminated included politically active economists.

D. To become familiar with the policy and institutional issues the next part of the course examines the relative merits of alternative institutions from the SNB perspective, the Paretian perspective, and others that are widely used for particular policy areas. We will take up institutional reforms towards the end of the block on policy choices.

II. An Overview of the Demand for Government Services

A. Many public finance courses and textbooks begin with an analysis of taxation.

i. This approach is useful if the government is considered to be a revenue maximizing leviathan and also for some pedagogic reasons (the tools used to analyze taxation are simpler than those used to analyze the demand for government services).

ii. Tax theory, however, is a less useful if the government is considered to be a more or less representative one, because in that case **the demand for services comes before** the demand for revenues. In such cases, taxes are "demanded" by voters as a means

of financing government services or addressing externality problems.

iii. We take up some issues in taxation later in this lecture and the more traditional analysis of optimal taxation after exploring why voters and other groups may favor of policies that address public goods and externality problems.

a. Both public goods and externality problems can be addressed through "proper" tax systems.

b. In addition, both sorts of problems can be addressed through various forms of regulation may also be applied.

c. Both tax and regulatory policies also require enforcement procedures, although we will not have much time to consider them in this course.

B. Economics provides a variety of explanations for the demand for government services.

i. An individual's demand for government services in most cases can be represented in the same way as his or her demand for other public services

a. That is consumer-taxpayers demand government services because they increase their net benefits (personal utility).

- When deciding how much of a government service they demand, price (opportunity cost) will be taken into account.

b. However, the actual "purchase" of government services is not often under the control of a single individual, but rather the outcome of elections and legislation.

ii. Cost considerations (for a majority) also are important consideration in whether particular service are produced or regulated by government or by the private sector.

a. Most services can be produced privately as well as by a government.

b. If consumers-voters have choices among alternative producers of a service they will normally choose to purchase from the organization offering the lowest prices.

- So it is likely that consumer-voters demand particular government services because they receive those services at better prices or better service levels than available in markets.
- This price depends partly on the tax system in the case of government services.

c. It turns out that there are several cases in which the government may be the least cost provider of a services which exhibit economies of scale and so are prone to natural monopolies in private markets.

iii. Within democracies the particular instituions of government will determin how voter demands affect public policies.

iv. Under dictatorship or leviathon governments the services provided will be those that somehow advance the ruler's interest.

- Extractive states may monopolize services because it can use monopoly profits as a source of income (McChesney 1987, Congleton and Lee 2008

C. In many cases, desired services are not efficiently or economically provided by firms that operate in normal market conditions.

i. Common examples include pure public goods and other services which are produced under fairly extreme economies of scale, as developed later in this set of notes.

a. The properties of pure public goods imply that they will often be under provided in private markets.

b. The government can use its coercive power to solve various free rider problems and/or to finance provision of services demanded by the median voter.

c. In the case of natural monopolies, services may be provided at a higher price in markets than by a representative government.

- This might be said, for example, of national defense, law enforcement, highways, fire protection, water sanitation, and standards.

d. Such cases are often said to be a **market failure**, but this term often seems too strong for many of the situations being described.

e. In the case of extractive states, a government may monopolize services because it can use monopoly profits as a source of income (McChesney 1987, Congleton and Lee 2008

ii. It bears noting that fiscal illusion may cause some services to be provided publicly that are actually cheaper when provided privately.

a. Mistakes can be made by voters, even in a group.

b. Tax contributions and/or other costs may be underestimated for some of the services.

iii. It also bears noting that an "extractive" state may monopolize services that would otherwise be competitive markets, simply so that it can charge a higher price.

a. Programs that are extractive (e.g. produce rents for a special interest group) may similarly cost persons who are not in the group (often, consumers) more than they generate as benefits for the favored groups (producers of products or services in "protected" markets). Such tends to be the case for sugar tariffs.

