I. Positive and Normative Theories of Taxation

A. Given that voters (or dictators) have interests in government services--the provision or subsidization of public goods such as the provision of security (national defense, law and order), solving significant externality problems, and social insurance--some means of paying for them is necessary.

i. We have already spent some time discussing taxation.

ii. Tax burden and deadweight loss calculation were reviewed in the second set of notes

iii. The Samuelson and Lindahl characterization of optimal levels of a pure public good also provide ideas about how to fund public services.

a. These ideas were based in part on conventional normative theories of taxation.

b. Both suggested broad based neutral taxes or lump sum tax systems applied in a particular way to fund the services provided.

iv. The Pigovian tax solutions to externality problems also link tax revenues to services.

a. The taxes are likely to be sufficient to fund the enforcement apparatus necessary to impose the taxes and may well generate a surplus that can be used to fund other services.

b. We noted that Pigovian taxes do not have a conventional (partial equilibrium) dead weight loss associated with them.

c. (However, excessive taxation of externalities or externality generating activities, would have a conventional deadweight loss.)

v. The cap and trade system can also produce revenue when auctions of effluent rights (or other similar rights, hunting lisences etc) are used.

B. So we have already spent quite a bit of time discussing taxation and revenues, although we have not until today focused on taxes per se--which is a core topic in Public Finance.

i. This handout focuses on more or less convention micro-economic analysis of taxation.

ii. There are both positive and normative theories of taxation--choices among tax systems are based partly on their positive effects and partly on their normative ones.

- This is true both for the purposes of theory (welfare economics) and for thinking about the behavior of voters and their elective representatives.

iii. All taxes are consequences of choices of one kind or another, and those choices can be modeled.

iv. For the most part, taxes are imposed as a method of generating revenue, although some taxes (Pigovian taxes) are assessed to change incentives at the margin.

C. Positive theories of taxation attempt to model the choice of tax systems (which are often quasi-constitutional choices) and tax rates within a given system of taxation.

i. Many of these choices will be affected by the effects of particular taxes, and so two levels of positive analysis are necessary:

a. The direct impact of a tax has to be assessed, which includes the revenues generated in the short and long run, and also relevant economic and political affects of the tax of interest.

b. The relevant economic and political effects varies with the institutional setting in which the tax decisions are made. Which effects are taken account of and their relative importance (salience) varies with the institutionally induced aspects of policymaker preferences.

- See for example: Hettich and Winer (1984, 1989), Buchanan (1967), Buchanan and Brennan (1977, 1980), Feldstein (1976).

c. Non-pragmatic aspects of taxation are also relevant for positive analysis insofar as policymakers are directly or indirectly (e.g. through voter preferences) influenced by such preferences.

D. With respect to normative theories,

i. taxes can be ranked in various ways, according to the normative and pragmatic objectives of the person conducting the analysis.

ii. From the perspective of a revenue maximizing dictator (leviathan), tax systems and tax rates can be ranked according to short, medium term, and long term revenues.

iii. From the perspective of voters, tax systems can be ranked by their burdens of taxation (and their normative appeal), given the value of the public services desired.

iv. Various philosophical perspectives can also be used to assess the relative merits of tax systems and tax rates. The most common of these are based on utilitarian and Paretian normative theories, although others involving equity (fairness), rights, and coercion are also occasionally used.

E. The mainstream normative theories used by economists ("welfare economics") has produced three broadly accepted norms about tax systems, which are mostly, but not entirely, self-consistent:

i. Tax systems should be broad based.
- Such tax systems as income taxes, sales taxes, and value-added taxes (VATs) are broad based.

ii. Tax systems should be neutral.
- A neutral tax system does not have direct effects on relative prices.
- Such tax systems are normally broad based, do not have loopholes (e.g. narrow exemptions), and treat all "base holders" similarly.

iii. Tax systems should minimize the total burden of the tax system.
- Tax burden is minimized through the choice of tax systems and through choice of tax rates.
- For example, if one uses an excise tax system, one can choose rates to minimize the sum of the excess burdens across markets ((Ramsay taxation [1927]).
- Under a Ramsay tax system, the most inelastic markets will be taxed at the highest rates and the most elastic markets at the lowest rates. [Why? Show Geometrically and/or Mathematically]

F. In addition to these widely accepted norms (principles of optimal taxation), there are many other less widely-held normative conclusions:

i. Utilitarians (which include most economists) argue that taxes should be allocated in a manner that maximizes a social welfare function (often of the Bentham variety).

a. Utilitarian systems will generally be progressive.
- If there is diminishing marginal utility of income, equal marginal sacrifice of utility will require larger tax payments by relatively wealthy persons.
- This is sometimes referred to as taxation according to "ability to pay."

b. Sophisticated utilitarian analysis takes account of incentive effects of taxation in the long run, whereas "simple-minded" analysis does not.
- Utilitarian normative theories of taxation are so widely used that "**Optimal Tax Theory**" can be said to be **broadly utilitarian**.
- (There are, however, other less widely used normative theories of taxation.)

ii. Contractarians argue that an ideal tax system is one that everyone within a given society is willing to accept. Determining the ideal tax system in this sense normally requires analyzing the entire fiscal system (expenditures and taxation) simultaneously.

a. As an analytical device contractarians often use the "veil of ignorance" or the "veil of uncertainty."

b. Rawls, for example, argues that such a fiscal system will maximize the welfare of the least advantaged, because of fairly extreme risk aversion on the part of the members of society who will make the decision.

c. Buchanan, on the other hand, suggests that one should take the status quo system as given and negotiate improvements in the fiscal system. These negotiations will necessarily take place from behind a "veil of uncertainty" because future events are so difficult to forecast.

d. In both cases, uncertainty is presumed to make agreements easier to negotiate..

iii. Perhaps surprisingly, given the differences in approach, the conclusions of utilitarian and contractarian analysis are fairly similar.

a. They often support the three principles mentioned above.

b. Many distributional conclusions are also similar (with the "appropriate" assumptions about weights in social welfare functions and risk averson among taxpayer-voters).

c. There are, however, many points of contention.

d. Buchanan, for example, argues contractarian theory is in principle operational.

