

I. Introduction to Models of Electoral Competition

- A.** Under a variety of somewhat restrictive, but evidently realistic assumptions, it can be shown that competition among office seeking candidates for the votes of policy oriented voters converge to common positions at the median of the distribution of voter policy ideal points.
- B.** There are a variety of implications of the simple median voter model in terms of government policies (some of which we will explore in detail today):
- i. Policies will tend to be moderate, e. g. drawn from the middle part of the political spectrum.
 - ◆ (The middle can be regarded as "moderate" essentially by definition.)
 - ii. Most people will be at least partially displeased with the policies chosen insofar as they have different ideal point, even in a perfectly functioning democracy, as long as peoples tastes, circumstances, or expectations differ.
 - ◆ (However, it is possible that most people are dissatisfied with government policy yet still prefer the use of majoritarian decision rules to any other.)
- C.** In the limit, at full equilibrium, government policies will maximize the median voter's expected utility, given his constraints, expectations, and goals.
- i. An implication of this "strong form of the median voter theorem" is that *any change in circumstance that changes the constraints of the median voter, or the identity of the median voter, is predicted to have systematic effects on the size and composition of government programs.*
 - ii. Another implication of "C" is that, increases dispersion of the distribution of voter preferences (increased radicalism) tends to have little, if any, effect on public policies unless it affects the median of the distribution of voter ideal points.
 - ◆ *This implies that median voter policies will be more stable than average voter policies.*
- D.** (The spatial voting model can be extended in various ways, for example it can account for non voters and rational abstainers, and even imperfect information to some extent.)

II. The strong form of the median voter theorem implies that particular policies can be modeled as the solution to one person's political optimization problem.

- A.** Such optimization problems are often very straightforward to characterize and perform comparative statics on.
- i. Consequently, the median voter model is widely used to analyze the level and growth of government service levels.
 - ii. It plays a significant role in both the theoretical and empirical public finance literature dealing with taxes and expenditure levels.
- B.** Consider electoral selection of a public services that is funded with a non-distorting "head tax."

- i. Each voter in his capacity as a policy "maker" looks very much like the standard consumer in a grocery store, except that in addition to private budget constraints, he has a "public" budget constraint to deal with.
- ii. Suppose that voter's have the same utility function defined over private consumption (C) and some public service (G).
- iii. Suppose further that each voter has a different amount of money, W_i , to allocate between C and G, and, further, assume that the government faces a balanced budget constraint, and that all expenditures are paid for with a head tax, T. Assume that there are N tax payers in the polity of interest.
- iv. In this case:
 - ◆ $U = u(C, G)$
 - ◆ $W_i = C + T$
 - ◆ $g(G) = NT$
- v. Note that T can be written as $T = g(G)/N$ and substituted into the private budget constraint to make a single unified budget constraint:
 - ◆ $W_i = C + g(G)/N$
 - ◆ which in turn can be solved for C and substituted into the utility function:
 - ◆ $U = u(W_i - g(G)/N, G)$
- vi. Differentiating with respect to G yields a first order condition that characterizes the median voter's preferred government service level:
 - ◆ $-U_C (g_G/N) + U_G = 0 = H$ or equivalently as $U_C (g_G/N) = U_G$
 - ◆ The right hand side of the latter is the subjective marginal benefit (marginal utility) of the government service, the left-hand term is the subjective marginal opportunity cost of government services in terms of lost private consumption.
 - ◆ Note that the subjective marginal cost of the service is determined by both preferences (marginal utility of the private good C) and objective production or financial considerations, c_G/N . The latter can be called the median voter's marginal cost share, or price for the government service.
- vii. An implication of the first order condition together with the implicit function theorem is that each voter's demand for public services can be written as:
 - ◆ $G_i^* = \gamma(W_i, N)$ that is to say, as a function of his own wealth (holding of the taxable base) and the population of tax payers in the polity of interest.
 - ◆ The implicit function differentiation rule allows one to characterize comparative statics of how changes in wealth, W_i , and number of tax payers, N, affect a voter's demand for government services.
 - ◆ Specifically $G_W^* = H_W / -H_G$ and $G_N^* = H_N / -H_G$ where H is the first order condition above.
- viii. Recall that solving for these derivatives requires using the partial derivative version of the composite function rule and paying close attention to the location of all the