D. We begin by considering public goods problems and why governments (or clubs) may be asked to address such problems.

i. The economics of "market failure," can be used to provide an efficiency rationale for taxpayer demand for government services.

ii. For the most part, we will use **utilitarian** (social net-benefit maximizing) normative theory today both as a source of **normative** "efficiency" claims and also as part of a **contractarian rational for voluntary collective action** concerning the problems identified.

iii. Normative and positive theories of redistribution and social insurance provide rationales for other policies such as social insurance and redistribution. These will be taken up later in the course.

III. In What Sense are Pure Public Goods Under Provided?

A. Some useful definitions:

i. **DEF**: A **pure public good** is a good that is perfectly sharable. A pure public good can be simultaneously consumed by "as many people as want" simultaneously. Examples include gravity, national defense, the air (quality), and gravity.

ii. **DEF** A **pure private good** is a good that cannot be shared without proportionately reducing everyone's consumption of the good.

a. Essentially, a pure private good can only be consumed by only one person at a time.

b. Examples include a jelly bean, a pair of shoes, a shirt, a hat, or a nap. (Most economic analysis assumes that goods and services are pure private goods.)

iii. **DEF** A **club good** is a good that is sharable within limits. A club good can be shared by several people, but the "quality" of the consumption falls with the number of people sharing the good, although less than proportionately. On

a. Club goods are "congestable," the benefits that a person recieves from the good falls as the number of users increases.

b. Examples include this lecture, the highways, swimming pools, parking lots, parks, etc.

iv. Note that these definitions imply that a **spectrum of good types** can be defined based on the degree of "shareability."

a. Pure public goods are at one end of this spectrum and pure private goods at the other. In between are various sorts of "club goods" which are shareable to some extent, but not perfectly so.

b. Note that this spectrum does not require "excludability" which is often included in definitions of club and private goods.

c. Excludability makes a good more likely to be available under private provision, but does not affect its shareability.

d. Excludability does, however, affect whether it will be shared or not.

B. One explanation for the under provision of public goods is the **"free rider problem."**

i. A relatively **extreme form** of the free-rider problem can be illustrated with a 2x2 game matrix.

- Other continuous geometric and mathematical representations of free rider problems as developed later in these notes.

The Free Rider Problem

Contributions to Providing a Pure Public Good

	Bob: Contributes	Bob: Free Rides
Al: Contributes	A, B 3, 3	A, B 1, 4
Al: Free Rides	4, 1	2, 2

(Individual payoffs are in (ordinal) utility terms. Note that "rank order" is sufficient to illustrate the essential logic of this social dilemma.

ii. Note that both Bob and Al are better off if the public good is produced (3>2), yet each prefers that the other person provide it (4>3).

- The payoffs to such social settings resemble those of a "prisoner's dilemma" game.

iii. If each person independently chooses his benefit maximizing strategy, each will choose to free ride.

a. Note that if Bob contributes, Al gets 3 if he/she also contributes, but gets 4 if he/she free rides.

- Note that if Bob free rides, Al gets 1 if he/she contributes, but gets 2 he/she free rides.
- Thus, **regardless of what Bob does, Al is best off if he/she free rides**.

b. Note that essentially the same logic applies to Bob's choice of strategy.

- The same logic applies to public goods settings in which there are many persons who must contribute in order to produce the service.

iv. The **Nash equilibrium** of the free-rider game is **mutual free riding**.

- The result is individually rational, **but each person would be better off if the public good is produced. (3>2)**
- Privately optimal behavior in this setting, leads to an outcome that is agreed by each to be worse than the case in which they both contribute to providing the public good.
- (3, 3) is Pareto superior to (2, 2).
- (Note that this provides a reason to expect collective action to attempt to solve the problem, and implies that resources might be available to construct institutions that can address it. why?)