e. A Pareto superior move will have essentially unanimous support, essentially by definition (although there may be strategic reasons for voter-taxpayers to lie about whether one is made better or worse off by a given reform).
   - The preference revelation problem is also faced by utilitarians.
   - The demand revealing tax (Clarke tax) is said to induce people to reveal their true valuations (Tideman and Tullock 1976).
   - Note that Buchanan's approach implies that normative questions can be answered empirically--if one accepts a particular class of normative theories.

iv. Other theorists use national income accounts to approximate average welfare and argue that taxes should be allocated in a manner that maximizes the long run growth of per capita national income.

a. (Maximizing median or total income are also occasionally mentioned.)

b. Such persons often favor consumption taxes in order to encourage saving and investment.

c. The effects of a consumption tax on investment can be illustrated with indifference curves and budget constraints, but the intuition behind the effect is based on simple supply and demand analysis.
   - If the price of saving falls relative to consumption, individuals will consume less and save more. And if savings increase, capital will be more rapidly accumulated, which leads to higher income levels and growth rates.
   - A consumption tax is thus not neutral.
   - [However, indifference curve analysis demonstrates that consumption taxes are more neutral than their proponents believe. Show this.]

v. There are also a variety of normative principles of fairness, horizontal and vertical equity, that are often mentioned in discussions of tax systems and tax rates.

a. A tax system is said to exhibits **horizontal equity**, if all persons holding a given level of the taxable base are treated in the same manner.
   - See, for example, Atkinson (1979) or Musgrave (1990).

b. A tax system is said to exhibit vertical equity, if all persons, regardless of their holding of the taxable base, are treated equally.
   - See for example McDaniel and Repetti (1992) or Mooney and Jan (1997).

c. Buchanan and Congleton (1998) argue that political stability and efficiency (within democracies) is enhanced when all persons are treated equally on both the expenditure and taxation side.

vi. There are also rights and liberty based theories of taxation--a proper tax has been adopted through proper procedures, respects existing rights, and is implemented in a relatively non-coercive manner.

vii. Moreover, tax systems and tax rates can be assessed by their political effects. Some taxes may produce "better" political decisions than others.

a. For example, some systems are less transparent or more complex than others, and so are more likely to mislead consumer-voters about the marginal cost of government programs.
   - See, for example, Wagner (1976).

b. In addition, some tax systems are easier to administer and/or less subject to corruption than others.

c. Such tax systems may be less than optimal under economic norms, but may improve the quality of political decisionmaking sufficiently (reduce voter mistakes or regrets) to offset their economic disadvantages.

G. The rest of today's lecture will explore in more depth and rigor a subset of the above normative theories of taxation.

II. A Review of the Geometric and Algebraic Foundations of the Principles of Broad-Based Neutral Taxation

A. Recall that the behavioral affects of an excise tax can be analyzed both with demand and supply curves and with indifference curves and budget constraints, as we saw in lecture 2.

B. We can use some of the diagrams from that lecture to illustrate the geometric, normative, case for neutral-broad-based taxation.

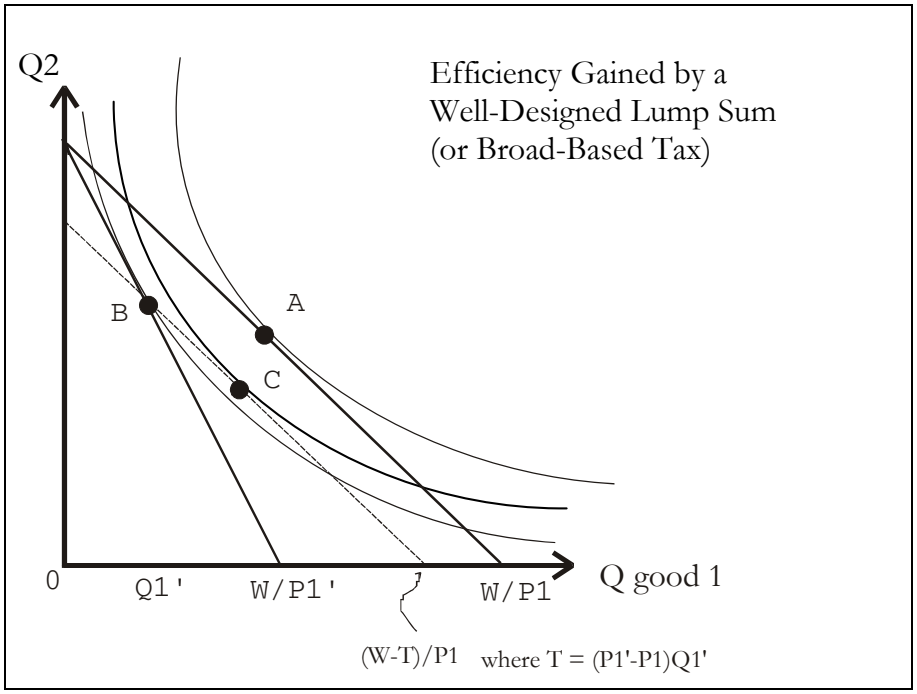
i. Suppose that there are two goods, 1 and 2, both of which the consumer normally uses.

ii. The effect of an excise tax on good 1 is to increase the price of the taxed good from P* to Pc .

a. [Show this using supply and demand or algebraic examples].

b. This increase in price affects the location of each consumer's budget set.

c. It rotates the budget constraint from the untaxed end of the budget constraint and generates a new budget constraint that lies inside the original one at all points where the consumer purchases positive quantities of the taxed good.



iii. Suppose that "A" is the original bundle consumed by this consumer.

a. In this drawing the tax has increased price of good 1 from P1 to P1' (this price effect is taken from a supply and demand diagram)

- In the case drawn, the new higher price causes the consumer to purchase bundle B instead of A. (Indeed, A is no longer feasible.)
- Note that had the same revenue been generated from a lump sum tax equal to Q1'P1'

b. Such a tax, would have allowed the individual to purchase a bundle like C which is on a higher indifference curve (not drawn) than bundle B.

- This loss in utility (from being on a lower indifference curve) is another measure of the excess burden of a non-neutral tax on consumers.

c. Again, much of the excess burden (deadweight loss) is a consequence of reduction in purchases of the taxed good, particularly that part which was generated by the "relative price" effect of the excise tax.