variables in the various functions included in "H," the first order condition. We find that:

$$\begin{aligned} \diamond G^*_W &= [-U_{CC}(g_G/N) + U_{GW}] / \\ & \quad -[U_{CC}(g_G/N)^2 - U_C(g_{GG}/N) - 2U_{CW}(g_G/N) + U_{GG}] > 0 \end{aligned}$$

and

$$\begin{aligned} \diamond G^*_N &= [-U_{CC}(g_G/N)(g(G)/N^2) + U_C(g_G/N^2) + U_{GW}(g(G)/N^2)] / \\ & \quad -[U_{CC}(g_G/N)^2 - U_C(g_{GG}/N) - 2U_{CW}(g_G/N) + U_{GG}] > 0 \end{aligned}$$

◆ That is to say, with head tax finance, **each voter's demand for a pure public service rises with personal wealth and with population.**

ix. Moreover, since demand is strictly increasing in W , it turns out that the median voter is the voter with median income.

- ◆ It is this voter, whose demand for public services will lie in the middle of the distribution.
- ◆ The voter with median income has a preferred service level G^{**} such that the same number of voters prefer service levels greater than G^{**} as those who prefer service levels lower than G^{**} .
- ◆ The comparative statics of a voter with median income can, in this case, be used to characterize the course of government spending through time, as other variables change (here, exogenous shocks to W or N , changes in tastes, etc.).

C. Other, somewhat richer, models can be built to analyze such problems as:

- i. The effects of different tax instruments: proportional and progressive tax instruments
- ii. Optimal redistribution motivated by narrow self interest and/or altruism
- iii. the effects of varying degrees of publicness on demand for services: club goods

D. For example, Meltzer and Richards (1981) provide a Spartan but sophisticated analysis of how the median voter model can be used to represent the equilibrium size of government in a pure transfer model of government policies.

E. It bears noting that not every median voter model has unambiguous predictions about the effects of changes in the parameters of the median voter's choice problem on the median voter's demand for a given public policy, but useful insights may be obtained about the relationships between those parameters of public policy formation are often obtained even in those cases.

III. On the Normative Properties of Median Voter Policies

A. Although the median voter model implies that the median voter gets what "he wants," it does not imply that public policies will be efficient in the usual Paretian sense.

i. This can be seen mathematically by comparing the service level in the above model with that which would be Pareto efficient in a society of three individuals with different tastes or wealth.

◆ [Recall that the Pareto Efficient level can be characterized with a social welfare function, or by maximizing one person's utility while holding the other's constant. See lecture notes.]

ii. Alternatively, one can develop a graphical illustration that demonstrates that the median voter will prefer an output (or other policy level) that is Pareto inefficient whenever the median and "average" voter have different ideal points.

B. The median voter model developed to this point has ignored information costs faced by all voters which might lead voter's to be less than perfectly informed about their tax burdens or the benefits of public programs.

- i. In the case where the median voter's expectations are unbiased, he/she will still on average get what he/she wants.
- ii. In cases where rational ignorance implies biased expectations about the consequences of policies (as for example when one remains entirely ignorant of some policy detail or implication) then the median voter may not even get what he/she truly wants.

C. Information problems open the door to interest groups and the bureaucracy who may manipulate voters by appropriately subsidizing various kinds of information and encourage malfeasance (agency costs, bribery) on the part of elected and unelected government officials which would be unlikely to be detected by rationally ignorant voters. [Later in the course, I will argue that essentially the whole special interest group/rent-seeking literature is predicated on informational problems of these kinds in open democratic societies.]

IV. Some Theoretical Weaknesses in the Median Voter Model

A. There is one nearly devastating weakness to the median voter model, namely "the median voter" does not always exist in even an analytical sense.