C. A similar, but often less severe, free-riding occurs in continuous representations of the production of pure public goods.

i. Suppose that G is a pure public good and X is a pure private good and that Al and Bob have similar utility functions and budget constraints, $U^A = u(G, X^A)$ and $W^A = G + PX^A$

ii. Al's utility maximizing quantity of the pure public good can be found by substituting for XA in her utility function, $X^A = (W^A - G)/P$ and for G with $G = G^A + G^B$

- Recall that if G is a pure public good, each person gain full benefits from the other's purchase of the good.
- Which yields: $U^A = u(G^A + G^B, (W^A - G)/P)$

iii. Differentiating with respect to G yields Al's ideal purchase of G:

- $U^A{}_G = u_G - u_X/P = 0$ at G^{A*}

a. Which implies that $G^{A*} = g(G^B, W^A, P)$

b. (This equation is A's demand for the public good, and also her "best reply function" for the free rider public goods game)

iv. The mathematics for Bob yields a similar result:

$G^{B*} = g(G^A, W^B, P)$

v. At the Nash equilibrium:

$G^{B*} = g(G^{A*}, W^B, P)$

and $G^{B*} = g(G^{A*}, W^B, P)$

vi. The **Pareto efficient level G** can be characterized using a Benthamite social welfare function (and other similar methods).

- $W = U^A + U^B$

a. Differentiating with respect to G^A and G^B yields the first order conditions that describe the Pareto efficient levels of G^A and G^B

- $W_{GA} = u^A{}_G - u_X/P + u^B{}_G$
- $W_{GA} = u^B{}_G - u_X/P + u^A{}_G$

b. Note that **these two first order conditions describe functions that are "outside" of Al and Bob's best reply functions**, because they each require the external benefits of the other person to be taken account of.

- [Draw the figure that illustrates this conclusion.]
- Thus, the Nash equilibrium of this continuous version of the free rider game is Pareto sub optimal.
- Too little of the public good is purchased by each.
- However, some of the public good does get privately produced!

vii. Because provision of pure public goods tends to generate positive externalities (analyzed in the next block of lecture material) pure public goods tend to be under provided in the absence of some kind of collective action (which is not necessarily governmental provision of the service).

IV. Solutions to Public Goods Problems

A. There are a variety of solutions to public goods problem, including explicit coordination among those free riding, the formation of "public goods" clubs, private contracts (agreements) to contribute to produce the public good, and government action.

i. In small number cases like that of the 2x2, the persons affected may form a small club (and perhaps hire a manager) to solve the problem.

ii. In large number settings it will of ten be cheaper to use the government that to form a new club for this purpose.

V. Pareto Optimal Provision of Pure Public Goods

A. Within democracies, many government services can be understood as attempts to solve various free rider problems.

i. In some cases, it will be easier for a group to take over production of a public good rather than to provide the proper Pigovian subsidies to encourage sufficient production.

ii. This may be done privately through clubs, or publicly through governments of one kind or another.

iii. In such ideal cases, **government can be thought of as a special kind of club with the power to tax**.

- (Such clubs are good examples of what would be justified under contractarian theories of the state.)

B. Samuelson in a classic 1954 paper on the optimal (utilitarian) supply and financing of a pure public good characterizes the ideal service level for a pure public goods and the ideal methods that can be used to finance the "collective" production of pure public goods.

C. In general terms, Samuelson's paper suggests that:

i. The government should provide public goods at a Pareto optimal level, or, equivalently, at the level that maximizes social welfare or social net benefits.

a. It bears keeping in mind that there are often more than one Pareto efficient outcome.