- You learned in micro economics that every price increase has both a (relative price) substitution effect and a wealth effect on purchases of the good whose price has increased.
- Similarly every excise tax that affects consumer prices has both a (relative price) substitution effect and an income effect on purchases of the good whose price has increased because of a tax.

C. The behavioral effect of a general tax and a lump sum tax tends to be smaller than that of an excise tax, because these taxes have only an income (wealth) effect.

i. A revenue neutral lump sum tax, a (neutral) general sales tax, and an income tax all shift each consumer's budget constraint towards the origin, but **these taxes do not affect the slope** of the consumer's budget constraint.

- Consequently, general taxes and lump sum taxes tend to have a smaller effect on behavior than excise taxes that raise the same amount of revenue. (There is no "substitution effect.")

ii. To see this, *calculate the slope of the budget lines for lump sum, sales and income taxes*.

a. Recall that slope is "rise over run."

b. In the case without taxes, the slope of the budget line is -(W/P2) / (W/P1), which simplifies to - P1/P2.

c. In the case of a lump sum tax, the endpoints of the new budget line are (W-T)/P1 and (W-T)/P2. The slope of the new budget constraint is -[(W-T)/P2]/[(W-T)/P1] which equals - P1/P2. (Show this algebraically.)

iii. In the case of a an income tax, where W is treated as income, the after tax income is (1-t)W, so the endpoints of the new budget line are ((1-t)W)/P1 and ((1-t)W)/P2.

- The slope of the new budget line is:
  $-[(1-t)W)/P1]/[((1-t)W)/P2] = -P1/P2.$

iv. In the case of a general sales tax the new after tax prices will be approximately (1+t)P1 and (1+t)P2.
- (What assumptions about supply and demand are sufficient for this to be exactly true?)
- The slope of the new budget line will be $-[(W)/(1+t)P2]/[((1-t)W)/(1+t)P1]$, which again can be shown to equal $-P1/P2$.

a. All three of these taxes are "neutral" with respect to the choice illustrated in our diagram.

v. None of these taxes change the relative prices of goods 1 and 2.
- The slope of the budget line remains -P1/P2 in each case..
- This is the geometry behind broad-based neutral taxation.

D. The geometric case is fairly week, insofar as the excess burden of non-neutral taxes is geometrically pretty small!

i. The neutral tax system required to produce a Pareto superior move from a non-neutral tax requires a lot information. [why?]
- A Pareto reform of an excise tax system is very likely to violate widely held norms of horizontal equity. [why?]

ii. It also bears noting that in some cases, the purposes of some taxes it to **change behavior**, as with Pigovian taxation.
- In such cases, excise taxes and other "marginal" taxes will be more effective at altering behavior than lump sum or general taxes.

iii. [**It also bears noting, however, that no tax can be completely neutral, because taxes can affect locational choices of firms and consumers.**]

III. Another influential normative theory of taxation was proposed by Frank Ramsay in 1927.

A. When we examined the burden of taxation with supply and demand curves, in most cases it turned out that the burden of taxation (reduction in consumer surplus and profits) was larger than the tax revenue collected.
- The difference is called either the deadweight loss of taxation or the excess burden of a tax.

B. Ramsay argued that a system of excise taxes should attempt to **minimize the total excess burden** of the tax system.

i. **A Ramsay tax** system, thus, imposes higher tax rates on markets with relatively inelastic supply and demand curves, and relatively lower tax rates on markets with relatively large price sensitivities.

ii. The Ramsay tax can be analyzed using the diagrams worked out in previous lectures that show excess tax burdens.

a. Note that if markets with perfectly inelastic demand or supply curves exist, government services can be financed without any deadweight loss at all, if taxes on such goods can generate sufficient revenues.

b. (Remember that taxes on products with inelastic supply or demand curves generate no deadweight losses.)

C. The Mathematics of a Ramsay excise tax system can be developed for any revenue target by writing down functions for individual market tax revenues and deadweight losses.

i. To do so, total revenue is expressed as a sum of tax revenues generated from every market, which serves as a constraint.

ii. The total deadweight loss is the sum of the DWL in every market taxed.

iii. Deadweight loss is minimized subject to the tax revenue constraint. The simplest representation is something like the following:
- Assume that market equilibria can be written as function of tax rates, given preexisting market supply and demand curves.
- This implies that both deadweight loss and tax revenues from a particular market can be written as a function of the specific tax applied in the market of interest.
- Let $D = \Sigma\, d_i(t_i)$
- and $T = \Sigma\, t_i\, Q_i\, (t_i)$
- Let $T = T^*$ (the ideal or desired tax revenue

- A Ramsay tax system is a vector of excise taxes that minimizes
  $$D = \Sigma\, d_i(t_i)$$
- subject to $T^* = \Sigma\, t_i\, Q_i\,(t_i)$
- Form a Lagrange objective function and take derivatives with respect to the individual tax rates and the Lagrangian multiplier. This yields:
- as $di_{ti} = \lambda[\, Q_i\,(t_i) + tQ_{iti}\,]$ for all "i" markets
- and $T^* = \Sigma\, t_i\, Q_i\,(t_i)$

iv. Dividing the first order conditions for markets "i" and "j" generates:

- $di_{ti} / dj_{tj} = [\, Q_i\,(t_i) + tQ_{iti}\,] / [\, Q_j\,(t_j) + tQ_{jtj}\,]$
- the ratio of the marginal deadweight losses (from an increase in taxes) in market's "i" and "j" should equal the ratio of their marginal tax revenues.

v. Since both tax revenue and deadweight loss can both be written as functions of elasticity (or slope), each markets supply and demand curves, the above can also be written in terms of elasticity (or slopes).

vi. As in the diagrams the least elastic markets get the highest taxes under a Ramsay tax system.

a. Note that this principle differs from the principle of neutrality, because the tax rates will not generally be equal across all markets.

b. Most Ramsay tax systems affect relative prices.

c. (Can you think of a Ramsay tax system that would not have this effect?)

