

I. The Burden of Taxation

A. Taxation is the primary method by which governments finance themselves.

- i. Essentially all taxes transfer resources from the private to public sector, where government decision makers (both elected and unelected) will choose how those resources will be allocated between providing services and redistribution.
- ii. Essentially all taxes shift resources to the government by threatening current resource holders (property owners, labor, international trading firms, etc.) with punishments of various sorts if they do not "give" their resources to the government's tax collectors.
 - a. In this sense, **all taxes are coercive at the point of collection.**
 - b. This contrasts with government bonds and ordinary fees for services, because such transactions are voluntary at the point of collection. Bond buyers and public service purchasers expect to be better off after the purchase, whereas tax payers normally feel worse off after paying the tax (although better off than had they not paid and been placed in jail).
 - c. (On the other hand, insofar as taxes are used to fund desired public services, taxation as a method of government finance can be regarded as voluntary in much the same sense that the amounts paid stores for their products can be regarded as voluntary. In such cases, voters prefer to "tax themselves" to pay for desired governmental services, rather than go without those services.)
- iii. To the extent that government borrowing (sales of bonds) are repaid, even borrowing can be regarded as an implicit tax, because the government promises bond buyers that they will collect tax revenue in the future to pay interest and principal on the bonds.
- iv. Alternative non-tax sources of revenue involve sales of government services or assets to the public..
 - a. For example, the U. S. central government (federal government) sells assets when it sells mineral, timber, and grazing rights to industry.
 - b. National parks are partially financed by charging admission, as are toll bridges and roads.

B. The most important taxes for the federal government of the United States today are:

- i. the income tax (1.168 trillion dollars in 2007)

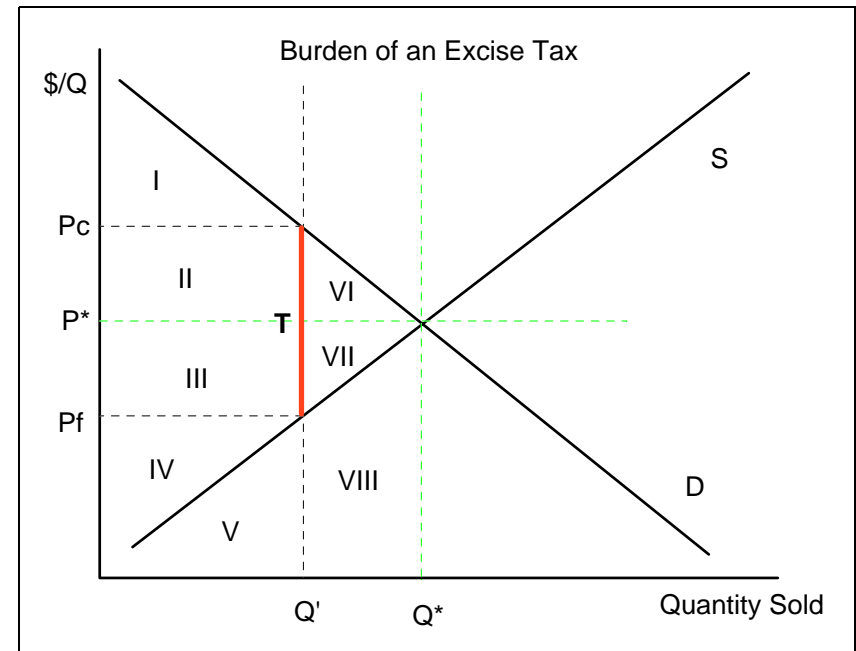
- ii. the payroll tax (used to finance social security) (873 billion dollars in 2007)
- iii. corporate income taxes (342 billion dollars in 2007)
- iv. excise taxes (57.1 billion dollars in 2007)
- v. At the state and local levels of taxation in the United States, the most important taxes in 2007 were: the sales tax (414 billion), the property tax (370 billion), the income tax (275 billion).
- vi. (Source: *2008 Statistical Abstract of the United States*)
- vii. (Tax receipts fell somewhat in the next two years because of the recession.)

C. The burden of particular taxes can be measured in two ways:

- i. First, [the accounting method] it can be calculated as a cash payment--in much the same way that payments for ordinary goods are calculated.
 - a. (This is the most widely used measure by macro-economists, accountants, and newspaper reporters.)
 - b. (It is also occasionally implicitly used by public finance economists. When data on the slopes of relevant demand and supply curves are unavailable. In such cases, its often convenient as a first approximation to assume that supply is horizontal--as in a Marshallian competitive long run equilibrium--and so the full economic burden falls on consumers, as shown below.)
- ii. Second, [the economic method] the burden of taxation can be calculated by determining the **losses** imposed on taxpayers as a consequence of the tax--that is to say the *opportunity cost of the tax*.
 - a. This is the approach used by most economists (most of the time).
 - b. From an economic perspective, the burden of an excise or income tax can be measured as the **reduction of consumer surplus and profits** induced by the tax.
 - ◆ (This measure how much worse off consumers and firms are because of the tax itself, which ignores any benefits they may receive from the tax financed services.)
 - ◆ (In an indifference curve representation of tax burden, burden is the change in utility associated with the tax.)
 - c. This measure of burden differs a bit from the money paid to the government in several ways.
 - ◆ First, the total burden of a tax is normally larger than the amount of money that taxpayers send into the treasury.