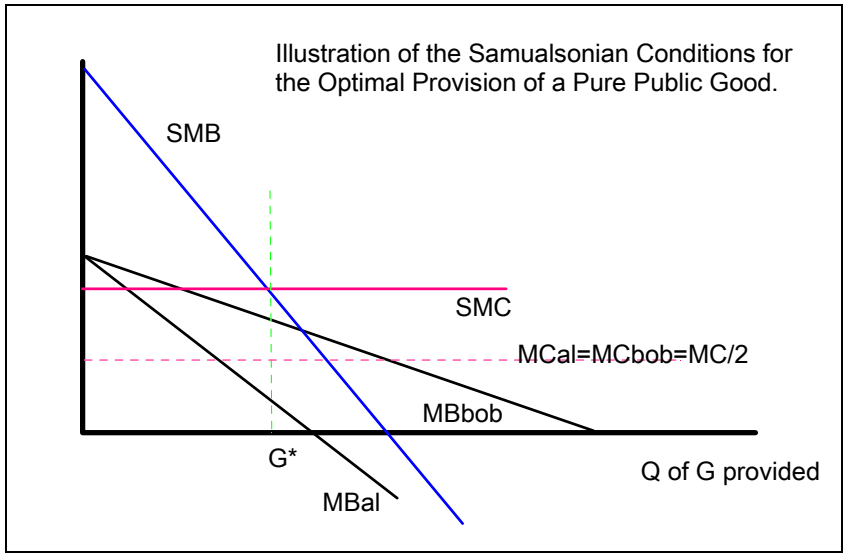
b. In cases in which more than a single Pareto efficient level occurs, social welfare functions will "recommend" that a particular Pareto efficient level be produced.

ii. The ideal output level of a pure public good is independent of the social welfare function used. It occurs at the output where the sum of the marginal benefits realized by users of the public good equals the marginal cost of producing them.

iii. The ideal tax system does not impose a deadweight loss (e. g. they should be broad-based or lump sum taxes)

iv. The tax system should raise just sufficient revenue to cover the cost of the Pareto efficient level of public services.

v. (These three conditions, optimal production of government services financed by an efficient tax system are sometimes called the **Samuelsonian conditions for the optimal provision of a pure public good**.)



D. The essential mathematics of the Samuelsonian characterization of the Pareto optimal collective provision of a pure public good.

i. First we need some private choice notation: Let G be the level of a pure public good, let Xi be the level of a pure private good received by individual *i*, let Ui = u(G, Xi) be the utility of individual *i* associated with a particular combination of the public good and private good received by *i*

ii. Second, we need some macro-choice notation. Let W be a social welfare function and let T(G,X) = 0 be the technological frontier of combinations of the public good and private goods, with X = ΣXi.. Suppose there are N persons in the society of interest.

iii. The problem of maximizing social welfare can be written as a Lagrangian:

- max $\ell = W(U_1, U_2, U_3 .... U_N) - \lambda (T(G,X))$

iv. Differentiating the Lagrangian with respect to G, X_1, X_2, X_3 .... X_N, and λ yields the first order condition for the social welfare maximizing level of G and for the distribution of private goods--which we will ignore for the purposes of this derivation.

- $\Sigma W_{Ui} Ui_G = \lambda T_G$
- $W_{Ui} Ui_X = \lambda T_X$ for all i = 1 ... N (This represents N equations)
- T(G,X) = 0

v. After obtaining the Lagrangians first order conditions, the next step is to manipulate the first order conditions into a form that is both economically interesting and useful. Samuelson uses a rather clever series of steps to do so.

a. First, divide the first and second condition to eliminate the lamda.

- $[\Sigma W_{Ui} Ui_G] / W_{Uj} Uj_X = T_G / T_X$

- (I have used the jth of the foc's for the private good to avoid confusion with "i" the counter for the summation in the numerator)

b. Since the denominator does not change with "i" it can be brought inside the brackets--because it is essentially a constant as far as this fraction is concerned.

- $[\Sigma\ W_{Ui}\ Ui_G\ /\ W_{Uj}\ Uj_X\ ] = T_G\ /\ T_X$
- Now note that the foc's for the private goods imply that
  $W_{Ui}\ Ui_X = W_{Uj}\ Uj_X$
- This condition holds for all "i" and "j" (for every person's private good).