D. A special case of such a tax system is a tax on land--which is sometimes called a **Georgist tax**, after Henry George (1885) who proposed financing government entirely with land taxes.

i. The supply of land, after all, is perfectly inelastic (ignoring dikes and dumps).

a. (Analyze the limitations, if any, of a Georgist land tax. Where does the value of a piece of land come from? Would there be allocative affects across different types of land?

b. Would a Georgist land tax system be a neutral Ramsay tax?)

- See Tideman (1982)

ii. Given the possibility of international emigration, can their actually be a tax that has no dead weight loss? Discuss.

IV. Pigovian Taxation, and the Double Dividend

A. Another tax system that is consistent with the Ramsay norm of minimizing the excess burden of taxation is the Pigovian tax, which we discussed as a possible solution to externality problems.

B. A Pigovian tax is designed to change behavior, however, the change induced INCREASES social net benefits, rather than diminishing them as do other excise-like taxes.

C. In this manner it is sometimes argued that taxes and other fees used to internalize externalities generate a "double dividend" they both solve externality problems and produce revenue in an efficient manner.

D. The geometry of externalities and externality problems is straight forward.

i. In a supply and demand (market) diagram, we introduce a new curve that represents the **external** marginal costs (or marginal benefits) of the activity of interest.

- The predicted market outcome (Q*) is not affected by the existence of the new marginal external cost curve, because both firms and consumers are assumed to ignore the externality generated.
- (Note that this positive prediction plays an important role in the entire exercise and is an assumption that can be tested.)

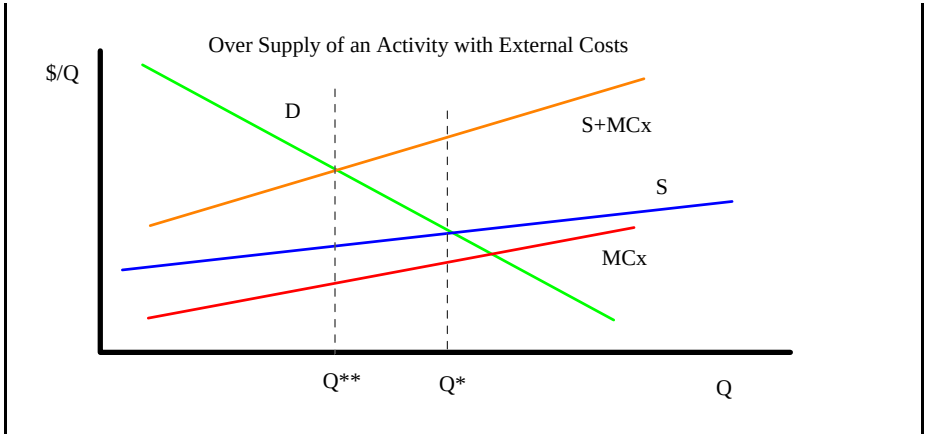
ii. To find out whether an externality generating activity or output is over or under supplied, we find the social marginal benefit and marginal cost curves, and use them to characterize the social net benefit maximizing activity level (output, Q**).

a. To find the SMB and SMC curves, recall that the Demand curve is approximately the same as the marginal benefits received by consumers and the supply curve is approximately the industry's marginal cost. To these we add the external marginal benefits and/or external marginal costs to find the social marginal benefit and social marginal costs curves--now taking account of the spillovers.

b. Because an externality generating activity generates benefits or costs for a wide range of people simultaneously, the social marginal benefit and marginal cost curves for such activities are "vertical" sums of the relevant consumer, firm, and spillover MB and MC curves.

E. The level of the activity that maximizes social net benefits is generally found where the social marginal benefit of the activity equals its social marginal cost curve. If Q* does not equal Q**, there is an externality problem.

i. EXAMPLE: In the figure below, the market supply and demand cross at Q*, but the SMB and SMC curves (here D and S+MCx) cross at Q**. Since Q* is not equal to Q** there is an externality problem.



a. The inefficiency (market failure) conclusion of this diagram can be reached using several normative theories.

- For example, the activity level chosen fails to maximizes social net benefits, then there is an externality (or public good) problem.
- The activity fails to realize all potential gains to trade and so there are Pareto superior moves possible.
- From behind a veil of ignorance, the community would (or may) agree to implement an institution that generates Q** rather than Q*.
-

F. Pigovian Taxes: Internalizing Externalities

i. A Pigovian tax attempts to change incentives at the margin by imposing a tax (or subsidy) on the activity that generates the externality.

ii. Notice that if the externality producer is subject to a tax equal to the marginal external cost (benefit) at the Pareto efficient level, the externality producer will now choose to produce the Pareto efficient output/effluent levels.

a. Such a tax (or subsidy) is said to internalize the externality, because it makes the externality producer bear the full cost of his actions (at Q**).

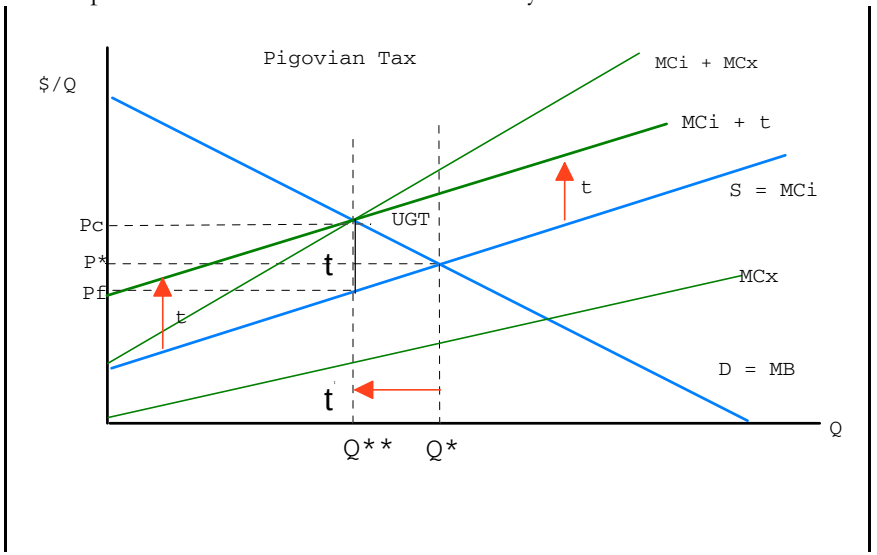
b. (In principle, Pigovian tax schedules can have a variety of shapes, but for the purposes of this class we will assume that they are all "flat taxes" that assess the same tax on every unit of the product (or emission) produced.