- i. Duncan Black is the modern (re) discoverer of the idea of electoral cycles in one dimensional policy spaces. In some, fairly unlikely, one dimensional arrays of voter preferences, the majority rule preference ordering may be non-transitive and no median voter would exist. [Single peaked preferences are sufficient to guarantee the existence of a median voter in one dimensional issue spaces. See also Arrow's generalization of this point in his well known Impossibility Theorem.]
- ii. In 2-dimensional cases, a median voter exists ONLY in cases where voter tastes are symmetrically arrayed (see Plott, 1969). In most plausible looking 2D policy space diagrams, cycles are endemic even if voter preferences are single peaked! E.G., in most illustrating examples, every policy has a non-empty *win set*. (Def: The win set of policy z is the set of policies which could beat z in a majority rule election or referendum)

B. Fortunately for advocates of "lean models" of politics, there is a body of evidence that suggests voter preferences over policies are (largely) of the sort which can be mapped

into a single issue space while retaining "single peakedness" (See, for example, Poole and Daniels, 1985).

- i. Moreover, the median voter model has a good empirical track record in Public Finance as a model of government program size across states and through time.
- ii. Thus in once sense the model is very frail, but in another appears to be quite robust.

C. (Buchanan has argued that "cycling" can, perhaps surprisingly, be a *good property* of majority rule systems insofar as it promotes equity. With cycling, everyone eventually gets to be a member of the majority coalition at some point and so will not be perpetually exploited.)

V. Another Electoral Model

- A. The only widely used alternative to the median voter model of electoral equilibrium is the stochastic voting model. This model uses a stochastic representation of the voter's behavior that is not entirely consistent with the usual economic representations of choice--or at least assumes that candidates for elective office believe that voters act in this somewhat non-rational manner.
- B. The stochastic voting model is widely used in the rational politics literature in Political Science and is widely *referred to* by many pieces in public economics that want to model government as maximizing a social welfare function.

VI. The Logic of Stochastic Voting

- A. As a Consequence of the Voter's Trembling Hand
 - i. The behavior of a stochastic voter is represented with a probability function. One common and simple way of representing the probability that a particular voter chooses candidate X over candidate Y assumes that this probability is simply the ration of the expected utility generated by candidate X's policies divided by the sum of the expected utilities of both (all) candidates.

$$P(X|Y) = \frac{U^e(X)}{[U^e(X) + U^e(Y)]}$$
 - ii. The most straightforward interpretation of voting in a stochastic voting model is that voters make mistakes. That is to say, voters do not always maximize utility, but rather *more often maximize utility than not*.
- B. As a Consequence of Candidate Ignorance
 - i. Another, and now more common interpretation of the model, is that candidates do no fully understand the preferences of voters and so try to imagine the consequences of their policies on the welfare of individual voters. These estimates are assumed to resemble the $P(X|Y) = f(U(X), U(Y))$
 - ii. Note that the non-stochastic voter model developed previously is a special case of this more general representation of the voter's behavior. $P(X|Y) = 1$ if $u(X) > u(Y)$ and $P(X|Y) = 0$ if $U(X) < u(Y)$ and $P(X|Y) = 0.5$ if $U(X) = U(Y)$.
 - iii. However, the stochastic voting literature generally assumes that function f is continuous and differentiable, which the former is not.

C. Candidates Choose Policies to Maximize Expected Votes

- i. As in the median voter model, candidates choose policies to maximize their prospects for election. Here they can either maximize expected voters--the usual assumption--or the probability of being elected (which is not always the same as the former, but more difficult to represent.)

VII. The Positive Properties of Electoral Equilibrium in a Stochastic Voting Model

- i. Given the position of the other candidate, and the probability functions for each voter, the expected voter for candidate X taking policy G^X is

$$V^e = \sum f_i(U_i(G^X), U_i(G^Y))$$
 - ii. Differentiating with respect to G^X allows candidate X's best reply function to be characterized:
 - ◆ $V^e_{G^X} = \sum (dF_i/dU_i) (dU_i/dG^X) = 0$
 - ◆ which implies that $G^{X*} = g^X(G^Y)$
 - iii. Note that candidates take account of the welfare of EVERY voter, and that the extent to which a voter's interest is taken account of by is a combination of the marginal utility to "voter i" of the policy variable of interest and the extent to which "voter i" will be *more likely* to vote for X as his utility increases.
 - iv. Another contrast with the median voter model, there is, in principle, a specific ideal policy position for each policy position a candidate's opponent(s) might take.
- A. In a symmetric election game, the Nash equilibrium is also symmetric. That is to say, each candidate takes the same policy position and iia holds for each candidate simultaneously.
 - B. Note that in cases where the joint (across all voters) probability distribution is symmetric that the **median** voter outcome can be an electoral equilibrium in this model as well, but more generally the policy positions taken will reflect a "weighted average" of all voter interests.
 - i. The weights are determined as in iia above: e.g. by the a combination of voter preferences (marginal utility) and electoral responsiveness (marginal increase in probability of voter for X as $U(G_x)$ increases).
 - C. The comparative statics of electoral equilibrium reflects changes in the (unstated) constraints of voters and/or changes in the technology of delivering government services.
 - D. Give the assumptions made, an electoral equilibrium always exists. That is to say there are no electoral cycles in the classic stochastic voting model.

