

I. There are a variety of policy implications associated with the median voter model.

- A. First, policies tend to be moderate, e. g. drawn from the middle part of the political spectrum.
 - ▶ The middle can be regarded as "moderate" essentially by definition.
- B. Second, many, indeed most, people will be less than completely satisfied with the policies chosen, because their ideal points differ from those of the median voter.
 - ▶ "Unhappiness," in this sense tends to be present in perfectly functioning democracies, as long as peoples tastes, circumstances, ideologies, and/or expectations differ.
 - ▶ It bears noting, however, that most people may be dissatisfied with government policy, yet still prefer the use of majoritarian decision rules to all the others that they are aware of.
- C. Third, in cases in which the strong form of the median voter theorem applies, government policies tend to maximize the median voter's expected utility, given his constraints, expectations, and goals.
 - i. An implication of "C" is that increases dispersion of the distribution of voter preferences (increased radicalism) tends to have little, if any, effect on public policies unless it affects the median of the distribution of voter ideal points.
 - ▶ *This implies that median voter policies will be more stable than average voter policies.*
 - ii. Another implication is that *any change in circumstance that changes the constraints of the median voter, or the identity of the median voter, tends to change the size and composition of government programs.*
- D. Property "C" implies that public policies can be modeled as the solution to one person's political optimization problem, whenever the strong form of the median voter theorem is likely to hold.
 - i. Such optimization problems are often very straightforward to characterize and perform comparative statics on.
 - ▶ Consequently, the median voter model is widely used to analyze the level and growth of government service levels.

- ▶ It plays a significant role in both the theoretical and empirical public finance literature dealing with taxes and expenditure levels.
- ii. (As developed later in the course, the median voter model is often augmented by taking the effects of interest groups and institutions into account)
- iii. Note that the strong form of the median voter theorem seems likely to be more relevant to two-party electoral contests held under plurality rule than to the multiparty contests held under proportional representation.
 - ▶ However, the weak form of the median voter theorem is likely to hold for PR systems of governments. The party supported by the median voter is likely to be a member of the ruling coalition.
- iv. **Some food for thought: Are any of these properties of electoral equilibrium sensitive to the assumption that everyone votes? How would rational abstention affect the results? Does it matter whether abstentions are predictable or not?**

II. Examples of Median Voter Models

- A. Consider a referendum for a public service that is funded with a non-distorting "head tax."
 - i. Each voter in his capacity as a policy "maker" looks very much like the standard consumer in a grocery store, except that in addition to private budget constraints, he has a "public" budget constraint to deal with.
 - ii. Suppose that voter's have the same utility function defined over private consumption (C) and some public service (G). But suppose further that each voter has a different amount of money, W_i , to allocate between C and G, and, further, assume that the government faces a balanced budget constraint, and that all expenditures are paid for with a head tax, T. Assume that there are N tax payers in the polity of interest.
 - iii. Thus:
 - a. $U = u(C, G)$
 - b. $W_i = C + T$
 - c. $g(G) = NT$
 - iv. Note that T can be written as $T = g(G)/N$ and substituted into the private budget constraint to make a single unified budget constraint:
 - ▶ $W_i = C + g(G)/N$

v. This in turn can be solved for C and substituted into the utility function:

► $U = u(W_i - g(G)/N, G)$

vi. Differentiating with respect to G yields a first order condition that characterizes the median voter's preferred government service level:

► $-U_C (gG/N) + U_G = 0 = H$

► or equivalently as $U_C (gG/N) = U_G$

- a. The right hand side of the latter is the subjective marginal benefit (marginal utility) of the government service, the left-hand term is the subjective marginal opportunity cost of government services in terms of lost private consumption.
- b. Note that the subjective marginal cost of the service is determined by both preferences (marginal utility of the private good C) and objective production or financial considerations, cG/N . The latter can be called the median voter's marginal cost share, or price for the government service.

vii. An implication of the first order condition together with the implicit function theorem is that each voter's demand for public services can be written as:

- a. $G_i^* = \gamma(W_i, N)$ that is to say, as a function of his own wealth (holding of the taxable base) and the population of tax payers in the polity of interest.
- b. The implicit function differentiation rule allows one to characterize comparative statics of how changes in wealth, W_i , and number of tax payers, N , affect a voter's demand for government services.
- c. Specifically $G^*_W = H_W / -H_G$ and $G^*_N = H_N / -H_G$ where H is the first order condition above.
- d. Recall that solving for these derivatives requires using the partial derivative version of the composite function rule and paying close attention to the location of all the variables in the various functions included in "H," the first order condition. We find that:

$$G^*_W = [-U_{CC} (gG/N) + U_{GW}] /$$

$$-[U_{CC} (gG/N)^2 - U_C (gGG/N) - 2 U_{CW} (gG/N) + U_{GG}] > 0$$

and

$$G^*_N = [-U_{CC} (gG/N)(g(G)/N^2) + U_C (gG/N^2) + U_{GW}(g(G)/N^2)] /$$

$$-[U_{CC} (gG/N)^2 - U_C (gGG/N) - 2 U_{CW} (gG/N) + U_{GG}] > 0$$

- e. That is to say, with head tax finance, each voter's demand for a pure public service rises with personal wealth and with population.
- viii. Moreover, since demand is strictly increasing in W , it turns out that the median voter is the voter with median income.
 - a. It is this voter, whose demand for public services will lie in the middle of the distribution.
 - b. The voter with median income has a preferred service level G^{**} such that the same number of voters prefer service levels greater than G^{**} as those who prefer service levels lower than G^{**} .
 - c. The comparative statics of a voter with median income can, in this case, be used to characterize the course of government spending through time, as other variables change (here, exogenous shocks to W or N , changes in tastes, etc.).

III. Other Applications and Illustrations

- A. Consider the following model of a voter's preferred level of environmental regulation.
 - i. Let $U = u(Y, E)$ where Y is material consumption (income) realized by the median voter, and E is the (perceived) level of environmental quality. Suppose that environmental quality is a function of regulatory stringency R and national income, $E = e(R, Y)$.
 - ii. To simplify a bit, suppose that the median voter gets a constant fraction "a" of national income which is decreasing in regulatory stringency, $Y = y(R)$ and $Y_m = aY$
 - iii. The constraints and definitions can be substituted into the median voter's utility function: $U = (ay(R), e(R, y(R)))$
 - iv. This can be differentiated with respect to R to characterize the median voter's ideal stringency of environmental regulation R^* .
 - v. R^* will satisfy $U_Y aY_R + U_E (E_R + E_Y Y_R) = 0$
 - vi. The first time is the median voter's marginal cost and the last is his marginal benefit from more stringent environmental regulation. (Explain why.)

- vii. The implicit function theorem (see class notes) can be used to determine the comparative statics of environmental regulation with respect to parameters of the median voter's optimization problem. The results are (qualitative) forecasts of public policy in this area.
- B. A somewhat more realistic electoral model of the demand for pure public goods can be developed by changing the fiscal constraints a bit.
- i. Suppose that G is a public service received by the median voter (which may or may not be a pure public good). Again let his utility level be defined over other consumption, here C , a variable affected by the policy of interest, here the level of G provided.
 - ii. Let the level of G be an increasing function of the taxes collected, $G = g(T)$ and suppose this time that the total tax revenue be a function of national income.
 - ▶ Instead of a head tax, assume that a proportional tax is used. ($T = a + tY$ is a surprisingly good first approximation for aggregate taxation in the US.)
 - ▶ Suppose further that national income, Y , is itself negatively affected by the marginal tax rate t , here $Y = y(t)$. (That is to say assume that the tax has a deadweight loss.)
 - iii. Again assume that each voter's personal income is some constant fraction of national income, a_i , so that voter i 's private consumption is $C = (1-t) a_i Y$.
 - iv. To characterize public policy using the strong form of the median voter theorem, substitute the median voter's constraints into the median voter's utility function.
 - ▶ This yields an optimization problem with one control variable (here t , which implicitly determines both G and y).
 - ▶ $U = u((1-t) a Y, G(t y(t)))$
 - ▶ [Puzzle: why is there just one degree of freedom in this characterization of the median voter's optimization problem?]
 - v. Differentiating with respect to t characterizes his ideal tax level, t^* will satisfy:
 - ▶ $U_C [(-1)aY + (1-t)aY_t] + U_G G_T (Y + tY_t) = 0$
- ▶ Given all the assumptions, the median voter's ideal tax rate, public service level, and private consumption level are all determined by the median voter's parameter a , that is the share of gdp received by the median voter.
 - ▶ That is: $t^* = f(a_m)$ where m denotes the median voter.
- vi. Given t^* , the median voter's ideal public service level can be written as:
- ▶ $G^* = g(t^* Y(t^*))$
 - ▶ National income as $Y = Y(t^*)$
 - ▶ and the median voter's private consumption as: $C_m = (1-t^*) a_m Y(t^*)$
 - ▶ (t^* in all these equations is the the tax rate that satisfies the first order condition above.)
- vii. The implicit function differentiation rule can be used to characterize the comparative statics of the median voter's choice.
- ▶ This allows us to make forecasts about the course of public policy in this area.
 - ▶ However, the model is still a fairly lean one, in that there is just one exogenous parameter, namely the median voter's share of national income.
 - ▶ (In a competitive economy, this would reflect his or her value marginal product.)
- viii. As an exercise, construct a somewhat more "concrete" model where there is a balanced budget constraint, and G is produced via constant returns to scale, the cost of G is just cG . (This will produce another exogenous parameter of the median voter's choice problem.)
- ▶ One could also model national income in a more elaborate way, by for example including technology or capital in the aggregate production function.
 - ▶ Other income tax schedules could also be used, E. G. a linear one $T = a + tY$
- C. Note that all of the above models have implication about the proper way to estimate the demand for services.

