Chapter 2A: Exogenous Taxation

I. Introduction

The usual approach that economists take when analyzing the effect of a tax is to assume that the tax in question is "exogenous," which is to say, not caused or even partly determined by the individuals interacting in the markets affected. Its as if the taxi of interest was imposed by a King, independent fiscal authority, or from outer space, rather than selected by a parliament, congress, or elected executive branch of government. The latter clearly affect which taxes we actually observe. However, for appraising the effects of alternative tax systems and rates that may be imagined, the assumption of exogeneity makes perfect sense. Analyzing such taxes and, perhaps, simulating their effects on well calibrated models of the relevant economy are, in a sense, experiments initiated by the person or persons undertaking the analysis.

In principle, analyzing the effects of taxes on economic activity requires a general equilibrium analysis, but such analyses cannot be undertaken without a many simplifying assumptions. For example, labor and capital are often assumed to be homogeneous goods, and relatively simple functional forms for the various relationships are assumed—such as linear ones, as for example in many macroeconomic models. Such highly simplified models are sometimes referred to as computable general equilibrium models. Rather than undertake such an analysis, we'll use a somewhat more general, but still limited partial equilibrium model for the analysis.

There are tradeoffs involved here, but in cases in which the effects of taxes are mainly on particular types of markets and their close substitutes (or complements) a partial equilibrium model can characterize most of the effects of a new tax or change in tax rates without assuming that all relevant functions take specific algebraic form.

II. A Digression on the Logic of Demand and Supply

Partial equilibrium analysis looks at a small subset of markets and traces the effects of market conditions and changes in market condition on the sales and prices of a the goods in the markets considered. In this case, we'll focus a single market that is linked to other markets through effects on the typical consumer's budget set and a typical producer's production process and its associated total cost function

The consumer (Al) maximizes a utility function U=u(X, X2) subject to the budget constraint $Y = P_1X_1 + P_2X_2$.. We'll assume that good X1 is of special interest, because it is to be taxed as with an excise tax. Although excise taxes are not the most common source of government revenue, there are a variety of excise taxes as for example for special hotel taxes, cellphone taxes, gasoline taxes, and fares on jet liners. The two-good case allows us to use the "substitution trick" to shorten the derivation of demand. Note that the budget constraint implies that X_2 can be written in terms of X_1 as $X_2 = (Y - P_1X_1)/P_2$. We can substitute that relationship into the utility function as below:

$$U=u(X1, (Y - P_1X_1)/P_2) \qquad (2.1)$$

Differentiating with respect to X1 and setting the result equal to zero characterizes Al's best utility maximizing choice for good X_1.

$$u_{X_1} + u_{X_2}\left(-\frac{P_1}{P_2}\right) = 0 \;\; at\; X_1^* \qquad (2.2)$$

Partial derivatives are denoted with subscripted variable names. The implicit function theorem allows X_1^* to be written as:

$$X_1^* = x_1(P_1, P_2, Y) \qquad (2.3)$$

This is Al's demand function and if Al is the average consumer then the overall market demand for good X_1 is simply N times that expression, where N is the number of consumers in the market.[1] If Al is not the average, then market demand would be the sum of the N individual demand curves. As long as the essential utility maximizing problem is the same for each (e. g. two goods and similar shaped but not necessarily identical utility functions), the

[1] The implicit function theorem shows that if a differentiable function H=h(a, b, c, d) = 0 exists, then another function g exists that can describe a (or any other of the variables) as a function of all the other variables in H, as with a = g(b, c, d). Here we use this theorem to write down the general form of a demand function.

comparative statics of market demand will be qualitatively similar to that of Al's demand function.