- ◆ Most taxes have a **deadweight loss**, which can be measured as the extent to which "social surplus" is reduced by the existence of a particular tax, which as we will see depends partly on how the taxes are spent.
 - ◆ Second, the distribution of the tax burden varies with market conditions (the slopes of the relevant supply and demand curves) rather than with who signs the check sent to the treasury.
- iii. In this sense, tax payments are often made by persons or firms who do not write checks to the treasury, and who may not be "obviously" affected by a particular tax law..
- a. For example, sales taxes are paid by firms in the sense that firms (or firm owners) actually write the checks deposited in the government's treasury. Thus, calculated as cash payments, one could say that the burden of a sales tax falls entirely on firms.
 - b. However, if firms simply increase their prices to pay for the tax, which is what they appear to do at the cash register, then the tax burden has really been "shifted" forward onto their customers, even though consumers never actually write checks for sales taxes and send them into the treasury.
 - c. In many cases, the persons most affected by a tax are **not** the persons who "directly" pay the taxes by writing out a check to the treasury or IRS!
 - d. Note also that one can not easily assure that tax payments are shared equally by firms and consumers or by employees and employers by simply requiring each to send in a check for half of the tax collected.
 - e. Ultimately, the distribution of tax burden depends on the slopes of the demand and supply curves in the markets of interest.
 - f. (See the illustration below.)
 - g. (Similar results hold for monopolists, although the diagrams are a bit different.)

D. Illustration of the economic burden of an excise tax:

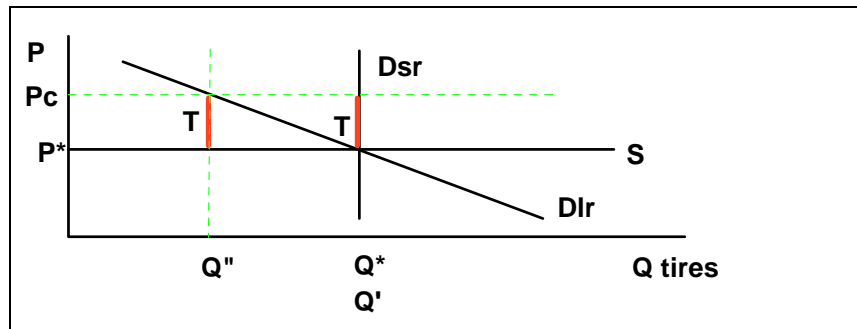
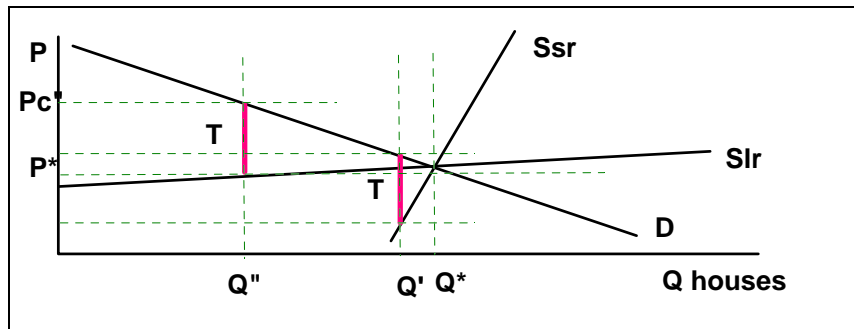


- i. Suppose that a market is initially in an equilibrium without taxes, so that demand equal supply at P^* . In this case, there is no "tax wedge" between the price paid by consumers, P_c , is the same as that received by firms, P_f ; so $P_f = P_c = P^*$.
 - a. Now, suppose that an excise tax of T is imposed on each unit of the good sold in this market, as for example is done with tire sales in the US.
 - b. After the tax is imposed, P^* is no longer the market clearing price:
 - c. If T is simply added to P^* by firms, consumers will purchase too little at their new price ($P_c = P^* + T$) to match supply, which would remain at Q^* .
 - d. On the other hand, if firms simply "ate" the tax, they would provide too little of the good to meet demand (at their after tax price of $P_f = P^* - T$). Supply would fall and demand would remain at Q^* if $P_c = P^*$ and $P_s = P^* - T$.
 - e. To clear the market, thus, firms have to receive less than P^* per item sold, and consumers have to pay more than P^* .

- f. At the new equilibrium output, the demand curve will be exactly T dollars above the supply curve, and $Q_d(P_f + T) = Q_s(P_f)$.
- ii. This equilibrium output is shown in the diagram. At Q' , supply equals demand, if the price paid by consumers is exactly T dollars higher than the amount firms receive ($P_f = P_c - T$).
- ◆ Q' units of the good are sold, with $Q' < Q^*$.
- a. At this equilibrium, there is a sense in which the tax has simply been passed onto consumers, because $P_c = P_f + T$.
- ◆ However, there is another sense in which the **burden of taxation is shared** by firms and consumers, because both consumer surplus and profits have been diminished by the tax!
 - ◆ **Consumer Surplus falls** from area I + II + VI (before the tax at Q^*) to just area I after the tax is imposed and output falls to Q' .
 - ◆ Similarly, **Profit falls** from III + IV + VII (before the tax at Q^*) to area IV (after the tax at Q').
- b. The burden on consumers is II + VI, and that on firms is III + VII.
- c. Note that **this distribution of the loss of consumer and firm net benefits occurs regardless of who actually writes the check to the state or federal treasury.**
- ◆ Price movements ultimately determine the actual division of burden between firms and consumers.
 - ◆ If firms send in the check, their effective "payment" is reduced by the increase in price paid by consumers.
 - ◆ If consumers write out the checks, their effective "payment" is reduced by the price decrease absorbed by firms.
- iii. The amount of revenue raised by the tax is $T \cdot Q'$.
- a. Q' units are sold and each pays a tax of T dollars.
- b. The total tax revenue, TQ' , can be represented in the diagram area II + III in the diagram.
- ◆ (Note that II + III is the area of a rectangle T tall and Q' wide.)
- iv. Notice that the tax revenue is smaller than the "surplus" lost by taxpayers (firms and consumers in the affected market).
- a. The reduced profit plus the reduced consumer surplus equals $\{II + VI\} + \{III + VII\}$.
- b. The total burden of this tax is VI + VII larger than the tax revenue.