- ▶ They indicate which variables are endogenous and exogenous.
 - ▶ They indicate which relationships belong in a simultaneous system of equations.
 - ▶ They indicate variables that should be included in both the system of equations and in reduced form estimates.
- D. Median voter models can also be used to characterize different “kinds” of median voters and different conceptions of state enterprises.
- i. For example, Meltzer and Richard (1981) provide a Spartan but sophisticated analysis of how a median voter model can be used to represent the equilibrium size of a **pure transfer model** of government policies.
 - a. Their model assumes that a demogrant program is financed with a proportional income tax and that the tax has a deadweight loss.
 - b. Their analysis can be characterized with just two fiscal assumptions and one economic assumption that can be summarized in two equations:
 - ▶ voter i 's after tax income is: $Y_i = (1-t) y(t) + G$
 - ▶ the demogrant, G , received is $G = 1/N (t \sum Y_i)$ where N is the number of demogrant recipients
 - ▶ Voter's choose t and G to maximize the income (or utility from income).
 - ii. In many cases, such a model implies corner solutions of complete or no redistribution.
 - a. Ignoring deadweight losses from taxation, all voters with below average income favor a tax of 100% and all voters with incomes above the average prefer no redistribution.
 - b. So, “extremism” is common among voters in such models.
 - iii. In the case they analyze, however, the deadweight loss of the tax system induces the median voter to redistribute at an intermediate level, because intermediate levels maximize the demogrant received.
 - ▶ The deadweight loss of tax systems allows intermediate solutions.
 - ▶ Intermediate solutions may also arise if voters are altruistic and the median voter's permanent income is above average.
 - ▶ [illustrate]
- E. It bears noting that not every median voter model has unambiguous predictions about the effects of changes in the parameters of the median voter's choice problem on the median voter's demand for a given public policy.
- i. In such cases, the model helps to identify relationships and variables for the purposes of empirical tests.
 - ▶ It is often surprising that some parameters cannot be easily signed without making fairly strong assumptions about voter preferences, incomes or tax systems.
 - ii. Counter intuitive results (from plausible models) can also **improve our intuition** and also our understanding of the assumptions required to be able to qualitatively predict political outcomes without estimating median voter demand functions.
- F. As mentioned earlier, **median voter models can also be augmented** to examine the effects of institutions, ideology, information problems, and interest groups.
- i. The expected democratic consequences of different assumptions about human nature can also be explored.
 - ii. Policy preferences like preferences for most economic goods, include both subjective factors, many of which can be represented with the geometry and math worked out in the public choice literature.
 - iii. Subjective factors may be further analyzed and extended by including ideology, altruism, and other behavioral factors into the models of voter preferences and behavior.
- G. Overall median voter models are powerful “analytical machines” that can be used to analyze the consequences of political competition in democratic states populated by more or less rational individuals.
- i. The results provide useful qualitative predictions about democratic government policies.
 - ▶ The models and predictions can be estimated and tested using conventional statistical techniques.
 - ii. Although such models are clearly simplifications of far more complex phenomena, they capture an important, fundamental, property of political competition within democratic states.