If Al's utility function has positive first derivatives (e.g. X_1 and X_2 are both goods), negative second derivatives (both goods exhibit diminishing marginal utility) and cross partials of the utility function are either zero or positive, it can be shown that Al's utility function is downward sloping and that X_1 is a normal good in the economic sense that more of it will be purchased if Al's income increases.[2]

The market supply function can be derived in a slightly simpler way. Suppose that Apex produces and sells good X_1 . Its production function implies a cost function of the form $C=c(X_1, w, r, T)$ where Q is the quanity of good X_1 produced and sold by Apex, w is the wage rate for labor, r is the rental cost of capital, and T is the level of technology. All the arguments except X_1 are assumed to be determined outside the firm, either by competitive input markets or because of the state on knowledge about the physical and social environment Apex produces in.[3]

Assume that the market for X_1 is sufficiently competitive that Apex can be regarded as a price taker. In this case, Apex's profit can be written as:

$$\Pi = P_1 X_1 - c(X_1, w, r, T) \tag{2.4}$$

Differentiating Apex's profit function with respect to X_1 and setting the result equal to zero characterizes Apex's profit maximizing output X_1*.

[2] These derivatives can be calculated and signed using the implicit function differentiation rule. Given function H = h(a, b, c, d), and the implicit function a = g(b, c, d) the derivative of a with respect to b can be calculuated as da/db = H_b/-H_a where H_b is the derivative of H with respect to b and H_a is the derivative of H with respect to a. The effect of an increased income on Al's demand for X1 is just dX1*/dY = H_Y/-H_{X1} .

[3] Note that w and r would be vectors of wage rates and rent rates for capital goods in relatively complex production processes. We greatly simplify, but without much loss, by assuming just two inputs. Likewise technology could also be thought of as a vector of technologies used for various stages of production.

$$\frac{d\Pi}{dX_1} = P_1 - C_{X_1} = 0 \text{ at } X_1^* \tag{2.5}$$

The first term is Apex's marginal revenue and the second is its marginal cost function. This "first order condition" characterizes Apex's output as long as the profit function is strictly concave, as it will be if the cost function increases with output at and increasing rate and also increase with wages and interest at a rate greater than or equal to zero. Production costs fall as technology improves as do marginal cost.

There is a sense in which X_1* can be thought of as the quantity that Apex will supply at price P_1, other things being equal, but it is really the characterizing a relationship that varies with market price, input prices and technology. The implicit function theorem implies that equation 2.5 can be used to characterize X_1* as a function of the other arguments in equation 2.5. (Recall that C_{X_1} is a function that includes the same arguments as its parent function, namely X_1*, w, r and T.) That function can be written as:

$$X_1^* = s_1(P_1, w, r, T) \tag{2.6}$$

Equation 2.6 is Apex's supply function. Given the assumptions about the shape of Apex's total cost function, the implicit function differentiation rule implies that supply increases as price and technology increase, but falls as input prices increase.

If there are M firms in this market and Apex is an average firm market, then market supply will be MX_1^*, and if not then will be sum of the similar but not identical supply curves of all the firms potentially in the industry. In either case, if the firms in or potentially in this market and all use more or less similar production methods, then the comparative statics of the industry supply curve resembles that of Apex. (Note that assumption that there are a finite potential number of firms that may participate in this market differs from the usual Marshallian characterization, but is consistent with a Ricardian one in which firms use similar but somewhat different production methods or have access to inputs of somewhat different quality. Some firms may realize profits greater than zero in the long run because of their higher quality capital, slightly better technology, manement, or labor force,) This mathematical derivation of demand and supply implies that market demand functions slope downward as in the graphical derivation using net benefit maximization and market supply curves slope upwards as in the graphical derivations included in the condensed review of undergraduate public finance provided as part of chapter 1.

III. Taxation: A Partial Equilibrium Analysis

If we assume that Al is the average consumer and Apex is the average firm, then the price that tend to emerge, P_1^*, is the one where supply equals demand, which can be characterized using the derivations above.