- c. This area of "excess burden" is sometimes referred to as the **deadweight loss of an excise tax.**
- E. Both the extent of the deadweight loss and the distribution of the tax burden vary with the slopes of the supply and demand curves.
- i. Generally, more of the burden falls on the side of the market with the least price sensitive curves.
- a. If the demand curve is less elastic than the supply curve, more of the burden falls on consumers than on firms.
- ◆ In the extreme case in which market demand is completely inelastic or the industry supply curve is completely elastic, all of the burden falls on consumers!
- b. On the other hand if the demand curve is very elastic, because good substitutes exist, or the supply curve is relatively inelastic then more of the burden tends to fall on the firm.
- ◆ In the extreme case in which the market supply of the product of interest is completely inelastic or consumer demand is perfectly elastic, all of the burden falls on suppliers.
- ii. The excess burden of a tax tends to increase with the price sensitivity (slope or elasticity) of the demand and supply curves.
- F. Both supply and demand tend to be **more elastic in the long run than in the short run**, consequently, the excess burden of taxation tends to be larger in the long run than in the short run.
- i. In cases in which long run and short run demand are the same, the fact that long run supply is relatively more price sensitive (elastic) than short run supply implies that the burden of a new tax or increase in tax tends to be gradually shifted from firms to consumer in the long run.
- ◆ Many competitive markets have perfectly elastic supply curves in the long run, which implies that narrow taxes on such products are shifted entirely to consumers in the long run.
- ii. There are, however, also cases in which consumer demand is more price elastic in the long run than in the short run (as when demand for a good is determined in part by consumer capital goods, like automobiles).
- ◆ In such cases, a tax such as a gasoline tax may be gradually shifted from consumers to firms (owners of capital and natural resources) in the long run.

- iii. In cases where both sides of the market (firms and consumers) are more price elastic in the long run than in the short run, the shift of burden will reflect their relative ability to adjust.
- iv. However, all such long run adjustments imply that **deadweight losses to narrow taxes, such as an excise tax, are larger in the long run than in the short run.**
- v. Illustrations: effects of an excise tax in the short run and long run for different kinds of markets



- vi.
 - a. Note that in the first case, supply is more elastic in the long run than in the short run, so the initial effect of the tax is largely on firms, but in the long run the burden is shifted mostly to consumers.
 - ◆ The after tax price falls at first for firms, but rises back to P^* .
 - ◆ The price to consumers rises just a bit at first, but rises to $P^* + P_c$ in the long run.
 - b. The second case is an unusual case where demand is more price sensitive (elastic) in the long run than in the short run, but because supply is completely elastic in both the long and short run, the burden falls entirely on consumers in both the short and long run.

- c. As an exercise, construct a case in which the burden falls entirely on firms in both the long and short run.
- vii. In some cases, losses of consumer surplus may occur in other markets as a result of excise taxes.
 - ◆ For example high gasoline taxes encourage bicycle purchases, which tend to increase the price paid for bicycles, at least in the short run.
 - ◆ However, profits rise by nearly as much as the consumer surplus falls, so we will ignore these secondary effects in most of our analysis of tax burdens in this class.

II. Normative Principles of Taxation

- A. The ideas summarized in these diagrams are have often been used to characterize "normative theories" of taxation.
- B. For example, one normative theory of taxation was proposed by Frank **Ramsay** in 1927. He argued that a system of excise taxes should attempt to **minimize the excess burden** of the tax system.
 - i. A **Ramsay tax** system imposes higher taxes on markets with relatively inelastic supply and demand curves, and relatively lower taxes on markets with relatively large price sensitivities.
 - ii. If markets with perfectly inelastic demand or supply curves exist, government services can be financed without any deadweight loss at all, if taxes on such goods can generate sufficient revenues.
 - a. (Remember that taxes on products with inelastic supply or demand curves generate no deadweight losses.)
 - b. A special case of such a tax is a tax on land--which is sometimes called a Georgist tax after Henry George who proposed financing government entirely with land taxes. The supply of land, after all, is perfectly inelastic (ignoring dikes and dumps).
 - c. (Analyze the limitations, if any, of a Georgist land tax. Where does the value of a piece of land come from? Would there be allocative affects across different types of land? Would a Georgist land tax be neutral even if it is a Ramsay tax?)
 - d. (Given the possibility of international emigration, can their actually be a tax that has no dead weight loss? Discuss.)
- C. Another normative principle argues that a tax system should not directly affect *relative prices* across markets (see the indifference curve analysis in the appendix).