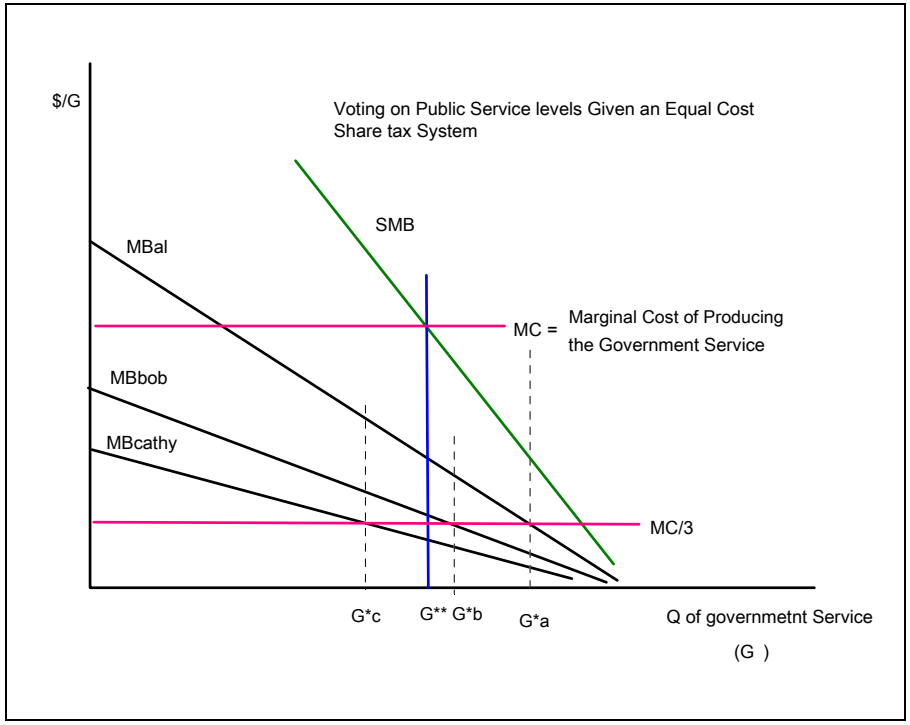
c. This equivalence means that you can rewrite the equation under part b as:

- $[\Sigma\ W_{Ui}\ Ui_G\ /\ W_{Ui}Ui_X\ ] = T_G\ /\ T_X$
- by substituting the various "i-terms" for the "j" term that we started with.
- This allows us to simplify a bit:
- $\Sigma\ [Ui_G\ /\ Ui_X\ ] = T_G\ /\ T_X$

d. The ideal level of a pure public good will set the **sum of the marginal rates of substitution between the private and public good equal to the technological rate of transformation** between them.

- (In the diagram above, this condition is represented by setting the sum of the marginal evaluation curves equal to the marginal cost of the pure public good.)

vi. Note that **the optimal provision of a pure public good is completely independent of the social welfare function** used.



VI. Lindahl Taxes as an Important Special Case of the Samuelsonian Solution.

A. There are a variety of practical problems with this characterization of the Pareto optimal provision of a pure public good, but the political one is of particular interest for the purposes of this course.

a. Most Pareto efficient provisions of a public good make individual tax payers "unhappy" with the amount of the public service provided, given their tax costs.

b. Moreover, most tax systems for financing Pareto efficient levels of a public service yield majorities that prefer non-Pareto to Pareto levels of the service.

c. In the figure below, the median voter outcome is G*b (Bob's ideal point under that tax system) rather than G** (the SNB maximizing service level).

B. There is, however, a special case of the Samuelsonian Solution that avoids this problem, namely the **Lindahl tax** system.

- (Lindahl, a Swedish economist and student of Wicksell, surprisingly figured out this solution decades before Samuelson figured out how to characterize the Pareto efficient level of a pure public good.)