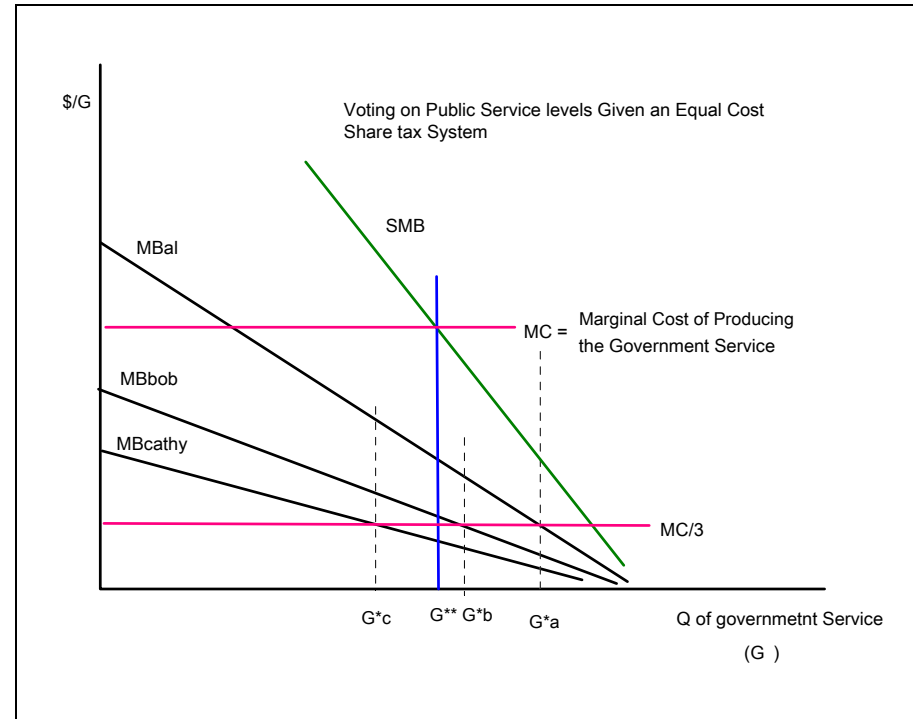
- ▶ Nonetheless, both the median voter model and the stochastic voter model suffer from a number of limitations, many of which are similar to the limitations of the perfect competition models of neoclassical economics.

IV. On the Normative Properties of Median Voter Policies

- A. Although the median voter model implies that the median voter gets what "she wants," it does not imply that public policies will be efficient in the usual Paretian sense.
- This can be seen mathematically by comparing the service level in the above model with that which would be Pareto efficient in a society of three individuals with different tastes or wealth.
 - It turns out that Pareto Efficient level can be characterized with a social welfare function, or by maximizing one person's utility while holding the other's constant.
 - If a Benthamite social welfare function is used, it turns out that the ideal is pretty much what the "average" voter rather than the median voter wants.
 - ▶ A Benthamite social welfare function is simply $W = \sum U_i$.
 - Graphical illustrations work very well for showing a variety of cases in which a median voter's ideal point differs from the Pareto efficient outcome.
 - The illustration below shows how an equal share funding rule operates.
 - ▶ (Such rules are often used by private clubs.)
 - Note that the median voter in this case prefers a bit more of the public service than the Pareto efficient level G^{**} .
 - ▶ [How would you change the diagram to produce a case in which the median voter under demands the public service?]
 - Unfortunately, graphs are not very good at capturing simultaneity issues, but they can easily show how different cost sharing rules (tax systems) and distributions of preferences affect median assessments of "optimal" service levels.

V. Information, Agency, and Timing Problems

- A. The median voter model developed to this point has ignored information costs faced by all voters which might lead voter's to be less than



perfectly informed about their tax burdens or the benefits of public programs.