- i. That is to say, a tax system should be **NEUTRAL**.
 - a. A perfectly neutral tax system would not affect private sector decisions across markets for private goods and services, because a neutral tax does not affect relative prices faced by firms or consumers (although it does, of course, produce revenues for the government).
 - b. A Neutral system of excise taxes would impose excise taxes so that prices increased by the same proportion in every market taxed.
 - c. Alternatively, the government could look for somewhat broader or narrower tax sources that do not have relative price effects, such as a general sales tax, or a **lump sum tax (head tax)**.
 - ii. The geometry of lump sum taxes and other nearly neutral taxes can be illustrated a bit more easily using indifference curves and budget constraints, as is done in class and in the Appendix below.
 - ◆ (Systems of excise taxes that generate a proportionate increase in all prices faced by consumers can be illustrated with the tools developed above using demand and supply curves for several markets.)
- D.** The benefit normative theory of taxation argues that one cannot determine the proper division of the tax burden without knowing the distribution of benefits from the services that are provided.
- i. For example, Lindahl argues in favor of a **benefit tax**, that is a tax that imposes the greatest burden on those who receive the most valued services should pay the highest taxes.
 - ii. Under an ideal Lindahl tax system, each person's *marginal tax rate would be set equal to the marginal benefits* she or he receives from government services.
 - iii. James Buchanan (who won a Nobel Prize in economics while a professor at GMU, partly for his contributions to public finance) tends to agree with Lindahl.
 - a. Buchanan argues that proper accounts of tax burden--should focus on net tax burden--that is, they should take account of the services financed by taxes as well as the taxes paid.
 - b. For example, if a person receives an especially valuable service from the government, it is possible that his or her "true" net tax burden is negative. Others who receive no services of value, might have positive net tax burdens.
- c. Ideally, all citizens would bear "negative" tax burdens in the sense that each person should receive services that are considered to be more valuable than the taxes paid.
 - d. (We will take up the demand and supply of government services later in the course.)
- E.** Some economists argue that a tax system should be designed to maximize economic growth--or at least to minimize its impact on economic growth. Such economists often favor tax systems that include tax preferences for saving, income from capital, and promote capital formation.
- F.** Other normative principles of taxation come are rooted in social norms or political philosophies of various kinds--often dealing with **fairness** (or equity).
- i. For example, many argue that persons should pay based on their "ability to pay."
 - a. This notion of fairness tends to imply progressive income taxes.
 - b. For example, a "fair tax" might be one that caused all taxpayers should all sacrifice approximately the same "utility" (rather than net benefits) when they pay their taxes.
 - ◆ (Since the marginal utility of money tends to be smaller for rich persons than poor persons, more money would be collected from rich persons than from poor persons.)
 - ◆ (That is, taxes should be progressive rather than regressive)
 - ii. Others suggest that fairness requires all persons to pay be treated the same way under a tax system.
 - ◆ (This notion of fairness tends to imply a flat tax--a proportional tax on income.)
- G.** [Think about these alternative principles and try to identify ones that are more or less compatible with each other and ones that conflict with each other.]
- H. Useful Definitions for Thinking about Tax Schedules:**
- i. Equity based analysis of taxes often focus on the progressivity of a tax system. In those debates, progressivity is assessed relative to household wealth or income.
 - ii. However, progressivity or regressivity can be assessed relative to the tax system itself, which is the method used in this class.

- a. A **progressive tax** is a tax whose average burden increases as the taxable base owned by an individual increases. [Such taxes often have marginal tax rates that increase with the base (increase with income), although not all progressive taxes have this property. Most income tax systems in industrialized countries are somewhat progressive.]
- b. A **proportional tax** is a tax whose average tax burden does not change with income. (Such taxes normally have a constant marginal tax rate, as true of most sales taxes and some income taxes. A flat (proportional) tax on income has the form: $T = tY$.)
- c. A **regressive tax** is a tax whose average tax burden falls with income. Such taxes often have declining marginal tax rates with ownership of the taxable base, however, not all regressive taxes have this property. An example of a regressive tax in the US is the social security tax--which has a cap on taxable income.