C. Lindahl adds another condition to the three Samuelsonian conditions for Pareto efficient provision of a pure public good.

a. **Lindahl** suggests that the *taxes used to finance public services should* ***equate marginal benefits and marginal costs for individuals at the desired output*** **of government services**.

- Lindahl taxes are, thus, said to be idealized **benefit taxes**.

i. They can also be applied to finance the Pareto efficient level of a pure public service, G*, characterized by Samuelson..

ii. **Under Lindahl taxation, everyone in the society of interest is prefers the Pareto optimal level of public goods to all others**.

- [See the figure below for an illustration of Lindahl taxation.]

a. Note that under a perfect benefit tax of this sort, each person "demands" the same output of the pure public good, namely G**.

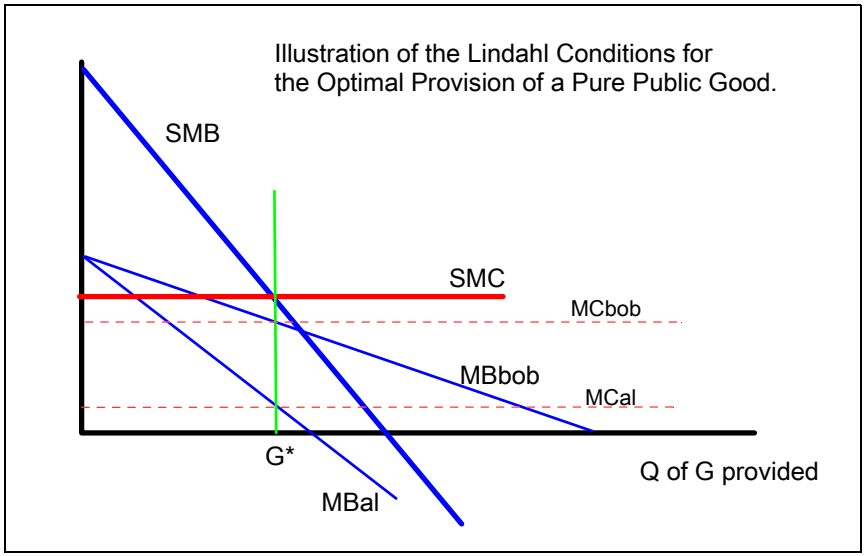
b. This contrasts with the less restricted Samuelsonian case, in which persons are very likely to disagree about the best service level to provide!

- (Consider for example the special case in which the cost of the service is shared equally among three persons with different marginal benefit curves.)
- Those whose marginal tax cost are below their marginal benefits from the service will demand more, whereas those whose marginal tax cost is above their marginal benefits will want less!

iii. Lindahl taxes would induce unanimous agreement about the level of a pure public good to be provided. This implies that sincere voting to solve public goods problems that are financed via Lindahl taxes will produce Pareto efficient output levels of the pure public good.

a. See the figure below.

b. This would, of course, require providing institutions that assure that taxes are paid. Free riding on contributions to the public good remain rational even under Lindahl taxes.]



VII. An Illustrative (and Feasible) Lindahl Tax System for the case in which residents have similar Cobb-Douglas utility functions.

A. There is **an interesting special case of Lindahl taxation** that arises when all persons on the polity have identical tastes that can be represented with a Cobb-Douglas utility function, but they have different wealth endowments.

i. Let G be the pure public good and C be private consumption and assume that "t" is the price of the pure public good and "P" is the price of the pure private good.

ii. In this case, "Mr/Ms i" maximizes $U_i = C_i^a G^{1-a}$ subject to $W_i = tG + PC_i$

B. Mr/Ms i's demand for the government services given this tax-price system for financing the public service can be derived by (i) forming a Lagrangian (which works well for C-D functions), (ii) differentiating with respect to C_i, G, and λ, and (iii) doing some clever algebra.