- In the case where the median voter's expectations are unbiased, he/she will still on average get what he/she wants.
 - In cases where rational ignorance implies biased expectations about the consequences of policies (as for example when one remains entirely ignorant of some policy detail or implication) then the median voter may not get what he/she *truly* wants.
 - For example, in public finance, there is a fairly large literature on "fiscal illusion."
- B. Several strands of the rational-choice literature on elections, and the next few lectures, analyze such problems.
- C. Information problems open the door to interest groups and a variety of principle agent problems both among elected representatives and the bureaucracy.

- ▶ Essentially the whole special interest group/rent-seeking literature is (implicitly) predicated on informational problems of these kinds in open democratic societies.
- ▶ It is information problems that potentially allow candidates, interest groups, and the bureaucracy to “manipulate” voters by appropriately subsidizing various kinds of information and encourage malfeasance (agency costs, bribery) on the part of elected and unelected government officials which would be unlikely to be detected by rationally ignorant voters.

VI. Appendix: the Stochastic Voting Model

- A. The only widely used alternative to the median voter model of electoral equilibrium is the stochastic voting model. This model uses a stochastic representation of the voter's behavior that is not entirely consistent with the usual economic representations of choice--or at least assumes that candidates for elective office believe that voters act in this somewhat non-rational manner.
- B. The stochastic voting model is widely used in the rational politics literature in Political Science and is widely *referred to* by many pieces in public economics that want to model government as maximizing a social welfare function.
- C. Stochastic voting can be regarded as a consequence of the voter's “trembling hand.”
- i. The behavior of a stochastic voter is represented with a probability function. One common and simple way of representing the probability that a particular voter chooses candidate X over candidate Y assumes that this probability is simply the ratio of the expected utility generated by candidate X's policies divided by the sum of the expected utilities of both (all) candidates.

$$P(X|Y) = \frac{U^e(X)}{[U^e(X) + U^e(Y)]}$$
 - ii. The most straightforward interpretation of voting in a stochastic voting model is that voters make mistakes. That is to say, voters do not always maximize utility, but rather *more often maximize utility than not*.
- D. As a Consequence of Candidate Ignorance

- i. Another, and now more common interpretation of the model, is that candidates do not fully understand the preferences of voters and so try to imagine the consequences of their policies on the welfare of individual voters. These estimates are assumed to resemble the $P(X|Y) = f(U(X), U(Y))$
- ii. Note that the non-stochastic voter model developed previously is a special case of this more general representation of the voter's behavior. $P(X|Y) = 1$ if $u(X) > u(Y)$ and $P(X|Y) = 0$ if $u(X) < u(Y)$ and $P(X|Y) = 0.5$ if $U(X) = U(Y)$.
- iii. However, the stochastic voting literature generally assumes that function f is continuous and differentiable, which the former is not.

E. Candidates Choose Policies to Maximize Expected Votes

- i. As in the median voter model, candidates choose policies to maximize their prospects for election. Here they can either maximize expected voters--the usual assumption--or the probability of being elected (which is not always the same as the former, but more difficult to represent.)

F. The Positive Properties of Electoral Equilibrium in a Stochastic Voting Model

- i. Given the position of the other candidate, and the probability functions for each voter, the expected voter for candidate X taking policy G^X is

$$V^e = \sum f_i(U_i(G^X), U_i(G^Y))$$
 - ii. Differentiating with respect to G^X allows candidate X's best reply function to be characterized:
 - a. $V^e_{G^X} = \sum (dF_i/dU_i) (dU_i/dG^X) = 0$
 - b. which implies that $G^{X*} = g^X(G^Y)$
 - iii. Note that candidates take account of the welfare of EVERY voter, and that the extent to which a voter's interest is taken account of by is a combination of the marginal utility to "voter i" of the policy variable of interest and the extent to which "voter i" will be *more likely* to vote for X as his utility increases.
 - iv. Another contrast with the median voter model, there is, in principle, a specific ideal policy position for each policy position a candidate's opponent(s) might take.
- G. In a symmetric election game, the Nash equilibrium is also symmetric. That is to say, each candidate takes the same policy position and *iii* holds for each candidate simultaneously.