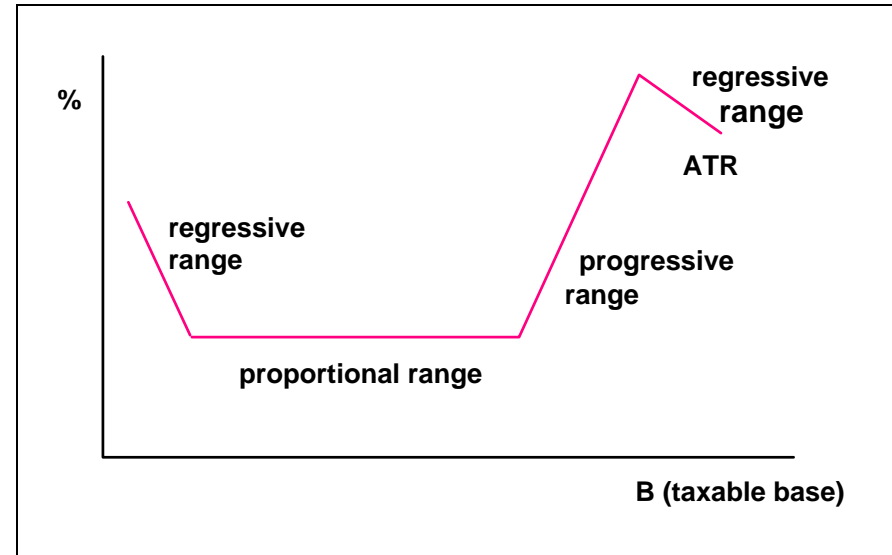
iii. **Definitions and Relationships:**

- a. The **tax base**, B , is that which is taxed. (taxable income, sales of final goods and services, profits, property, gasoline, etc.)
- b. The **average tax rate** of a particular tax often varies with an individual's holding of the taxable base. If an individual pays tax T_i on a holding of B_i , his average tax rate is B_i/T_i . (If $T_i = \$50$ and $B_i = 200$, the average tax rate for this tax is $50/200 = 0.25$ or 25%.)
- c. The **marginal tax rate** of a particular tax is the change in taxes owed for a one unit increase in holdings of the taxable base, $\Delta T/\Delta B$. (So, if a tax payer earning 50,000/year pays a tax of 10,000 and a taxpayer earning 50,001 pays a tax of 10,000.50, his or marginal tax rate is $0.50/1 = 50\%$. Fifty percent of each additional dollar earned is taken from the "last" dollar of income earned by a taxpayer earning 50,000/year.)

d. **Diagramming average and marginal tax schedules:**

- ◆ If MTR is above ATR, then that ATR curve will be rising (the marginal tax rate will be pulling the average up).
 - ◆ If MTR is below ATR, then the ATR curve will be falling (the marginal tax rate will be pulling the average down).
 - ◆ -If the $MTR = ATR$, the ATR will be neither rising nor falling.
- e. Since individual decisions are determined by marginal cost and marginal benefits at various quantities, **it is the marginal tax rate rather than the average tax that directly affects tax payer behavior** in most cases.

- ◆ (Thus, one argument in favor of proportional, or indeed, regressive taxes, is that they may have smaller effects on economic activities than a revenue equivalent progressive tax.)



- I. (Peckman's estimates of the effective average and marginal tax rates faced by a typical American tax payer, often look a bit like this odd tax schedule.)
 - i. (As an exercise try to determine what the marginal tax schedule that corresponds to this average tax schedule looks like.)
 - ii. (Explain briefly why Peckman finds regressive ranges of taxation at both the highest and lowest ranges of income.)
- J. Other normative tax theories are unconcerned with the fairness of the tax system and argue that a tax system should attempt to promote economic growth--or at least minimize the reduction in growth associated with raising a given amount of revenue.
 - i. Such persons often favor consumption taxes in order to encourage saving and investment.
 - ii. (The effects of a consumption tax on investment is easiest to illustrate with indifference curves and budget constraints, but the intuition behind the effect is simply based on supply and demand. If the price of saving falls relative to consumption, individuals will consume less and save more. And if savings increase, capital will be more rapidly accumulated, which leads to higher income levels and growth rates.)

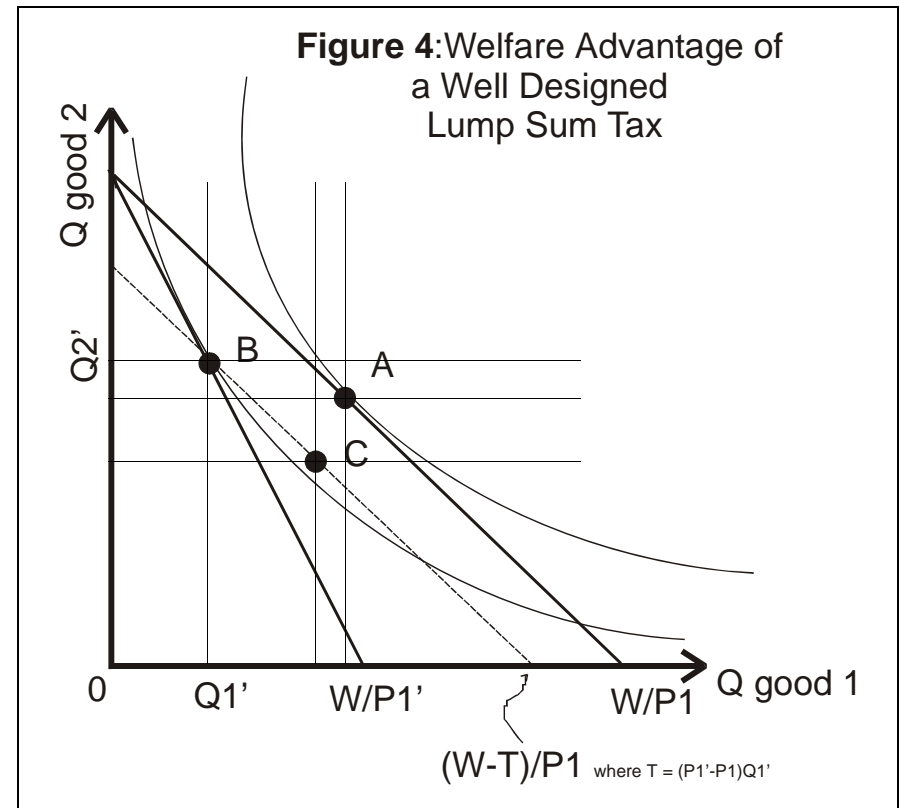
III. Analysis of Tax Neutrality Using Indifference Curves: The Case for Broad Based Taxation

A. The behavior affects of alternative tax systems can also be analyzed with indifference curves and budget constraints.

- i. The indifference curve approach is better able to characterize the problems associated with narrow tax bases and differential taxation of goods.
- ii. It can also illustrate the effects of pure wealth taxes, as with a lump sum tax, more directly than can the net-benefit maximizing models, because consumer wealth is only indirectly represented in MB curves.
- iii. (Wealth affects the height of a person's MB curve by affecting his or her willingness to pay for goods and services.