i. $\mathcal{L} = C_i^a G^{1-a} + \lambda (W_i - tG - PC_i)$

ii. Differentiating with respect to C_i, G, and λ, setting the results equal to zero, and some simple algebra yields:

a. $(1-a)C_i^a G^{-a} = \lambda t$

b. $aC_i^{a-1} G^{1-a} = \lambda P$

c. $W_i = tG - PC_i$

iii. Some more "easy" algebra on these three first order conditions allows Mr/Ms i's demand for G to be characterized.

a. Dividing "a" by "b" yields: $[(1-a)/a]\ [C^*/G^*] = t/P$

b. Solving for C_i^* yields: $C_i^* = G^*\ (t/P)\ [a/(1-a)]$

c. Substituting for C in the budget constraint: $W = tG^* - PC_i^*$ yields $W_i = tG^* - P\ G^*\ (t/P)\ [a/(1-a)]$

d. Simplifyng and solving for G_i^* yields $G_i^* = (1-a)W_i/t$

- As normally the case with C-D utility functions, each person spends a **particular fraction of their wealth** on the goods of interest, here (1-a), and the amount purchased varies with the price, here (t).
- In the special case of interest here, everyone spends the same fraction of their wealth on each type of good.

iv. A Lindahl tax system has the property that it induces each person to demand the same quantity of the public good. (See the diagrams above.)

a. We can use this property to characterize the Lindahl tax prices for this polity.

b. Each person pays a different price under a Lindahl system, here t_i.

c. Those taxes induce each to purchase the same quantity of goods so for persons "i" and "j," $Gi^* = Gj^*$ at their respective Lindahl taxes.

d. Assume that G** is the Pareto optimal quantity of the pure public good and that "t" satisfies the Samuelsonian condition and, so, is just sufficient to pay for the pure public good [that is: $\Sigma tiG^{**}=c(G^{**})$].

- In this case the Lindahl taxes satisfy: $(1-a)Wi/ti = (1-a)Wj/tj$
- and $ti/tj = Wi/Wj$
- That is to say if "i" has twice as much wealth as "j", i's marginal tax price should be twice as high as j's.

e. **In this special case [identical Cobb-Douglas tastes and balanced budgets], a progressive tax that has (marginal) tax prices equal to relative wealth (or income) is a Lindahl tax system.**

v. Notice that a **proportional tax on wealth** sufficient to pay for the services provided has this property if preferences are C-D and fundamentally similar.

- Let $\Sigma t_i Wi=c(G^{**})]$, with equal wealth taxes $t = ti = tj$
- This implies that proportional wealth tax rate is $t = c(G^{**})/\Sigma Wi$
- So the marginal tax cost of G** for Mr j is
- $c(G^{**})\ [Wj\ /\Sigma Wi\ ]$
- differentiating by G gives the marginal tax cost for j
- $[dC/dG]\ [Wj\ /\Sigma Wi\ ]$
- Note that if one divide's Ms i's marginal tax cost by Mr j's tax cost one gets Wj/Wi because all the other terms are the same.

C. Of course, preferences differ and are not likely to all take the form of a Cobb-Douglas utility functions, and this tax financing scheme might not be sufficient to pay for the pure public service.

i. However, it is important to note that Lindahl taxation in such "easy circumstances" does not require determining each person's unique marginal benefits schedule (or marginal rate of substitution).

   a. Note that many communities in the past used flat real estate taxes to finance public services. If most people hold their wealth as real estate, this implies that there should have been more consensus about the level of services in such communities under that tax system. Was there?

   b. If there is not greater consensus, this would suggest that taste differences were important or wealth is not principly in real estate. People of similar income may have quite different demands for bicycle paths, day care centers, parks etc.. Explain.

ii. Proportional wealth taxes would not be a Lindahl tax system if there are differences in tastes and economies, diminishing marginal utility of income, and/or diseconomies of scale in producing government services.