- H. Note that in cases where the joint (across all voters) probability distribution is symmetric that the **median** voter outcome can be an electoral equilibrium in this model as well, but more generally the policy positions taken will reflect a "weighted average" of all voter interests.
- The weights are determined as in iii above: e.g. by the a combination of voter preferences (marginal utility) and electoral responsiveness (marginal increase in probability of voter for X as $U(Gx)$ increases).
- I. The comparative statics of electoral equilibrium reflects changes in the (unstated) constraints of voters and/or changes in the technology of delivering government services.
- J. Give the assumptions made, an electoral equilibrium always exists. That is to say there are no electoral cycles in the classic stochastic voting model.

VII. Appendix: A Digression on Social Welfare Functions

- A. Work that examines normative properties of electoral equilibrium relies either upon the Pareto Criteria or some social welfare function.
- Pareto Optimal/Efficient: A state of the world is Pareto Optimal if there is no change possible which would make at least one person better off and no one is worse off.
 - Pareto Superior. State A is Pareto Superior to state B, iff at least one person prefers A to B and no one prefers B to A.
 - (Note that a state C is Pareto Optimal is there are no feasible states which are Pareto superior to C.)
- B. A **general individualist social welfare function** has the property that social welfare increases in any case where one person gains and no one loses from a change in resource allocation. That is to say, a social welfare function is monotone increasing in the utility level of every individual in the society of interest.
- $$W = f(U_1, U_2, \dots, U_N)$$
- C. Interesting special cases of social welfare function include the utilitarian (Benthamite) **social welfare function** and the Nash social welfare function.

- Benthamite: $W = \sum U_i$
 - Nash: $W = \prod U_i$
- D. All individualistic social welfare functions have the property that constrained maximization yields a Pareto Optimal state. (Why?)
- For example, if a fixed stock of a pure private good were to be allocated to maximize social welfare, one would maximize W subject to the resource constraint $X = \sum X_i$. The result would be a unique social welfare maximum.
 - Note that many other states would also be Pareto optimal. In fact any distribution of private good X among non-altruists would be Pareto optimal including the one that maximized the social welfare function of interest.
 - From this illustration, you should be able to see that the weights of the social welfare function (consider the weighted Benthamite social welfare function, $W = \sum \alpha_i U_i$ where $\alpha_i > 0$) are very important determinants of the particular conclusions that emerge from maximizing social welfare.
 - If all individuals had the same utility functions, and all the weights were the same, than the Benthamite solution to the divide the pie problem would have recommended dividing the pie up equally. However other weights would yield different recommended "social welfare" maximizing policies.
- E. The normative properties of electoral equilibrium in stochastic voting models resemble those of a weighted social welfare function.
- Note that the functions being maximized by candidates in a stochastic voting model are some what odd special cases of a social welfare function where the weights are implicitly determined by voter responsiveness to policy variables.
 - That is to say, each candidate maximizes a social welfare function OVER THE VOTERS participating in the election of interest.
 - Consequently, if every one votes, the electoral policy result in such a model is ALWAYS Pareto optimal.
- F. The electoral result of a stochastic voting model would fail to be Pareto optimal, only if some individual interests were *entirely neglected*, or *possibly given negative weight*.
- Some people may never vote, or may not vote when policy positions of candidates are close to each other or very far from their own ideal policy.

- ii. (For example, some voters may be more likely to support a candidate that promised to punish a subset of the electorate: perhaps racists, specific ethnic groups, or smokers.)
 - iii. In these cases, the expected vote function is not a social welfare function and the electoral results would be unlikely to be Pareto optimal.
- G. An area of controversy involves the “cycle free” nature of electoral equilibria in stochastic voting models. It turns out that the nature of the probability function describing voter behavior is critical. (See Feldman and Lee (1988)).
- i. A simple illustration may suffice, suppose that voters always voted for the candidate with the more favorable program, but occasionally made errors. Suppose that the errors are not affected by the other candidate's position.
 - ii. In this case, one would have a stochastic voter who behaved in an expected value sense just like the non-stochastic voters of the median voter type model. Again cycles could and would tend to be endemic in a 2 or more dimensioned policy space. The particular representation of stochastic voting is important.