B. **ILLUSTRATION.** Suppose that there are two goods, Q and X, both of which the consumer normally uses.

- i. Indifference curves represent combinations of goods for which a person is indifferent.
- ii. They are analogous to the contour lines of a topographical map. But, rather than height above sea level, they represent utility levels.
- iii. Consumers want to reach the highest indifference curve possible, given their opportunity (budget) set.
 - i.
- iv. Our previous analysis implies that the effect of an excise tax on a typical consumer is to raise the price of the taxed good from P^* to P_c .
- v. This increase in price affects the location of each consumer's budget set.
- vi. It rotates the budget constraint from the untaxed end of the budget constraint and generates a new budget constraint that lies inside the original one at all points where the consumer purchases positive quantities of the taxed good.
- vii. Suppose that "A" is the original (pre tax) bundle consumed by this consumer.
 - a. In the figure below, the tax has increased price of good 1 from P_1 to P_1' (this price effect is taken from a supply and demand diagram)
 - ◆ In the case drawn, the new higher price induces the consumer to purchase bundle B instead of A.
 - ◆ (Indeed, A is no longer feasible.)



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- b. It turns out that replacing an excise tax with a broad based or lump sum tax that produces exactly the same revenue from this individual would make him or her better off!
- c. If instead of an excise tax on good 1 a lump sum tax (or wealth or sales tax) had been used, the budget constraint would have shifted toward the axis, but the new budget constraint would have the same slope as the original one.
 - ◆ The "revenue equivalent" lump sum tax passes through point B and is parallel to the original (pre tax) budget constraint.
- d. Note that a lump sum tax, would have allowed the individual to purchase a bundle like C which is on a higher indifference curve (not drawn) than bundle B.
 - ◆ This loss in utility (from being on a lower indifference curve) is another measure of the excess burden of a non-neutral tax on consumers.

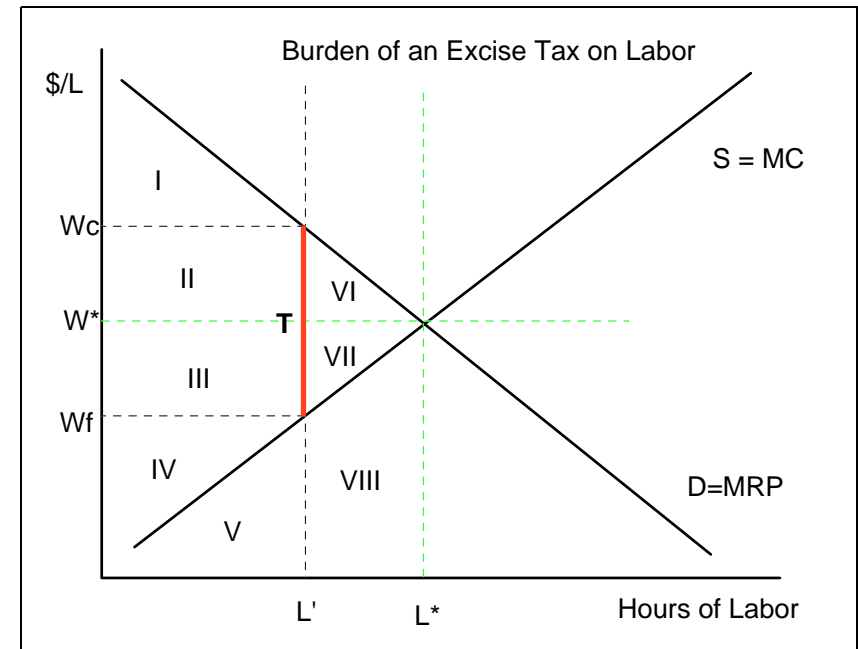
- e. Much of the deadweight loss is a consequence of reduction in purchases of the taxed good, particularly that part which was generated by the "relative price" effect of the excise tax.
- f. You learned in micro economics that every price increase has both a (relative price) substitution effect and a wealth effect on purchases of the good whose price has increased.
 - ◆ An excise tax that affects consumer prices has both a (relative price) substitution effect and an income effect on purchases of the good whose price has increased because of a tax.
 - ◆ A lump sum tax only has an income effect.

C. The burden (reduced utility) generated by a broad based tax or a lump sum tax tends to be smaller than that of an excise tax, because these taxes have only a wealth effect.

- i. A revenue neutral lump sum tax, a (neutral) general sales tax, and an income tax all shift each consumer's budget constraint towards the origin, but **these taxes do not affect the slope** of the consumer's budget constraint.
- ii. Consequently, general taxes and lump sum taxes tend to have a smaller effect on behavior than excise taxes that raise the same amount of revenue. (There is no "substitution effect.")