$$Nx_1(P_1^*, P_2, Y) = Ms_1(P_1^*, w, r, T) \tag{2.7}$$

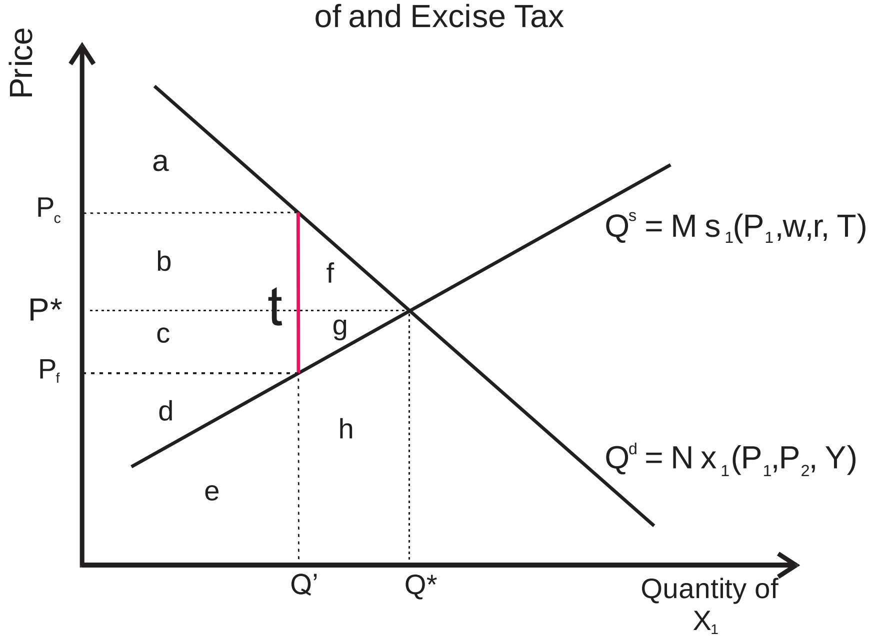
In the case, where a per unit excise tax, t, is imposed on purchases of X_1, the condition that equates demand with supply will require two prices to be characterized namely the one paid by purchasers and the one received net of taxes by sellers. Given that the consumer price is just T dollars per unit higher than that which firms receive, this can be written as:

$$Nx_1(P_1^*, P_2, Y) = Ms_1(P_1^* - t, w, r, T) \tag{2.8}$$

Where P_1^*now represents the price paid by consumers and $P_1^* - t$ represents the price received (net of taxes) by firms. These conditions are similar, but the second requires a particular pair of prices (one for consumers and one for firms) to be found rather than a single price as in the setting where sales of X_1 are not taxed.

The implied geometry of this solution is very similar to that of the diagram that developed geometrically in the review of undergraduate public finance—the only difference is that in this case the graphs are of functions derived using calculus rather than curves developed using geometric methods. The calculus derivations of supply and demand functions include various "shift variables" that would alter the equilibria—variables that are not obvious from the geometric derivation. Figure 2.1 illustrates the equilibria associated with equations 2.7 and 2.8.

Figure 2.1: Price and Output Effects of and Excise Tax



The tax revenue generated by the tax is tPQ' which is the area of the two rectangles b+c. Note that rectangle is Q' wide and t tall.

We've derived the demand curve from a utility function, so it is not exactly the same as the aggregate marginal benefit curve for consumers, but it can be used as an approximation of that curve. This allows us to consider area "a" to be the consumer surplus realized after that tax was imposed, whereas area a+b+f was the consumer surplus realized before the tax was imposed. So the lost net benefits for consumers (neglecting the benefits from any useful services that the government might fund with the tax revenue) is ΔCS = b+f. The industry profit realized after the tax is area d, whereas the profit realized before the tax was c+g+d, so the burden imposed on firms is $\Delta\Pi$ = c+g. The overall burden imposed on both sides of the market thus is (b+f) + (c+g) which is greater than the revenue generated (b+c). Thus, the tax can be said to generate an **excess burden** of (f+g).

The extent of that excess burden varies with the square of the tax in the case where the demand and supply curves are linear. (Both the base and the altitude of triangle vary with tax rate t, the area (A) of a triangle being ½ the base (B) times the altitude (H), which in this can be written as at^2, where B = t and H = at, the ratio of the altitude to the base of this family of triangles.)

Both the mathematical model and the diagram can be used to determine the qualitative effects of excise and other taxes that affect equilibrium prices in other markets. For example, if good 2 is a substitute for good 1, and good 2 is taxed rather than good 1, this will increase the price of a substitute for good 1 and so increase the demand for good X_1 and thereby its selling price. Similarly, a tax that affects the price of inputs used in the production of X_1 will increase the marginal cost of producing X_1 which causes each firm's marginal cost to increase and supply the shift back to the left, again increasing the selling price of X_1.