G. Illustration of the Pigovian Tax

i. Without a Pigovian tax, there are unrealized gains to trade (see triangle UGT) at Q*, between the firm and those affected by the externality.

a. The external cost at Q** is the vertical distance from MC to the MC + MCx curve.

b. This distance also represents the Pigovian the tax that should be put on production to internalize the externality. It is labeled "t."



ii. If a Pigovian tax of t dollars per unit is imposed on the firm's output (or emissions) the firm will now face a marginal cost for production equal to MC + t.

a. Given this tax augmented MC curve (which includes the tax that "internalizes" the externality) firms in the industry will change their individual supply decisions and the industry will now produce an output of Q**, the Pareto Efficient level.

b. Pigovian taxes induce firms to independently **internalize** the externality.

H. Pigovian taxes may yield substantial revenues although this is not their main purpose. Their main purpose is to change behavior.

- The tax above generate revenue equal to tQ**

i. **Pigovian taxes have no direct deadweight losses associated with them**.

a. Social net benefits increase, rather than decrease, when a Pigovian tax is imposed.

b. This implies that substituting Pigovian taxes for ordinary taxes tends to reduce deadweight losses from the overall tax system.

c. The latter is sometimes referred to as the **double dividend**.

- See Goulder (1995) for a useful overview of the literature on the double dividend. See also Blovenberg (1999), and Parry and Bento (2000)

d. The most important of Pigovian taxes under discussion during the past decade or two is the **carbon tax**, which would generate levels of revenue similar or greater than that of the Corporate Income tax.

- Carbon taxes have to be quite high to change CO2 emissions because the demand for most fuels (gasoline, diesel, etc) tend to be quite inelastic.

- Proposals to fund Swiss social security with a carbon tax, instead of an income or wages tax, was voted on through a national referendum several years ago. (It lost.)

V. Other normative principles of taxation argue that one cannot determine the proper division of the tax burden without knowing what services will be provided.

A. For example, Lindahl argues in favor of a **benefit tax**, that is a tax that imposes the greatest burden on those who receive the most valued services should pay the highest taxes.

i. Under an ideal Lindahl tax system, each person's *marginal tax rate is set equal to the marginal benefits* she or he receives from government services.

- Notice that such a tax system assures that the result is both Pareto efficient and **politically stable** in the sense that given the tax system, there is unanimous agreement about the optimal service level.

- (Illustration)

ii. James Buchanan (who won a Nobel Prize in economics while a professor at GMU, partly for his contributions to public finance) tends to agree with Lindahl.

a. Buchanan argues that proper accounts of tax burden--should focus on net tax burden--that is, they should take account of the services financed by taxes as well as the taxes paid.

b. For example, if a person receives an especially valuable service from the government, it is possible that his or her "true" net tax burden is negative. Others who receive no services of value, might have positive net tax burdens.

- Ideally, all citizens would bear "negative" tax burdens in the sense that each person should receive services that are considered to be more valuable than the taxes paid.

- Buchanan points out that most Western governments are very productive in the sense that a good deal of the wealth produced in a given nation state is affected by property rights, civil law, and public services--as well the taxes used to finance them.

B. Still other normative principles of taxation come are rooted in shared cultural norms--often dealing with **fairness** (or equity).

i. Some argue that fairness requires all persons to pay be treated the same way under a tax system.
   - This notion of fairness tends to imply a flat tax.
   - Examples include a proportional tax on income as developed in Buchanan and Congleton (1998, ch. 8).

ii. Others argue that taxation of persons should be based on their "ability to pay."
   a. This notion of fairness tends to imply progressive income taxes.
   b. For example, a "fair tax" might be one that caused all taxpayers should all sacrifice approximately the same "utility" (rather than net benefits) when they pay their taxes.
      - Since the marginal utility of money tends to be smaller for rich persons than poor persons, more money would be collected from rich persons than from poor persons.
      - This notion of fairness often plays an important role in policy debates over taxation in the US.
      - "Progressivity" if often argued to be desirable, while "regressivity" is often argued to be undesireable--although these ideas are not universally accepted among normative tax theorists.

iii. Perhaps the most extreme of the rational choice-based fairness analyses is that of John Rawls (1971, *A Theory of Justice*), who argues that fiscal packages should be designed so that the welfare of the least well off person in society is maximized.

VI. Some Useful definitions for characterizing and discussing tax equity and/or the equity of fiscal systems:

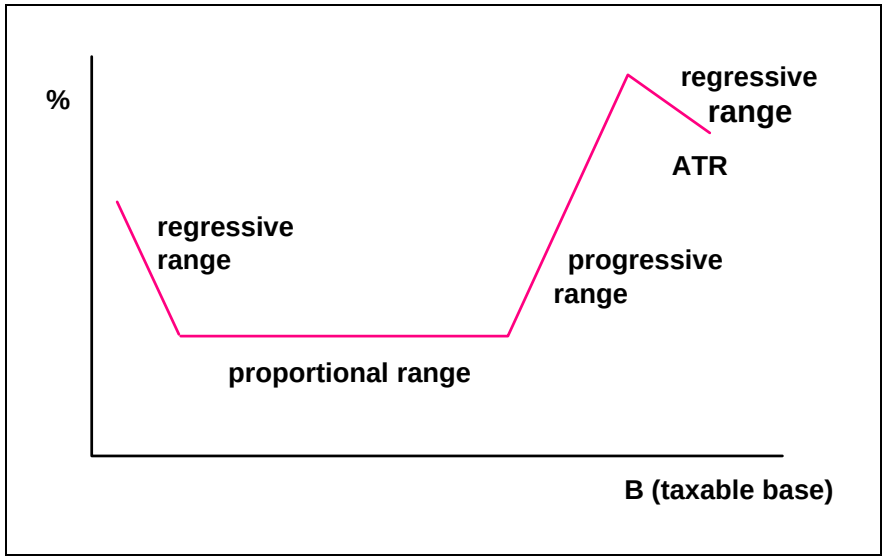
i. A **progressive tax** is a tax whose average burden increases as the taxable base owned by an individual increases. [Such taxes often have marginal tax rates that increase with the base (increase with income), although not all progressive taxes have this property. Most income tax systems in industrialized countries are somewhat progressive.]