IV. Application: The Taxation of Work and Labor Income

- A.** Excise taxes on alcohol and imports were once the largest revenue source for the central government of the United. After the 16th amendment was passed in 1913, income taxes of various kinds gradually became the most important tax revenue source for both then national and state governments.
- B.** All of the tools that we've developed to look at excise taxes can also be applied to income tax, if we choose the correct market (eg. label the axes correctly).
- C.** For example, one can model the effect of a tax on labor income as an excise on the market for labor.



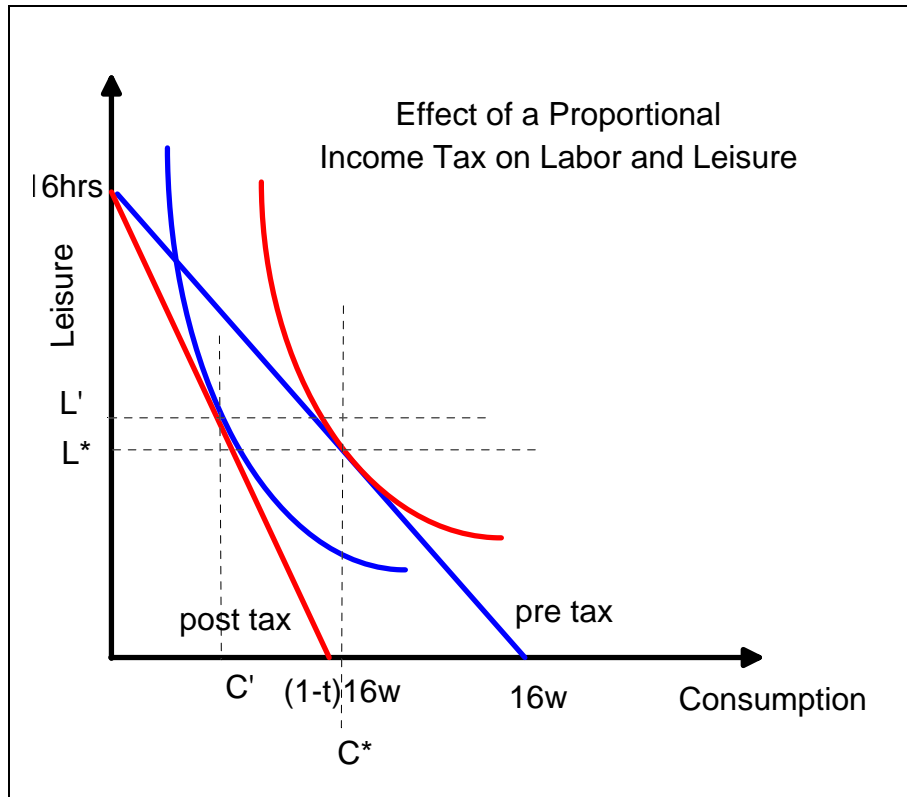
- i. Note that this diagram looks the same as the first one in this handout except for the axis labels.
 - ii. There are, however some slight differences in interpretation.
 - iii. The tax burden areas are all the same. Changes in the net benefits of those purchasing labor are still II + VI; Changes in the net benefits of those selling labor are still III + VII; the tax revenue raised is still II+III, and so the deadweight loss is still VI + VII.
 - iv. However, the buyers are now firms rather than "ordinary persons," and the supplier are "ordinary people" rather than firms.
 - v. So the net benefits that the buyers of labor get should probably be called profits, instead of consumer surplus (area I after the tax), and the area that represent the net benefits of labor (area IV after the tax) could be thought of as labor net benefits or possibly as consumer surplus.
- D.** One can also use indifference curves and budget constrains to look at the effects of an income tax on patterns of consumption or labor leisure choices.
- i. With respect to consumption choices, an income tax simply reduces the "W" at the ends of the budget constraints by the same proportion,

shifting the original line back to the left in a manner parallel to the original (pre tax) budget line.

- ii. Thus an income tax is neutral with respect to consumption choices.
- iii. However, and income taxes (and many other taxes) is not neutral with respect to labor leisure choices, as indicated by the above supply and demand analysis of the effect of labor taxes. It tends to discourage labor and increase leisure.

E. To analyze the effect of an income tax on hours worked with indifference curves requires some fresh thinking about how to label the axes. The best labels for this purpose are hours of leisure and income (or consumption of other goods in dollars).

- i. In this case the end points of the opportunity set (budget line) are hours available for leisure on the vertical axis and hours available for work times wage rates (dollars of potential consumption) on the horizontal axis.



- ii. (I've assumed 8 hours of sleep and eating time is necessary each day, which leaves 16 for leisure or work.)
- iii. Notice that an income tax changes the slope of the budget line in this case because it reduces one's after tax wage from w to $(1-t)w$. The slope of the post tax budget line (in red) is steeper than the pre tax budget line (in blue). After the tax is imposed one has to give up more leisure to obtain a given income or consumption level.
- iv. Because of this effect the income tax is not neutral with respect to leisure (which is hard to tax at the same rate [why?]).
- v. The person modeled cuts back on consumption, but slightly expands hours of leisure. This effect is consistent with the demand and supply analysis.
- vi. [In such diagrams, it is quite possible that the person will choose to work longer hours, reducing his or her leisure as well as her consumption. Draw such a diagram.]
- vii. Note that the logic of lump sum taxes would apply in this case as well, but the tax would have to be on available hours (reducing available time from 16 to 14 hours, for example) rather than on income alone. Simply reducing income by amount T , would also change the slope of the line.
- viii. Progressivity of an income tax in this type of diagram shows up as changes in the slope of the after tax budget line. The budget line may become a curve (or series of line segments) that becomes steeper and steeper as the tax becomes more and more progressive.