Similarly, a tax on labor income places a tax wedge between the price paid by purchasers of labor (employers) and suppliers of labor (employees), which tends to reduce after tax wage rates for labor suppliers, but it tends to increase pre-tax wages for firms. To see this simply relabel figure 2.1 with labor along the horizontal axis and wage rates on the vertical axis—and keep in mind that labor sellers of individuals and those buying labor tend to be firms. This effect would reduce after tax income for consumers of good X_1 while the supply effect (a higher input price) would reduce supply. The effect on demand would be to decrease demand for normal goods. The effect on supply is to decrease the supply of goods using the taxed input, with a larger effect on labor intensive industries than capital intensive ones. The effect of such a tax on the selling price of X_1 is ambiguous. In contrast, sales of X_1 tend to be reduced by both the shift in supply and in demand.

Extensions of the Core Partial Equilibrium Model of the Effects of Taxes

Notice that by explicitly taking account of linkages among markets, the mathematics of supply and demand helps to demonstrate how taxes in one market or for one type of product or input tend to have effects that go well beyond the market taxed. Markets are networks of exchange, production, and innovation, and a tax placed on any single or subset of markets tends to affect prices and outputs in many others.

Note also that equation 2.8 can easily be modified so that the equilibrium consumer or firm's price can be characterized as a function of all of the other "exogenous" variables and partial derivates with respect to those variables can be calculated (and some, but not all cases, signed using the implicit function differentiation rule. To do so, one merely subtracts the supply function from the demand curve to produce a "zero" equation, that can be used as the basis for an implicit function representation of consumer prices and for taking derivatives. For example:

$$P_1^* = f(t_1, N_1, P_2, Y, M_1, w, r, T_1) \tag{2.9}$$

"1" subscripts have been added to indicate variables that are likely to be associated exclusively with the production or demand for good X_1, which includes the number of consumers and firms and technology.

Moreover, tax schedules can also be introduced into the model by replacing t with either a vector of excise taxes (as in Ramsay taxation) or an income tax schedule with different average and marginal prices for different levels or kinds of income such as t=g(Y). In this manner the, the model developed provides a basis for a number of extensions that can be undertaken mathematically, but which are more difficult when figure 2.1 is derived geometrically, because relationships between variables are more difficult to rationalize from the purely geometric perspective.

However, once the mathematics behind the diagrams are understood, the diagrams provide a very useful and powerful way of illustrating the effects of targeted taxes on a single market or type of market and for illustrating tax effects on other related markets. In this way, the math can make one's "intuitions" about taxes more complete, precise, and internally consistent. Knowing how the system of variables is connected allows one to determine which variables are endogenous (output and prices) and which are exogenous, here taxes, consumer income, the prices of other goods relevant for consumers, input prices, and technology. Note that the shapes of the utility function and cost function do not explicitly appear, but they implicitly affect the shape of function f.

IV. Appendix: Some Tax Vocabulary

Tax Base: the things, activities, or services that are subject to a tax. A tax base is determined by legislation and interpretation. For example, an "income tax" does not usually tax all income, but only part of it. It is the part that is taxed that is the tax base for such income taxes, not all income. (Note that many news accounts and academics use an imaginary tax base to compute tax rates such as total income—as far as I know, no government taxes all of personal income.)

Average Tax Rate: the total tax paid under a tax system divided by one's holding of the tax base. In the case of an income tax, it is one's total tax payment divided by one's taxable income (e.g. that part of one's income that is in the tax base).

Marginal Tax Rate: the additional tax owed on an additional "dollar" of holdings of the tax base for a tax.

Progressive Tax: a tax which as an average tax rate that increases with holdings of the tax base.

Proportional Tax. a tax with a "flat" average tax rate. It's average tax rate is constant over the entire range, as most sales taxes and many property taxes are.

Regressive Tax. a tax with an average tax schedule that declines with holdings of the tax base.

Tax schedule: the function (usually in a table form) that determines how much tax one owes given a particular holding of the tax base.

Income tax schedules often have proportional and progressive ranges, a few such as the social security tax in the US have proportional and regressive ranges. The domain of a progressive tax has a marginal tax rate that is higher than the average rate. The domain of a proportional (or flat) tax has a marginal tax rate equal to the average tax rate. The domain of a regressive tax has a marginal tax rate that is below the average tax rate.

.

.