ii. A **proportional tax** is a tax whose average tax burden does not change with income. (Such taxes normally have a constant marginal tax rate, as true of most sales taxes and some income taxes. A flat (proportional) tax on income has the form: $T = tY$.)

iii. A **regressive tax** is a tax whose average tax burden falls with income. Such taxes often have declining marginal tax rates with ownership of the taxable base, however, not all regressive taxes have this property. An example of a regressive tax in the US is the social security tax--which has a cap on taxable income.

iv. [Instead of tax base, many analysts use income, which allows them to think in terms of "ability to pay" and apply utilitarian "fairness" norms to tax systems.]

v. **Definitions and Relationships:**
   a. The **tax base**, B, is that which is taxed. (taxable income, sales of final goods and services, profits, property, gasoline, etc.)
   b. The **average tax rate** of a particular tax often varies with an individual's holding of the taxable base. If an individual pays tax Ti on a holding of Bi, his average tax rate is Ti/Bi. (If Ti = \$50 and Bi = 200, the average tax rate for this tax is 50/200 = 0.25 or 25%.)
   c. The **marginal tax rate** of a particular tax is the change in taxes owed for a one unit increase in holdings of the taxable base, $\Delta T/\Delta B$. (So, if a tax payer earning 50,000/year pays a tax of 10,000 and a taxpayer earning 50001 pays a tax of 10,000.50, his or marginal tax rate is 0.50/1 = 50%. Fifty percent of each additional dollar earned is taken from the "last" dollar of income earned by a taxpayer earning 50,000/year.)
   d. In a **diagram of tax** schedules. If MTR is above ATR, then that ATR curve will be rising (the marginal tax rate will be pulling the average up). If MTR is below ATR, then the ATR curve will be falling (the marginal tax rate will be pulling the average down). If the MTR = ATR, the ATR will be neither rising nor falling.
   e. Since individual decisions are determined by marginal cost and marginal benefits at various quantities, **it is the marginal tax rate rather than the average tax that affects tax payer behavior**.
   f. (Thus, one argument in favor of proportional, or indeed, regressive taxes, is that they may have smaller effects on economic activities than a revenue equivalent progressive tax.)



vi. (Peckman's estimates of the effective average and marginal tax rates faced by a typical American tax payer, often look a bit like this odd tax schedule.)

a. (As an exercise try to determine what the marginal tax schedule that corresponds to this average tax schedule looks like.)

b. (Explain briefly why Peckman finds regressive ranges of taxation at both the highest and lowest ranges of income.)

VII. The Importance of Marginal Tax Rates

A. The supply and demand diagrams of lecture 2 provide very useful ways to illustrate the burden of an excise tax, tariff, or other tax that can be represented in more or less "flat" per unit terms.

B. However, they are less useful for examining the impact of more complex taxes such as a progressive income tax.

i. The taxes examined in the diagrams all had a constant tax rate, which implied that their average and marginal tax rates were essentially the same.

ii. In many cases, however, the marginal and average tax rates will differ, in which case it is the marginal tax rate rather than the average tax rate that is most important for predicting the impact of the tax on persons and markets.

iii. In such cases, one gains more insight into the effects of a tax by using indifference curve analysis or a bit of mathematics.

A. Consider the following decision setting in which Al can work to earn money for goods available only in markets or engage in leisure.

i. To simplify, assume that their H hours a day that can be worked and that Al is free to work as much or as little as he or she wants to.

ii. Also assume that work produces neither pleasure nor pain, but is s imply a means of obtaining the desired market basket of consumption goods, C.

iii. Leisure, L, is assumed to be a good subject to diminishing marginal returns as usual..

- Hours worked are denoted W and the wage rate is w.
- So, income is Y=wW.
- The income tax schedule is T = t(wW) or T =t(w(H-L)) with $T_Y > 0$.
- (Assume for convenience, that the entire burden of the tax is borne by Al.)

a. Given all this, we can write down the optimization problem that characterizes Al's labor-leisure choice:

- U = u(L, C)
- with C = wW - T or C = w(H-L) - t(w(H-L))

b. Both leisure and consumption are ordinary goods subject to diminishing marginal utility and have positive or zero cross-partials.

- $U_L > 0$, $U_C > 0$, $U_{LL} < 0$ and $U_{CC} < 0$ with $U_{CL} > 0$

c. To simplify the math a bit, note that one can substitute the budget constraint for C into the utility function.

- This allows Al's utility can be written either entirely in terms of leisure (L)
- or entirely in terms of hours worked W, if we also substitute for L = H - W.

d. In the latter case: U = u(H-W), wW - t(wW))

e. Differentiating with respect to W (the only control variable available to Al in this representation), we obtain the first order condition that characterizes Al's optimal work day:
- W* satisfies: $U_L(-1) + U_C(w - T_Y w) = 0$
-
- or $w(1 - T_Y) U_C = U_L +$
-
- $w(1 - T_Y) U_C$ is the marginal benefit of an hour worked (in utility terms) net of taxes
- U_L is the marginal opportunity cost of working (also in utility terms)
-
- A utility maximizing person will work at the point where his marginal increase in income $[w(1 - T_Y)]$ times the marginal utility of income equals the marginal utility of leisure.

iv. Note that it is the **marginal tax rate**, T_Y, rather than the average or total tax rate, that affects Al's decision.
- (In cases in which Al bears less than the full burden of the tax, it will be his or her effective marginal tax rate that affects behavior.)