V. APPENDIX: Some useful algebra for describing the effects of taxes on budget constraints

- i. The slope of the budget lines can be calculated *for the lump sum, sales and income taxes.*
 - a. A budget line has the form $W = P_1Q_1 + P_2Q_2$
 - b. The place where the budget line intersects the Q_1 axis is that where $W = P_1Q_1$ or where $Q_1 = W/P_1$
 - c. The place where the budget line intersects the Q_2 axis is that where $W = P_2Q_2$ or where $Q_2 = W/P_2$
 - d. Recall that slope is "rise over run."
 - e. In the case without taxes, the slope of the budget line is $-(W/P_1) / (W/P_2)$, which simplifies to $-P_2/P_1$.

- f. One can also find the slope using a bit of algebra. Write the budget line as Q_2 as a function of Q_1 and W .
- ◆ One algebraic route is to subtract P_1Q_1 from both sides to obtain: $P_2Q_2 = W - P_1Q_1$,
 - ◆ or $Q_2 = W/P_2 - (P_1/P_2)Q_1$
 - ◆ Note that the budget line is now in the form $Y = C + BX$, where B is the slope of the line, here $-(P_1/P_2)$.
- ii. Applications of slope to assess tax neutrality.
- a. In the case of a lump sum tax, the endpoints of the new budget line become $(W-T)/P_2$ and $(W-T)/P_1$.
- ◆ The slope of the new budget constraint is $-[(W-T)/P_1]/[(W-T)/P_2]$ which equals $-P_2/P_1$.
- b. In the case of a proportional income tax, where W is treated as income, the after tax income is $(1-t)W$, so the endpoints of the new budget line are $((1-t)W)/P_2$ and $((1-t)W)/P_1$.
- ◆ The slope of the new budget line is: $-[(1-t)W)/P_1]/[((1-t)W)/P_2] = -P_2/P_1$.
 - ◆ A broad based income tax (e.g. one without deductions) is a neutral tax.
- c. Similarly, in the case of a general sales tax the new after tax prices will be approximately $(1+t)P_1$ and $(1+t)P_2$. (What assumptions about supply and demand are sufficient for this to be exactly true?)
- ◆ The slope of the new budget line will be $-[(W)/(1+t)P_1]/[((1-t)W)/(1+t)P_2]$, which again can be shown to equal $-P_2/P_1$.
 - ◆ So a broad based sales tax is neutral.
- d. All three of these taxes are "neutral" with respect to the choice illustrated in our diagram.
- ◆ None of these taxes change the relative prices of goods 1 and 2. It remains $-P_2/P_1$ in each case.
 - ◆ (Note however that sales taxes have an effect on a consumer's decision to save, and income taxes have an effect on a consumer's decision to work.)
- iii. In some cases that we will examine later in the course (Pigovian taxes), the purposes of a tax may be to change behavior.
- ◆ In such cases, excise taxes and other "marginal" taxes will be more effective at altering behavior than lump sum or general taxes.
 - ◆ We will discuss the use of (Pigovian) taxes to "internalize" externalities after the midterm exam.
- iv. **The algebra that shows how to calculate a lump sum tax that generates the same revenue as an excise (narrow base) tax.**
- a. The pretax budget line is $W = P_1Q_1 + P_2Q_2$
- ◆ The slope of this budget line is $-P_1/P_2$
 - ◆ See above
- b. Suppose that a tax on good 1 raises its price to $(1+t)P_1 > P_1$
- c. The new budget line is: $W = (1+t)P_1Q_1 + P_2Q_2$
- ◆ The slope of this budget line is $-(1+t)P_1/P_2$
 - ◆ See above
- d. Suppose that (Q_1', Q_2') is the bundle chosen after the tax is imposed.
- e. The amount collected from the individual from the tax is tP_1Q_1' .
- f. The equivalent lump sum tax is thus $T = tP_1Q_1'$
- g. Under that tax, the taxpayer's budget line would be:
- $$W - tP_1Q_1' = P_1Q_1 + P_2Q_2$$
- ◆ The slope of this budget line is $-P_1/P_2$
 - ◆ See above
- h. Intuitively, this line should go through the point (Q_1', Q_2') because the consumer should still be able to afford that bundle. Let's see if we can prove that it does.
- i. Note that with the original post tax budget line,
- $$W = (1+t)P_1Q_1' + P_2Q_2' \text{ (the bundle bought was feasible)}$$
- j. This can be rewritten as: $W = P_1Q_1' + (t)P_1Q_1' + P_2Q_2'$
- k. Subtracting yields: $W - tP_1Q_1' = P_1Q_1' + P_2Q_2'$
- l. Notice that this expression simply describes a point on the lump sum budget constraint, under "g".
- m. That is to say this particular point is common to both budget lines.
- n. Thus the revenue-equivalent lump sum tax creates a new budget constraint that is parallel to the original budget line and passes through the bundle chosen under the excise tax.
- o. QED