VIII. A Digression on Social Welfare Functions

- A. Work that examines normative properties of electoral equilibrium relies either upon the Pareto Criteria or some social welfare function.
- Pareto Optimal/Efficient: A state of the world is Pareto Optimal if there is no change possible which would make at least one person better off and no one is worse off.
 - Pareto Superior. State A is Pareto Superior to state B, iff at least one person prefers A to B and no one prefers B to A.
 - (Note that a state C is Pareto Optimal if there are no feasible states which are Pareto superior to C.)
- B. A **general individualist social welfare function** has the property that social welfare increases in any case where one person gains and no one loses from a change in resource allocation. That is to say, a social welfare function is monotone increasing in the utility level of every individual in the society of interest.
- $$W = f(U_1, U_2, \dots, U_N)$$
- C. Interesting special cases of social welfare function include the utilitarian (Benthamite) **social welfare function** and the Nash social welfare function.
- Benthamite: $W = \sum U_i$
 - Nash: $W = \prod U_i$
- D. All individualistic social welfare functions have the property that constrained maximization yields a Pareto Optimal state. (Why?)
- For example, if a fixed stock of a pure private good were to be allocated to maximize social welfare, one would maximize W subject to the resource constraint $X = \sum X_i$. The result would be a unique social welfare maximum.
 - Note that many other states would also be Pareto optimal. In fact any distribution of private good X among non-altruists would be Pareto optimal including the one that maximized the social welfare function of interest.
 - From this illustration, you should be able to see that the weights of the social welfare function (consider the weighted Benthamite social welfare function, $W = \sum \alpha_i U_i$ where $\alpha_i > 0$) are very important determinants of the particular conclusions that emerge from maximizing social welfare.
 - If all individuals had the same utility functions, and all the weights were the same, then this divide the pie problem would have recommended dividing the pie up equally. However other weights would yield different recommendations/"social welfare" maximizing policies.

IX. On the Normative Properties of Electoral Equilibrium

- A. Note that the functions being maximized by candidates in a stochastic voting model are some what odd special cases of a social welfare function where the weights are implicitly determined by voter responsiveness to policy variables.

- B. That is to say, each candidate maximizes a social welfare function OVER THE VOTERS participating in the election of interest.
- C. Consequently, if every one votes, the electoral policy result in such a model is ALWAYS Pareto optimal.

X. Normative Controversy.

- A. The electoral result of a stochastic voting model would fail to be Pareto optimal, only if some individual interests were *entirely neglected*, or *possibly given negative weight*.
- Some people may never vote, or may not vote when policy positions of candidates are close to each other or very far from their own ideal policy.
 - (For example, some voters may be more likely to support a candidate that promised to punish a subset of the electorate: perhaps racists, specific ethnic groups, or smokers.)
 - In these cases, the expected vote function is not a social welfare function and the electoral results would be unlikely to be Pareto optimal.
- B. Another area of controversy involves the cycle free nature of electoral equilibria in stochastic voting models. It turns out that the nature of the probability function describing voter behavior is critical. (See Feldman and Lee (1988)).
- A simple illustration may suffice, suppose that voters always voted for the candidate with the more favorable program, but occasionally made errors. Suppose that the errors are not affected by the other candidate's position.
 - In this case, one would have a stochastic voter who behaved in an expected value sense just like the non-stochastic voters of the median voter type model. Again cycles could and would tend to be endemic in a 2 or more dimensioned policy space. The particular representation of stochastic voting is important.