VIII. Appendix: On the Mathematics of Leviathan: Taxation and Rent Extraction as a Ramsay System of Revenue (from Congleton and Lee 2009)

A. Consider the case where national governments determine the general extent of monopoly power, M, through broad polices such as anti-trust enforcement which affect the range of possibilities for monopolization across the economy as a whole.

i. A government's net revenues, N, in this case can be characterized as:

$$N = y(G, M, t, L, R)\, t - c(G) + \alpha r(M) \qquad (1)$$

ii. Where y is the national production function, G is the government service level, M is the degree of monopolization encouraged, L is the exogenous labor stock, R is the exogenous natural resource base, t is the proportional sales or income tax, c(G) is the cost of government services, and $\alpha r(M)$ is the revenue generated from would be monopolists. N is assumed to be strictly concave.

iii. Differentiating with respect to government service level G, t, and M, allows us to characterize the net revenue maximizing combination of government services, tax rates *and monopoly policies*.

$$tYt + Y = 0 \qquad (2.1)$$

$$tYG - CG = 0 \qquad (2.2)$$

$$tYM + \alpha rM = 0 \qquad (2.3)$$

iv. Subscripts denote partial derivatives of the variables subscripted.

A. The revenue maximizing government selects its policies over government services, tax rates, and monopolization policies to satisfy the three first order conditions simultaneously.

i. Equation 2.1 implies, as in the Buchanan-Brennan model, that tax rates will be set to maximize tax receipts (with ideal government service levels and monopolization throughout the economy).

ii. Equation 2.2 implies, as in the Olson-McGuire model, that productive government services will be provided by a revenue maximizing dictatorship up to the point where marginal tax revenues equal the marginal cost of those services.

iii. It bears noting that Leviathan produces *fewer* government services than required to maximize national income when optimal marginal tax rates are less than one hundred percent.
- (The later reinforces the Buchanan-Brennan argument favoring progressive income taxation under Leviathan.)

iv. Equation 2.3 implies that *monopolization will be encouraged* up to the point where the marginal loss of tax receipts equal the marginal gains from rent-seeking receipts induced by those policies.

a. A net revenue-maximizing Fisc has a direct interest in the industrial organization of its domain that is not entirely benevolent.
- The marginal increase in revenues generated by increased monopolization, αrM , varies with the institutional setting, characterized by α, and with the extent to which increased monopolization induces rent-seeking by would be monopolists, rM.

- The marginal cost of inducing rent-seeking revenues varies with effectiveness of the tax system, tYM , and the rate at which national income is reduced by the monopoly grants conferred, YM.[1]

b. Given optimal government service levels, G*, and tax rates, t*, equation 2.3 implies that the larger is the marginal increase in rent seeking revenues received by those with policy making power and the smaller the marginal tax loss, the greater is the government's ideal extent of monopolization.

B. It bears noting that the inequality forms of equation 2.3 allow the possibility of two corner solutions.[2]

i. First, there is a corner solution where no inefficient monopolization takes place.

ii. National income maximizing monopoly policies are adopted when the marginal tax cost of rent seeking is larger than marginal receipts, - tYM > αrM , for all M.

a. In this case, the Fisc's "encompassing interest" in the size of the tax base causes monopoly power to be allowed or promoted only insofar as it adds to national income.

b. Tradable copyrights, patents and exclusive land grants might be created, but other monopolies would be prevented by state action as with antitrust enforcement.

c. This is the only case where Leviathan will adopt the policies recommended in textbook discussions of optimal patent, trade and antitrust policies.

iii. The other extreme policy analogous to the Ekelund-Tollison interpretation of mercantilism is adopted when the marginal receipts from induced rent seeking exceed tax losses over the entire range of interest, e.g. when tYM < αrM for all M.

a. Complete monopolization of the economy can arise when the tax losses induced by monopolization are relatively small or when tax instruments are relatively ineffective sources of revenue (possibly because of shift of activities into the underground economy as in Marcouiller and Leslie, 1995).

b. In cases where tax losses are insignificant, the net revenue maximizing state attempts to *maximize* the size of rent seeking expenditures whenever $\alpha > 0$. Olson (1993) and Anderson and Boettke (1997) suggest that a good deal of the industrial policies of the former Soviet Union can be understood as such a corner solution.

iv. The intermediate cases between these corner solutions are the focus of the present analysis.

a. In this range, governments use a combination of tax, government services and monopolization policies to maximize net receipts.

b. Potential rent-seeking revenues lead government to adopt policies that induce *greater* monopolization than is consistent with maximizing national income, YM < 0 at M*, but the economy is not completely monopolized.

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[1] We interpret t as the effective tax rate, which may differ from both the statutory tax rate and the marginal tax burden. Opportunities to avoid paying taxes vary with the ability of the Fisc to police the tax law and opportunties to legally avoid paying taxes. It also bears noting that in some tax systems, tax revenues may actually increase as monopoly profits increase. For example, sales, value added, and profits tax revenues tend to increase as prices and monopoly profits increase. In such cases, rent-seeking possibilities may be expected to affect the choice of tax system as well as the degree of monopolization. We leave consideration of Leviathan's preferred tax *system* for future analysis. The income-based tax used in our analysis has been widely used in previous Leviathan models.

[2] We assume that the Fisc's objective function is strictly concave and that his constraint set is convex; consequently the Arrow Enthoven sufficiency conditions are satisfied. These imply that the corner solutions to the optimization problem with inequality constraints can be completely characterized using the Kuhn-Tucker first order conditions. The Kuhn-Tucker first order conditions imply that in cases where the conditions for an internal maximum or tangency condition are not satisfied, e.g. - tYM $\neq \alpha$rM for $0 \leq M \leq 1$, the maximal values of the objective function lie along the constraints as discussed above.

v. The implicit function theorem allows the relationships describing the Fisc's preferred vector of tax, government service and monopoly policies to be characterized as:

$$G^* = g(L, R, \alpha) \quad (3.1)$$

$$T^* = t(L, R, \alpha) \quad (3.2)$$

$$M^* = m(L, R, \alpha) \quad (3.3)$$

C. Proposition 1: *The greater is the possibility of obtaining additional revenues from rent-seekers, the more inclined the Fisc is to adopt policies that promote "inefficient" monopolization, e.g. to use rent-seeking games as a source of government revenue even though such policies reduce national income.*

i. The ideal monopolization policy, as characterized by equations 3.3, is of special relevance for the purposes of this paper.

ii. Using the implicit differentiation rule to differentiate M* with respect to α yields:

$$M^*_\alpha = \frac{\begin{vmatrix} tYGG - CGG & tYtG + YG & 0 \\ tYtG+YG & tYtt + 2Yt & 0 \\ tYMG & tYtM + YM & -t\alpha rM \end{vmatrix}}{\begin{vmatrix} tY_{GG} - C_{GG} & tYtG + YG & tYMG \\ YG+ tYGt & tYtt + 2Yt & tYtM + YM \\ tYGM & tYtM + YM & tYMM + \alpha rMM \end{vmatrix}} = \frac{[(tY_{GG}-C_{GG})(tY_{tt}+2Y_t) - (tYtG+YG)2\,](t\alpha rM)}{|H|} > 0 \quad (4)$$

i. The equation 4 is unambiguously greater than zero in the case where the net revenue function is strictly concave.

a. (The second order condition of the original optimization problem requires $|H|<0$ and the bracketed term of the numerator to be greater than zero.)

b. The last term in the numerator is also negative under the assumption that greater rent's induce greater rent seeking revenues.

c. Consequently, the leviathan model unambiguously implies that policies oriented toward increasing monopolization expand as the government's ability to profit from induced rent seeking efforts, α, increases.

D. Proposition 2: *regulations or monopoly grants that provide protection in output markets are generally more valuable to prospective rent-seekers than are protected production processes (patents) for firms in a given industry.*

i. A monopoly privilege that grants the exclusive right *to sell* a specific product allows a firm to profit from production within its protected sphere, without fear of price competition from close rivals.

ii. Grants of patent protection for specific production processes similarly allow firms to realize extra-ordinary returns by ensuring their position as a low cost producer.

a. A patented production process yields a Ricardian rent or inframarginal profits if the patented process is more cost effective than those not protected.

b. However, the rent associated with a patent is smaller than the profit associated with a monopoly in the same output market(s) insofar as the profitability of any production process clearly increases if one is able to manipulate price as well as output.[3]

iii. The most valuable patents are those which generate such dramatic cost savings over other available methods that a monopoly results in the specific output markets, as patents on specific production processes occasionally do.

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[3] This can be demonstrated mathematically as follows. Profit is revenue less cost. Consider the maximal profit associated with a given degree of monopoly power, M, and production technology, T. $\Pi^* = R(Q^*, M) - C(Q^*, T)$ Totally differentiating and appealing to the envelope theorem yields: $D\Pi^* = dM\,(\delta R/\delta M) - dT\,(\delta C/\delta T) > 0$ .Maximal profit rises as production technology improves (allowing lower production costs) and as monopoly power increases allowing greater revenues.

a. Moreover, output monopolies are more readily enforced than production methods are insofar as sales of outputs usually take place in public whereas production normally takes place in private.[4]

iv. **Consequently, a revenue maximizing Fisc will be inclined to grant monopoly protection to output markets rather than production processes, other things being equal.**

E. Proposition 3, **Ramsay Monopolization**: *the markets granted the most protection by the Fisc are those in which the demand for goods and services is least price sensitive. Consequently, the revenue maximizing pattern of monopolization tends to resemble a Ramsey tax.*

i. Monopolization of the least price sensitive markets maximizes the level of rent seeking induced because it maximizes the profits generated by a given degree of protection while minimizing the tax revenues lost by reduced output.

ii. To see this, we now disaggregate the original model of monopoly power within a market as a whole and focus on individual markets and revenues.

iii. Suppose there are *n* final goods markets that can potentially be granted a degree of monopoly power.

a. We represent the extent of monopolization generated by government policies in a particular industry as "monopoly mark up," *mi* , while retaining our assumption that government output is a pure public good and that the tax system is a broad based sales or proportional income tax.

b. We assume that in the absence of monopolizing regulation, the markets in question would be conventional competitive markets with constant marginal and average costs, $Ai = ai(t, G)$.

c. Tax rates and government services affect the average cost of producing output in market *j*.

iv. Average cost is increased by tax rates which reduce the effective real return to capital and labor, and is decreased by government services which lower transactions and transport costs.

a. Industry *i*'s output can thus be represented as, $Q^*i = qii(Pi,t,G)$ *where* $Pi = Ai + mi$.

b. Monopoly profits and total rent-seeking efforts in market *i* are miQ*i. Net revenue for the Fisc is now:

$$R = \Sigma i\ (t\ PiQ^*i + \alpha\ miQ^*i\ ) - c(G) \qquad (5)$$

v. In the case of a sales tax, monopolization can increase nominal tax receipts by increasing the value of output in the affected markets if total revenues or industrial income increases with price. Differentiating with respect to t, G and mi yields the first order conditions that characterizes the government's vector of taxation, services, and monopoly policies.

$$\Sigma i\ (\ PiQ^*i + t\ PiQ^*it\ ) = 0 \qquad (6.1)$$

$$\Sigma i\ (t\ PiQ^*iG\ ) - CG = 0 \qquad (6.2)$$

$$\Sigma i\ [t\ (Q^*i + PiQ^*iPi\ ) + \alpha\ (Q^*i + miQ^*iPi\ )] = 0 \qquad (6.3)$$

i. Given t* and G*, equation 6.3 is satisfied when mi is such that:

$$\alpha m^*i\ +\ t^*Pi\ =\ (\ t\ +\ \alpha)(Q^*i\ /\ -\ Q^*iPi\ )\quad for\ all\ i \qquad (7.1)$$

or

$$m^*i/\ Pi\ =\ [(\ t\ +\ \alpha)/\alpha\ ]\ [Q^*i\ /\ -\ Pi\ Q^*iPi]\ \ -\ t^*/\alpha \qquad (7.2)$$

ii. Given ideal tax and service policies, equation 7.2 indicates that the revenue maximizing vector of monopoly mark ups (as a percentage of the original price) is proportional to the price elasticity of demand in every market. (Recall that $\eta i = Q^*i\ /\ -\ Pi\ Q^*iPi$.)

[4] A patent for a production process that can be used to produce products for *several* markets can be more valuable than an output market in any *single* market. Thus, to the extent that the Fisc protects production processes, we would expect such *broadly applicable* processes would attract the interest of a revenue maximizing Fisc before narrower markets. Protecting production methods does have the political advantage of being less observable than output protections. Of course, as noted above, this also makes patented production methods more difficult to protect.